Vedic Mathematics

Lecture Notes – 2

Division

By
Prof. C. Santhamma


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References
Vedic Mathematics

DIVISION

Chapter I

I. By Nikhilam Rule:

(a) Special cases of dividing with 9, 8, 7 and 6 are dealt with here

Special Case 1: (Divisor is 9)

Vedic Method Steps are as follows

1) Partition the given number (dividend) into two parts. The second part is to be provided one 
digit place. This represents the remainder part whereas the first part gives the quotient. The 
first part may contain more than one digit. 

\[
\begin{align*}
\text{First} \quad & \quad \text{Second} \\
\text{(Quotient)} \quad & \quad \text{(Remainder)}
\end{align*}
\]

2) The second step is to put down the first digit in the first part as it is as a part of the answer

3) Then it is carried out to the next digit lying either in the quotient part or the remainder part 
as the case may be. After this carrying out, an addition takes place The process is 
continued till the addition finally takes place in the remainder column

Examples clearly show the above method.

I) Remainder is less than the divisor, 9.

**Examples:**

i) \[32 \div 9\]

**Current Method**

\[
\begin{array}{c}
9) 32 \\
- 27 \\
\hline
5
\end{array}
\]

**Vedic Method**

\[
\begin{array}{c}
9) 3 / 2 \\
\hline
\hline
3 / 5
\end{array}
\]

Quotient = 3
Remainder = 5
Example:

(i) \[ 27 \div 9 \]

**Current Method**

\[
\begin{array}{c}
9) 27 (3 \\
\underline{27} \\
0
\end{array}
\]

Quotient = 3
Remainder = 0

**Vedic Method**

\[
\begin{array}{c}
9) 2 \div 7 \\
\quad 1 \div 2 \\
\quad 2 \div 9 \\
\quad \underline{1 \div 0}
\end{array}
\]

(Vilokanam)

Quotient = \[2 + 1 = 3\]
Remainder = 0

1) In case the remainder is more than the divisor and has two or more digits, then it has to be treated as new dividend

2) The first process of partitioning the new dividend into the quotient and remainder parts is continued which is followed by division until finally the remainder comes out as a value less than the divisor (Refer example ii below)

3) All the additional quotients thus obtained in series are to be added to the original quotient (Refer example iii page 4)

4) If we get two digits as a single unit in the answer, then the first digit is added to the previous one. This is clearly shown in examples below (Whenever two or more than two digits are obtained as a single unit in Vedic Mathematical operations, retaining only the last digit, all the remaining digits are transferred to the immediate left hand position by addition) Refer example iii page 4.

(ii) \[ 368 \div 9 \]

**Current Method**

\[
\begin{array}{c}
9) 368 (40 \\
\underline{360} \\
8
\end{array}
\]

**Vedic Method**

\[
\begin{array}{c}
9) 36 \div 8 \\
\quad 3 \div 9 \\
\quad 39 \div 1 \div 7 \\
\quad \underline{1 \div 8}
\end{array}
\]

Quotient = \[39 + 1 = 40\], Remainder = 8
Vedic Mathematics

(iii) $40357 + 9$

<table>
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<th>Vedic Method</th>
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</thead>
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<tr>
<td>9) 40357 (4484</td>
<td>9) 4 0 3 5 / 7</td>
</tr>
<tr>
<td>36</td>
<td>4 4 7 / 1 2</td>
</tr>
<tr>
<td>43</td>
<td>* 4 4 7 12 / 9</td>
</tr>
<tr>
<td>36</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Quotient = 4482 + 1 + 1 = 4484, Remainder = 1

* If in the quotient one gets more than one digit then one has to carry to the previous digit all the digits excepting the last digit.

i.e., 4 4 7 12 = 4482

Special Case 2: (Divisor is 8)

(1) First two steps: (a) concerned with the partition of the dividend and (b) for obtaining the first digit in the quotient are common as for the divisor 9.

(2) The third step is to carry out twice the first quotient digit to the next digit either in the quotient place or the remainder place as the case may be.

(3) Then the process is continued as explained in the first case (divisor 9). Examples are given below

(i) $31 + 8$

<table>
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<th>Vedic Method</th>
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</thead>
<tbody>
<tr>
<td>8) 31 (3</td>
<td>8) 3 / 1</td>
</tr>
<tr>
<td>24</td>
<td>/ 6</td>
</tr>
<tr>
<td>7</td>
<td>3 / 7</td>
</tr>
</tbody>
</table>

Quotient = 3
Remainder = 7
Vedic Mathematics

(ii) \( 42567 \div 8 \)

**Current Method**

\[\begin{array}{c}
8) 42567 (5320 \\
40 \\
25 \\
24 \\
16 \\
16 \\
07
\end{array}\]

Quotient = 5320
Remainder = 7

**Vedic Method**

\[\begin{array}{c}
8) 4 2 5 6 / 7 \\
\underline{4 2 0 0} / 11 2 \\
\underline{4 0 0} / 56 / 11 / 9 \\
\underline{1 0 0} / 56 = 5306 \\
\underline{4 0 0} / 13 / 1 / 5 \\
\underline{2 0 0} / 1 / 2 \\
\underline{1 0 0} / 7
\end{array}\]

Quotient = 5306 + 13 + 1 = 5320
Remainder = 7

**Special Case 3: (Divisor is 7)**

(1) Also in the case of divisor 7, the first two steps (a) concerned with the partition and (b) for obtaining the first digit in the quotient are common as in the case of divisor 9.

(2) In the third step the first quotient is multiplied by 3. Then the process of carrying over the result of multiplication to the quotient part / reminder part is continued.

(3) Then the process is continued as given in the first case (divisor 9). This is clearly shown in examples below.

**Examples:**

(i) \( 29 \div 7 \)

**Current Method**

\[\begin{array}{c}
7) 29 (4 \\
20 \\
1
\end{array}\]

Quotient = 4
Remainder = 1

**Vedic Method**

\[\begin{array}{c}
7) 2 / 9 \\
\underline{2 / 6} \\
\underline{2 / 1 / 5} \\
\underline{2 / 3} \\
\underline{1 / 8} \\
\underline{1 / 1} (Vilokanam)
\end{array}\]

Quotient = 2 + 1 + 1 = 4
Remainder = 1

* Simplification \[\begin{array}{c}
4 \underline{10} \underline{25} 56 \\
5 2 10 6 \\
5 3 0 6
\end{array}\]
(ii) 31589 + 7

**Current Method**

\[
\begin{array}{c}
7) 31589 (4512 \\
28 \\
35 \\
35 \\
08 \\
7 \\
19 \\
14 \\
5 \\
\end{array}
\]

Quotient = 4512
Remainder = 5

**Vedic Method**

\[
\begin{array}{c|c|c|c|c|c|c}
7) & 3 & 1 & 5 & 8/ & 9 \\
\hline
9 & 30 & 105/ & 3 & 3 & 9 \\
3 & 10 & 35 & 113/ & 3 & 4/ & 8 \\
\hline
9/3 & 2 \\
\hline
3 & 13 & 4/ & 7 \\
\hline
4/1 & 2 \\
\hline
4/1 & 3 \\
\hline
1/1 & 3 \\
1/5 & 3 \\
\hline
\end{array}
\]

Quotient = 4463 + 43 + 4 + 1 + 1 = 4512
Remainder = 5

Special case 4: (Divisor is 6)

1. Also in the case of divisor 6, the first two steps are same as in the case of divisor 9
2. But the corresponding multiplier in the third step is 4
3. This is again followed by the same procedure as in the case of the divisor 9. The following examples are self-explanatory

**Examples:**

(i) 47 + 6

**Current Method**

\[
\begin{array}{c}
6) 47 (7 \\
42 \\
5 \\
\end{array}
\]

Quotient = 7
Remainder = 5

**Vedic Method**

\[
\begin{array}{c|c|c|c|c|c|c}
6) & 4/ \\
\hline
1 \\
\hline
4/2/ \\
\hline
2/1/1 \\
\hline
4/ \\
\hline
1/5 \\
\hline
\end{array}
\]

Quotient = 4 + 2 + 1 = 7
Remainder = 5
Vedic Mathematics

Current Method

6) 4392 (732

42
19
18
12
12
0

Quotient = 732
Remainder = 0

Vedic Method

6) 4 3 9 /
16 76/ 3 4 0
4 19 85 / 3 4 /
12 6 1 2
3 16 / 6 3 /
6 3 /
1 0

Quotient = 675 + 46 + 6 + 3 + 1 + 1 = 732
Remainder = 0

The proofs for these divisions are worked out by following the polynomial form in \( x \) (\( x = 10 \))

(A) \( (x - a) ) bx^3 + cx^2 + dx + e = (bx^3 + x(c + ab) + d + a(c + ab) \)

\[
\begin{align*}
&= bx^3 - abx^2 \\
&= (c + ab)x^2 + dx \\
&= (c + ab)x^2 - ax^2(c + ab) \\
&= dx + ax(c + ab) + e \\
&= dx - ad \\
&= ax(c + ab) + e + ad \\
&= ax(c + ab) - a^2(c + ab) \\
&= e + ad + a^2(c + ab) \\
&= e + a(d + a(c + ab))
\end{align*}
\]

(B) \( (x - a) ) bx^2 + cx + d = (bx + (c + ab) \)

\[
\begin{align*}
&= bx^2 - abx \\
&= (c + ab)x + d \\
&= (c + ab)x - (c + ab)a \\
&= d + (c + ab)a
\end{align*}
\]

(C) \( x-a) ) bx+c(l \)

\[
\begin{align*}
&= bx-ab \\
&= c+ab
\end{align*}
\]

Depending on the remainder further division takes place.

Proof:

where \( x \) is base, i.e., 10

In case of 9, 'a' becomes 1, i.e., the value obtained on application of Nikhilam Sutram to 9

(*) refer to Lecture notes I Vedic Mathematics on Multiplication.

In case of 8, 'a' becomes 2, i.e., the value obtained on application of Nikhilam Sutram to 8

In case of 7, 'a' becomes 3, i.e., the value obtained on application of Nikhilam Sutram to 7

In case of 6, 'a' becomes 4, i.e., the value obtained on application of Nikhilam Sutram to 6
Vedic Mathematics

Division

So we are multiplying in the third step the first quotient digit by 1, 2, 3 and 4 respectively
Considering example (ii) in the special case 4 when divisor is 6 (page No 7).
Applying equation (A) \( a = 4, b = 4, c = 3, d = 9, \ e = 2 \)
The quotient is 675
The remainder is 342
This is \( 3x^2 + 4x + 2 \), and Applying equation B. \( b = 3, c = 4, d = 2 \)
Quotient is 46 remainder is 66.
This is again written as \( 6x + 6 \) and applying equation (C).
Dividing by \( x - a \), we get 30 as the remainder and 6 as the quotient

The remainder is \( 3x + 0 \)
When divided by \( x - a \) The quotient is 3 remainder is \( 3a = 12 = x + 2 \) Applying equation (C)

When divided by \( x - a \) the quotient is 1 and the remainder is 6
When 6 is divided by 6 the quotient is 1 and the remainder is zero

\[ \therefore 675 + 46 + 6 + 3 + 1 + 1 \text{ is explained} \]

(b) General Method of division by applying Nikhilam Rule:

Step 1:
First partition the dividend into two parts from right end such that the remainder part consists of as many digits as the divisor has.

Step 2:
Apply Nikhilam Sutram to the divisor to get the new divisor. Division is now carried out by the new divisor value so obtained After this, the procedure is as follows.
This is shown by a specific example.

Consider one example: \( 223 + 78 \).

Partition 223 as 2/23 (Divisor has two digits)
The value obtained by applying the Nikhilam Sutram to the divisor 78 is 22, which is the new divisor in operation i.e., we are dividing by a lesser number.

1) \( 223 + 78 \)

Current Method

\[
\begin{array}{c}
78) 223 (2 \\
\quad 156 \\
\quad \downarrow \\
\quad 67 \\
\end{array}
\]

Quotient = 2
Remainder = 67

Vedic Method

\[
\begin{array}{c}
\text{First Part (Quotient)} \\
\text{Original} \rightarrow 78 \rightarrow 2/23 \rightarrow \text{Second Part (Remainder)} \\
\text{New} \rightarrow 22 \rightarrow 44 \\
\text{Divisor} \rightarrow 2/67 \rightarrow \text{Answer} \\
\end{array}
\]

Quotient = 2
Remainder = 67
Vedic Mathematics

Step 3: Bring down the first digit of the first part (quotient part) of the dividend as it is, to the answer.

Step 4: Then multiply this digit with the new divisor, digit by digit and put down the result from the next digit onwards and below the dividend (it may enter into the remainder part).

Step 5: Then addition is performed between this Multiplication result and the corresponding dividend as shown in the example.

Step 6: If the result of this addition is to be placed in the quotient, then we have to repeat the process of multiplication of that value (pertaining to the quotient part) with the new divisor.

Step 7: Placement of this result followed by addition is similar to the one already explained.

Step 8: If in the partition, the quotient part of the dividend consists as more than one digit (eg. 2 onwards), then all the digits are to be first exhausted.

Step 9: The multiplication with the new divisor stops with the last quotient digit of the answer.

Step 10: If the remainder is more than the original divisor (eg. 4), then a fresh division is carried out with this remainder as the new dividend. This process is continued until the remainder is less than the original divisor.

Step 11: While in addition more than one digit is obtained as a single unit (eg. 2 and 4), then the usual carrying over of all digits (excepting the right hand most) to the immediate previous digit(s) is applied to obtain the answer.

Examples are given below

2) 31242 ÷ 898

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>898) 31242 (34</td>
<td>898) 31 / 2 4 2</td>
</tr>
<tr>
<td>2694</td>
<td>102 3 / 0 6</td>
</tr>
<tr>
<td>4302</td>
<td>/ 4 0 8</td>
</tr>
<tr>
<td>3592</td>
<td>34 / 6 10 10</td>
</tr>
<tr>
<td>710</td>
<td>34 / 7 1 0</td>
</tr>
</tbody>
</table>

Quotient = 34
Remainder = 710
Vedie Mathematics

- 1203423 + 98789

Current Method

98789) 1203423 (12
98789
215533
197578
17955

Quotient =12
Remainder = 17955

Vedic Method

98789) 12 / 0 3 4 2 3
01211
0 / 1 2 1 1
/ 0 2 4 2 2
12 / 1 7 9 5 5

Quotient =12
Remainder = 17955

- 45679 + 99

Current Method

99) 45679 (461
396
607
594
139
92
40

Quotient = 461
Remainder = 40

Vedic Method

99) 4 5 6 7 / 7 9
01 0 4 /
0 / 5
/ 0 10
4 5 10 / 12 19
4 6 0 / 1 3 9
/ / 0 1

Quotient = 460+1 = 461
Remainder = 40

(1) If the value obtained by applying Nikhilam Sutram to the given divisor is greater than the divisor (eg. 5), one should first go in for a computed divisor which can be a multiple or sub multiple of the original divisor.

(2) From this a new divisor, (less than the original divisor) is arrived by applying Nikhilam Sutram to the computed divisor.

(3) The division is carried out with the new divisor until one gets a remainder which is less than the computed divisor.

(4) At this end one has to multiply only the quotient by ratio of computed divisor to the original divisor to bring the result equivalent to working with original divisor.

(5) If the remainder is greater than the original given divisor, this has to be divided again by the original divisor to get the final result.

(6) The quotient values so obtained are to be added to the previous quotient.

(7) It can also be achieved by subtracting n times (n is positive integer) the original divisor from the remainder, so that the result of subtraction gives a (positive) value less than the divisor. In such a case, to get the final quotient one has to add the value n to the quotient obtained so far.
Vedic Mathematics

5) 11121 + 21

Current Method

21) 11121 (529

105

62

42

201

189

12

Quotient = 529
Remainder = 12

* Nikhilam is applied to the computed
Divisor and the result is used as new
divisor. The division is continued until the
remainder is found to be less than the
computed divisor at which stage, the
corresponding quotient is to be multiplied
or divided by the number which is used as
multiple or sub-multiple to get the
computed value. If thus obtained
remained is greater than the original
divisor, then one has to continue the
division with original divisor which gives
the corresponding quotient and the final
remainder. * The quotient so obtained is
added to the other quotient part, resulting
the final quotient.

Vedic Method

New Divisor applying Nikhilam Sutram to
21 is 79. 79 > 21. Hence, we consider
multiple of 21.
21 x 4 = 84 (computed divisor)
New divisor by applying Nikhilam Sutram
to the computed divisor 84 is 16 which is
less than the original divisor 21
4 x 21 = 84) 111 / 2 1

16 16 /

2 / 12

/ 9

129 / 23 5

129 / 2 / 8 5

/ 2 12

129 / 2 / 10 17

129 2 1 / 1 7

/ / 1 6

131 1 / 2 13

132 / 3 3

x 4

* 528 33 (Vilokanam)

529 12

Quotient = 529
Remainder = 12

Original *
Division (1 x 21
12 Remainder
Chapter – II

Straight Division:

(a) Application of Urdhva Tiryak Sutram for numbers and also by Vinculum method:

Vedic Method of straight Division:

The following steps are to be considered.

1. Partition of the divisor:

Partition the divisor into two parts, such that one part is called Dhwajanka (flag), which takes place in the multiplication in the problem, and the other part, representing as part divisor is active in dividing the dividend. The part divisor can have one digit, two digits, three digits, four digits, etc., so also the Dhwajanka can have one or more digits. The partition of the divisor is such that the division and multiplication can be carried out with ease as much as possible. However, a general method is also workable.

2. In case of single digit divisor, in order to apply this method, one has to convert it necessarily into vinculum to enable the partition into Dhwajanka and part divisor.

3. Relation between Dividend partition and Divisor partition:

Partition the given dividend into two parts. The left most part is the quotient region and the other is the remainder region. The remainder region should have number of digits equal to the Dhwajanka concerned with the divisor. Keeping this in view the partition is drawn by counting the digits from the right extreme, towards left which defines the remainder region. This is diagrammatically represented as follows.

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dhwajanka</td>
<td>First Part (Quotient region) : Second Part (Remainder Region)</td>
</tr>
<tr>
<td>(Flag D)</td>
<td></td>
</tr>
</tbody>
</table>

Part Divisor (PD): Working Details : Working Details

Quotient : Remainder (answer line)

‘:’ represents partition in the dividend

4. In the partition it is to be noticed that the position of the partition represents invariably the decimal point. The examples clearly show the types of partition of the Dividend consequent on the partition of the divisor. (In doing so an important point is to be taken into consideration). For example:

(I) When the number of digits in the divisor is equal or less than that in the dividend, the problem is simpler (when there are no decimals in the divisor and dividend) in partitioning the dividend, following the usual rules of the partition.

Some examples are given below for the partition of the divisors and dividends.
Vedic Mathematics

Division

Eg. (1) 236 + 78

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>78</td>
<td>236</td>
</tr>
</tbody>
</table>

**Quotient Region**

(FlagD)
i) Dhrajanka
ii) Part Divisor (PD)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Remainder

Partition

Eg. (2) 3689 + 123

i) One way of representation

<table>
<thead>
<tr>
<th>23</th>
<th>3 6 : 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

ii) Another way of representation

<table>
<thead>
<tr>
<th>3</th>
<th>3 6 8 : 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

(3) 98645 + 34567

Number of ways of representations

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>34567</td>
<td>98645</td>
</tr>
</tbody>
</table>

(i) 4567 | 9 : 8 6 4 5 |
| 3       |            |

(ii) 567  | 9 8 : 6 4 5 |
| 34      |            |

(iii) 67  | 9 8 6 : 4 5 |
| 345     |            |
(iv) \[
\begin{array}{c|cccc}
 & 7 & 9 & 8 & 6 & 4 & 5 \\
\hline
1456 & & & & & & \\
\end{array}
\]

(2) When the number of digits in the Dhwajanka is greater than that of the dividend showing a deficiency, the partition takes care of this deficiency by starting the quotient with decimal point followed by zeroes equivalent to the deficiency. A few examples of such partitions are shown.

For example:

Eg(i): \(789 + 23451\)

\[
\begin{array}{c|c}
3451 & 789 \\
2 & \\
\end{array}
\]

\[.0. \text{Quotient Digits} \]

\[Q_1\]

As there are four digits in the Dhwajanka and three digits in the Dividend, one zero is to be placed after the decimal point in the quotient is (after the partition of the divided).

Eg.(ii): \(89 + 23451\)

\[
\begin{array}{c|c}
3451 & 89 \\
2 & \\
\end{array}
\]

\[.00 \text{Quotient Digits} \]

\[Q_1Q_2\]

As there are four digits in the Dhwajanka and two digits in the Dividend, two zeroes are to be placed after the decimal point in the quotient. i.e (after the partition of the divided).

Eg.(iii): \(9 + 23451\)

\[
\begin{array}{c|c}
3451 & 9 \\
2 & \\
\end{array}
\]

\[.000... \text{Quotient Digits} \]

\[Q_1Q_2Q_3\]

As there are four digits in the Dhwajanka and one digit in the Dividend, three zeroes are to be placed after the decimal point in the quotient. (after the partition of the divided).

If an intrinsic decimal point is present in the dividend or divisor or both, then the following rules for partition are to be considered

(3) When dividend alone has intrinsic decimal, the partition of the dividend should be counted from its decimal point to the left side so that the number of digits is same as that in the Dhwajanka. The decimal in the quotient starts from the partition.

For example.

Eg(i): \(782.693 + 425\)

\[
\begin{array}{c}
\text{Dhwajanka} \\
\text{Part Divisor}
\end{array}
\]

\[
\begin{array}{c|c}
25 & 7:82,693 \\
4 & \\
\end{array}
\]

\[\text{Decimal starting in the Quotient}\]
Eg(iii): 2.693 + 425

\[
\begin{array}{c|c}
25 & 2.693 \\
4 & 0.0 \quad \text{Quotient Digits} \\
\end{array}
\]

Dividend has only one digit on the left of decimal, one zero has to be included after the decimal in the answer, as the Dhwajanka has two digits. (Refer example. page No. )

Eg(iv): 0.2693 + 425

\[
\begin{array}{c|c}
25 & 0.2693 \\
4 & 0.00 \quad \text{Quotient Digits} \\
\end{array}
\]

Two zeros are to be placed on to the right of the decimal in the quotient digits, as the Dhwajanka has two digits.

Eg(v): 0.2693 + 425321

\[
\begin{array}{c|c}
25321 & 0.2693 \\
4 & 0.0000 \quad \text{Quotient Digits} \\
\end{array}
\]

Five zeros are to be placed after the decimal of the quotient digits, as the Dhwajanka has five digits.

(4) If in the problem, the divisor only has intrinsic decimal, the partition of the dividend is carried out in the usual way but by not considering the decimal point in the divisor, in the first instance i.e., taking the divisor as a whole, then partition the divisor. Now the partition in the dividend is according to the general rule. At the end in the result, the decimal in the quotient is shifted towards the right side of the quotient by the number of digits after the decimal in the divisor. This is shown clearly in the worked out examples. (Refer working details of 15628 + 23.4 in example 16 page )

Eg(i): 9856 + 12.3

\[
\begin{array}{c|c}
3 & 9856 \\
12 & 6 \\
\end{array}
\]

or

\[
\begin{array}{c|c}
23 & 9856 \\
1 & 56 \\
\end{array}
\]

Decimal is to be shifted to the right by one digit in the quotient to get the final result

Eg. (ii) 1757 + 523.7

\[
\begin{array}{c|c}
37 & 1757 \\
52 & 57 \\
\end{array}
\]

or

\[
\begin{array}{c|c}
237 & 1757 \\
5 & 57 \\
\end{array}
\]
Vedic Mathematics

Eg. (iii) 98476 ÷ 0.423

\[
\begin{array}{c|c|c}
3 & 9.847 & 6 \\
42 & : & : \\
\end{array}
\]

or

\[
\begin{array}{c|c|c}
23 & 9.84 & 7.6 \\
14 & : & : \\
\end{array}
\]

Decimal is to be shifted to the right by 3 digits to get the final result.

Eg. (iv) 17574 ÷ 0.0012

\[
\begin{array}{c|c|c}
2 & 1757 & 4 \\
1 & : & : \\
\end{array}
\]

Decimal is to be shifted to the right by 4 digits in the quotient to get the final result.

(5) When both the dividend and the divisor have intrinsic decimal, consideration of the divisor as a whole and followed by its partition helps in partitioning the dividend as per clause (3). In the final result one has to take cognisance of shifting of the decimal appropriately as given in clause (4) (Refer working details of 134.289 + 2 76 and 2 1387 ÷ 0 312 in examples 17, 18 in page No. 16)

Eg. (i) 374.8 ÷ 98.2

\[
\begin{array}{c|c|c|c}
82 & 3 & 7 & 4.8 \\
9 & : & : & : \\
\end{array}
\]

In the final answer the decimal has to be shifted to the right by one digit.

Eg. (ii) 8972.2 ÷ 12.34

\[
\begin{array}{c|c|c|c}
34 & 8 & 9 & 72.2 \\
12 & : & : & : \\
\end{array}
\]

In the final answer the decimal has to be shifted to the right by two digits.

Eg. (iii) 0.8972 ÷ 1.34

\[
\begin{array}{c|c|c|c|c}
34 & 0.8 & 9 & 7 & 2 \\
1 & : & : & : & : \\
0 & 0 & Q_1 & Q_2 \\
\end{array}
\]

Two zeros are to be placed on the right of the decimal in the quotient digits as the Dhwajanka has two digits. In the final answer the decimal has to be shifted to the right by two digits.

Eg. (iv) 0.0089 ÷ 1.23

\[
\begin{array}{c|c|c|c|c}
3 & 0 & 0 & 8 & 9 \\
12 & : & : & : & : \\
0 & Q_1 \\
\end{array}
\]

One zero is to be placed after the decimal point of the quotient as the Dhwajanka has one digit. In the final answer the decimal has to be shifted by two digits.

Working Details:

The following general points need to be considered for division.

(1) The divisor partition, dividend partition and position of the decimal point are to be first determined.

(2) The division is carried out digit by digit of the dividend by the part divisor. While doing so, if the first digit of the dividend is not divisible by the part divisor, then one may consider the
minimum number of required digits for the divisibility to obtain the first quotient or one may report the division digit by digit. This procedure is adaptable only in the quotient part of the dividend. However in the remainder part it should be digit by digit division

(3) In case the division starts with the decimal after the effective partition, the division should be digit by digit.

(4) When the divisor consists of decimal or the dividend consists of decimal or in certain cases both may consists of decimals, the rules are clearly given while describing the partition and placement of the decimal. The exact working of the division giving various quotients and remainders, intermediate dividends and new dividends at each stage of division can be demonstrated as follows. However, the above rules are to be strictly adhered to.

Step 1: Divide the first digit of the dividend by the part divisor giving quotient $Q_1$ and remainder $R_1$. The quotient $Q_1$ is placed in the answer line. The remainder $R_1$ is kept between the first and the second digits of the dividend and below the dividend, leading to the formation of first intermediate dividend. If the first digit is not divisible by the part divisor (2nd clause in general points), then one may consider the minimum number of digits for the divisibility and the remainder $R_1$ is to be kept accordingly leading to the formation of first intermediate dividend (ID) and so on.

Step 2: The first intermediate dividend is formed by the remainder $R_1$ and the digit of the dividend immediately following the first dividend / first group of digits taken as the first dividend.

Step 3: Now the Urdhva multiplication of the allowed first digit of the Dhwajanka with the first quotient digit ($Q_1$) is carried out and the result so obtained is subtracted from the first intermediate dividend (ID) to get the first new dividend (ND) and the process is continued to obtain corresponding intermediate dividends and new dividends.

Step 4: For getting the remaining new dividends, the following principles are to be adopted. In case the Dhwajanka consists of more than one digit, then the new dividends are to be formed by subtracting the results of multiplication of the quotients $Q_1, Q_2, Q_3, \ldots, Q_n$ as per the Tiryak or Urdhva and Tiryak taking into consideration in succession the number of quotients according as the number of digits in the Dhwajanka $D_1 D_2 D_3 \ldots$. The procedure is indicated by means of a diagram in case Dhwajanka having 1 or 2 or 3 digits. The same is to be extended for any number of digits in Dhwajanka as follows. These are the steps required for subtraction in arriving the new dividends from the respective intermediate dividends. It is to be noticed that while the Dhwajanka remains constant, the quotient digits successively vary in the multiplication to get the new dividends.

Step 5: If the quotients after the decimal point are zeroes consequent on the deficiency, which is invariably due to the number of digits in Dhwajanka being greater than the dividend, then the formation of new dividends by subtractions are according to the following principles.

(a) Count zeroes also as quotients.

(b) All such zero quotients will be only passive and will not contribute anything for either intermediate dividend or for subtraction.

(c) If a zero quotient results due to division, such zeroes will not contribute to the subtraction.

The steps required for subtraction in arriving at the new dividends can be diagrammatically shown as given below.
i) Dhwanjanka has one digit:

If Dhwanjanka is $D_1$ and the quotient digits are $Q_1, Q_2, \ldots, Q_n$, then the subtracting quantities are as follows

$$D_1$$
$$Q_1$$

$$D_1$$
$$Q_2$$

$$D_1$$
$$Q_n$$

etc

ii) Dhwanjanka has two digits:

Dhwanjanka is $D_1D_2$ and quotients digits are $Q_1, Q_2, \ldots, Q_n$

$$D_1$$
$$Q_1$$

$$D_1$$
$$D_2$$

$$Q_1$$
$$Q_2$$

$$D_1$$
$$D_2$$

$$Q_2$$
$$Q_1$$

$$D_1$$
$$D_2$$

$$Q_{n-1}$$
$$Q_n$$

If one wants the absolute remainder, then we have to subtract the following from the total remainder region.

$$\left[ \begin{array}{c} D_1 \times D_2 \\ Q_{n-1} \times Q_n \end{array} \right] \times 10 + \left[ \begin{array}{c} D_2 \\ Q_n \end{array} \right] \times 1$$
(ii) Dhvajanka has three digits:

Dhvajanka is $D_1D_2D_3$ and quotients digits are $Q_1, Q_2, \ldots, Q_n$

If one wants the absolute remainder, then we have to subtract the following multiplications from the total remainder region.

\[
\begin{array}{c}
D_1 \quad D_2 \quad D_3
\end{array} \quad x \quad 100 \quad - \quad \begin{array}{c}
D_2 \quad D_3
\end{array} \quad x \quad 10 \quad - \quad \begin{array}{c}
D_3
\end{array} \quad x \quad 1
\]

and so on for the multidigit Dhvajanka problem

Step 6: While working the new dividends, one may come across a negative value as a consequence of subtraction in which case one has to reduce the quotient by 1. To quote one example for the negative dividend, refer example 3. In the example 3, we can come up to the 4th quotient, i.e., 2, we get the intermediate dividend as 24. On computation to obtain new dividend, we have to subtract \( \frac{4}{2} = 8 \) from 24, thereby we are left with 16.

Dividing this new dividend by 3 we get the
Vedic Mathematics

Division

quotient 5 and remainder 1 giving 15 as the intermediate dividend. Continuing the process of subtraction of multiplication of this quotient and Dhvajanka, one gets 20, which is greater than 15 (ID), resulting in negative new dividend, which is not accepted in this method. Hence one has to reduce the quotient 5 by 1 resulting in the modified value as 4.

Proceeding similarly we will get the new dividend as 45 - 16 = 29. Divide this by 3, we can try 9, but with this also one can see again a negative dividend and hence the quotient 9 is to be further reduced by 1 giving the value 8. This gives a remainder 5. Hence intermediate dividend is 56. We have to subtract $\frac{32}{8}$ from 56, giving a value of 24 as the new dividend.

Similar procedure is carried out in problems when a repeated occurrence of negative value results by this method. In all such cases one has to go on reducing the quotient by 1 step by step until the negative dividend ceases.

Another method is suggested when negative new dividends are formed. That is the negative result is written in vinculum form and the entire procedure is adopted with the vinculum number. At the end, one has to necessarily come out of vinculum to give the final result. Examples are given for this also.

In the examples the formation of intermediate dividends and new dividends are clearly shown. The same is to be understood for the other examples.

Step 7: The new dividends are subjected to division by part divisor and the procedure of earlier steps are repeated until one enters into the remainder region.

Step 8: At this stage the remainder so obtained together with the remainder part of the dividend can be considered as intermediate remainder. From this one has to subtract the value obtained by multiplying Dhvajanka and the last digit of the quotient to get the final remainder, as explained diagrammatically earlier in case of one digit two digits etc. in Dhvajanka (refer step 4 page).

The procedure is still extendable to the remainder part for obtaining the decimals. If the number of decimals is specified in the beginning itself, then the problem can be worked out until the specification is reached.

A number of examples are worked out to cover as many varieties as possible in this type of straight division. Keeping in view that the number of digits of the Dhvajanka is the criterion for the partition of the dividend, a number of problems are worked out.

The division is also extendable to the case where the dividend alone or divisor alone or both have decimals.

The division is also carried out by converting the numbers into vinculum forms.

The division is carried out for different cases such as the number of digits in the Dhvajanka is equal to or greater than or less than that in the dividend.

The division is worked out to obtain finally:

i. As quotient and remainder

ii. As quotient having decimal
Vedic Mathematics

Division

The division is also explained when the remainder is also subjected to division to include a specific number of decimals in the quotient.

The proof for all the above details is given in terms of a polynomial in x where x value is taken as 10 to identify the given number

Example 1: \(398775 + 75\)

Current Method

\[
\begin{array}{c}
75)
\begin{array}{c}
375 \\
237 \\
225 \\
127 \\
\textbf{75} \\
525 \\
525 \\
0
\end{array}
\end{array}
\]

Vedic Method

\[
\begin{array}{c|c}
\text{Divisor} & \text{Dividend} \\
75 & 398775
\end{array}
\]

\[
\begin{array}{c|c}
\text{Dhwajanka (1)} & \text{IDs} \\
\text{(flag)D} & 5 \\
\text{Part} & 7 \\
\text{Divisor} & \begin{array}{c}
R_1 \\
R_2 \\
R_3 \\
R_4 \\
Q_1 \\
Q_2 \\
Q_3 \\
Q_4 \\
R
\end{array}
\end{array}
\]

\[
\begin{array}{c}
5 \\
4 \\
2 \\
5 \\
3 \\
1 \\
7 \\
7 \\
0 \\
\text{Answer line}
\end{array}
\]

Quotient = 5317

Remainder, R = 0 (Exactly Divisible)

V.M.

In the above example, the part divisor 7 is active in division and 5, the Dhwajanka, is active in multiplication. The dividend is also to be partitioned from right end of the dividend such that the remainder part consists of as many digits as the Dhwajanka has.

The steps are as follows:

(1) The first division is to be carried out in the quotient region as \(3 + 7\). But as it is not divisible one should consider 39 as first dividend. 39 divided by the part divisor 7 gives 5 as first quotient Q₁ and 4 as the remainder R₁.

\[
\begin{array}{c}
7) 39 \ (Q_1) \\
\text{35} \\
\text{4} \ (R_1)
\end{array}
\]
Vedic Mathematics

Division

Quotient Q₁, i.e., 5, is kept in the answer. Remainder, 4, is placed between the first dividend 39 as a unit and the next digit 8 as shown in the example which can be read as 48, the intermediate dividend (ID). From this, the new dividend (ND) can be computed as given below.

2) The first quotient Q₁, 5, is multiplied by Dhwajanka, 5, and the result is subtracted from 48, intermediate dividend, i.e., $48 - 5 \times 5 = 23$. This is new dividend

$$
\begin{array}{c}
D \\
5 \\
\uparrow \\
5 \\
Q₁
\end{array}
$$

(ID) $48 - 
\begin{array}{c}
5 \\
Q₁
\end{array}
= 48 - 25 = 23 \text{ (ND)}$

The new dividend 23 is divided by the part divisor 7 giving 3 as the next quotient digit Q₂ and 2 as the remainder R₂.

$$
7) \begin{array}{c} 23 \quad (Q₂) \\
2 \quad (R₂)
\end{array}
$$

The placement of the remainder $R₂$ obtained in this step is similar as given above. The next intermediate dividend is 27

3) Next new dividend is calculated by subtracting the multiplication result of $Q₂$ and Dhwajanka 5 from the intermediate dividend 27

$$
\begin{array}{c}
D \\
5 \\
\uparrow \\
3 \\
Q₂
\end{array}
$$

(ID) $27 - 
\begin{array}{c}
5 \\
Q₂
\end{array}
= 27 - 15 = 12 \text{ (ND)}$

This new dividend is divided by 7 giving 1 as the next quotient digit $Q₃$ and intermediate dividend as 57

$$
7) \begin{array}{c} 12 \quad (Q₃) \\
2 \quad (R₃)
\end{array}
$$

4) The new dividend is calculated as

$$
\begin{array}{c}
D \\
5 \\
\uparrow \\
1 \\
Q₃
\end{array}
$$

(ID) $57 - 
\begin{array}{c}
5 \\
Q₃
\end{array}
= 57 - 5 = 52 \text{ (ND)}$

$$
\begin{array}{c}
7 \quad (Q₄) \\
42 \quad (R₄)
\end{array}
$$
5) Here the problem has entered into the remainder region. Remainder is calculated as given below.

\[
\text{D} \\
\text{(Intermediate Remainder)} \quad 35 - \left[ \begin{array}{c} 5 \\ 7 \end{array} \right] = 35 - 35 = 0 \\
\text{Q}_4
\]

\[ \therefore \text{Remainder} = 0 \]

Proof is given by converting the numbers into polynomials in \( x \) (\( x \) being 10) A little reorientation in placements of products of divisor and quotient and bringing down a part of the original dividend will explain the Vedic method of straight division.

For example, in the proof given below the product of the first quotient 5\( x^3 \) with the divisor, is subtracted from the original dividend. This is followed by bringing down a part of the original dividend so that it can be written as difference of two terms (refer A in the proof). The term with minus sign can be identified with the result obtained by Urdhva Multiplication of 5\( x^3 \) with 5 step 1 in the proof. For the following steps also, the term with minus sign can be identified with the corresponding Urdhva multiplications as shown in the problem 7\( x + 5 \) can be understood as the Dhvajanka 5 and the part divisor, 7. This subtraction can be seen throughout the working with \( D_1 \).

\[
\text{Proof:} \\
\text{Divisor} \\
D_1 \\
\text{Dividend} \\
7x + 5 \) \( 39x^4 + 8x^3 + 7x^2 + 7x + 5 \) \( 5x^3 + 3x^2 + x + 7 \\
25x^4 + 25x^3 \\
4x^4 + 8x^3 - 25x^3 \\
= x^3(4x + 8) - 25x^3 \\
= 48x^3 - 25x^3 \\
= 23x^3 \\
= 23x^3 + 7x^2 \\
21x^3 + 15x^2 \\
\text{Quotient} \\
Q_1, Q_2, Q_3, Q_4 \\
\rightarrow \quad (1) \\
\text{Subtracting} \\
D_1 \\
\text{5} \\
\uparrow = 25x^3 \\
\text{Q}_1 \\
\text{5} \\
\uparrow = 15x^2 \\
\text{Q}_2 \\
\text{5} \\
\uparrow = 5x \\
\text{Q}_3 \\
\text{5} \\
\uparrow = 35 \\
\text{Q}_4
\]

\[
\text{52x} + 5 \\
49x + 35 \\
3x + 5 - 35 \\
= 35 - 35 \\
= 0
\]

\[ \rightarrow \quad (4) \]
Some examples are given below:

### Example 2: 79335 ÷ 123

**Current Method**

<table>
<thead>
<tr>
<th>123</th>
<th>79335</th>
<th>(645)</th>
</tr>
</thead>
<tbody>
<tr>
<td>738</td>
<td></td>
<td></td>
</tr>
<tr>
<td>553</td>
<td></td>
<td></td>
</tr>
<tr>
<td>492</td>
<td></td>
<td></td>
</tr>
<tr>
<td>615</td>
<td></td>
<td></td>
</tr>
<tr>
<td>615</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Vedic Method**

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>79335</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|ccc}
|   & 7 & 9 & 3 \div 123 \Rightarrow 123 & | \frac{6}{5} |
|---|---|---|---|---|
| 3 & 7 & 7 \div 123 & | \frac{6}{5} |
| 12 & 6 & 4 & 5 \div 123 & | \frac{6}{5} |
| Q_1 & Q_2 & Q_3 & |
| \end{array}
\]

**Quotient = 645**

**Remainder = 0** (exactly divisible)

**Vedic Method Steps:**

**Step 1:**

\[12) 79 \div (6 (Q_1))\]

\[\frac{72}{7} \div (R_1)\]

\[Q_1 = 6\]

**Step 2:**

\[(ID) 73 - \frac{3}{6} = 73 - 12 = 61 \div (ND)\]

\[Q_1 = 4\]

**Step 3:**

\[(ID) 73 - \frac{3}{4} = 73 - 12 = 61 \div (ND)\]

\[Q_2 = 5\]

**Step 4:**

\[15 - \frac{5}{7} = 15 - 15 = 0\]

\[Q_3 = 5\]

\[\therefore \text{Remainder} = 0\]
**Example 3:** 7896456 ÷ 34 (The answer is represented as quotient and remainder)

**Current Method**

34) 7896456 (232248

\[
\begin{array}{c}
68 \\
109 \\
102 \\
76 \\
68 \\
84 \\
68 \\
165 \\
136 \\
296 \\
272 \\
24
\end{array}
\]

**Vedic Method**

\[\begin{array}{c|c}
D & 4 \\
Divisor & 34 \\
Dividend & 7896456 \\
\hline
D & 7 \\
(Pd) & 1 \\
R_1 & 1 \\
R_2 & 1 \\
R_3 & 2 \\
R_4 & 4 \\
R_5 & 5 \\
\hline
(m) & 2 \\
Q_1 & 3 \\
Q_2 & 2 \\
Q_3 & 4 \\
Q_4 & 8 \\
\hline
(m) & 24 \\
\hline
\end{array}\]

**Quotient = 232248**

\[\begin{array}{c}
Q = 232248 \\
R = 24 \\
\end{array}\]

**Vedic Method Steps:**

**Step 1:**

3) 7 (Q_1) \[\begin{array}{c}
D_1 \\
Q_1 = 2 \\
\end{array}\]

**Step 2:**

\[\begin{array}{c}
10 (Q_2) \\
Q_2 = 3 \\
\end{array}\]

**D - Dhvajanka**

\[\begin{array}{c}
24 \\
\hline
Q_6 \\
\end{array}\]

\[\begin{array}{c}
Remainder = 56 - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = 56 - 32 = 24 \\
\end{array}\]
Vedic Mathematics

Step 3:

\[ \begin{array}{c}
D_1 \\
\uparrow \\
4 \\
\hline
3 \\
\downarrow \\
\hline
2 \\
\end{array} \]

(ID) \( 19 - \frac{4}{3} = 19 - 12 = 7 \) (ND)

\[ \begin{array}{c}
Q_2 \\
\hline
2 \quad (R_3) \\
\end{array} \]

3) \( 7 \ (Q_3) \)

\( \frac{6}{1} \quad (R_3) \)

\[ Q_3 = 2 \]

Step 4:

\[ \begin{array}{c}
D_1 \\
\uparrow \\
4 \\
\hline
2 \\
\downarrow \\
\hline
1 \\
\end{array} \]

(ID) \( 16 - \frac{4}{2} = 16 - 8 = 8 \) (ND)

\[ \begin{array}{c}
Q_4 \\
\hline
2 \quad (R_4) \\
\end{array} \]

3) \( 8 \ (Q_4) \)

\( \frac{6}{2} \quad (R_4) \)

\[ Q_4 = 2 \]

Step 5:

\[ \begin{array}{c}
D_1 \\
\uparrow \\
4 \\
\hline
2 \\
\downarrow \\
\hline
1 \\
\end{array} \]

(ID) \( 24 - \frac{4}{2} = 24 - 8 = 16 \) (ND)

\[ \begin{array}{c}
Q_4 \\
\hline
2 \quad (R_3) \\
\end{array} \]

3) \( 16 \ (Q_3) \)

\( \frac{15}{1} \quad (R_3) \)

Conversion to vinculum at the stage of step 6

Step 6:

\[ \begin{array}{c}
D_1 \\
\uparrow \\
4 \\
\hline
5 \\
\downarrow \\
\hline
1 \\
\end{array} \]

(ID) \( 15 - \frac{4}{5} = 15 - 20 = -5 \) (negative value)

\[ \begin{array}{c}
Q_5 \\
\hline
1 \quad (R_4) \\
\end{array} \]

\[ \therefore \text{We reduce the quotient 5 by 1 giving the modified quotient } Q_5(m) \text{ value as 4.} \]

3) \( 16 \ [Q_5(m)] \quad (m = \text{modified}) \)

\( \frac{12}{4} \quad [R_3(m)] \)

\[ Q_5(m) = 4 \]
Vedic Mathematics

Division

\[
D_1
\]

(ID) \[45 - \left( \begin{array}{c} 4 \end{array} \right) = 45 - 16 = 29 \text{(ND)}
\]

\[Q_5(m)\]

3) 29 (Q_6)

\[\begin{array}{c} 27 \\ 2 \end{array}\]

(R_6)

Remainder part entry

Continuation of Vinculum

Step 7:

\[
D_1
\]

(II) \[26 - \left( \begin{array}{c} 4 \\ 9 \end{array} \right) = 26 - 36 = -10 \text{ (negative value)}
\]

\[Q_6\]

:: We reduce the quotient by 1

3) 29 (8 [Q_6(m)])

\[\begin{array}{c} 24 \\ 5 \end{array}\]

\[R_6(m)\]

\[Q_6(m) = 8\]

\[
D_1
\]

Remainder = \[56 - \left( \begin{array}{c} 4 \\ 8 \end{array} \right) = 56 - 32 = 24\]

\[Q_6(m)\]

:: Quotient = 232248, Remainder = 24

\[
\begin{array}{c}
\text{Vinculum.} \\
4 & 7 & 8 & 9 & 6 & 4 & 5 : 6 \\
/ & / & / & / & / & / \\
3 & 1 & 1 & 1 & 2 & 1 : 1 \\
\end{array}
\]

\[\begin{array}{c}
2 & 3 & 2 & 2 & 5 & \bar{2} : 24 \\
2 & 3 & 2 & 2 & 4 & 8 : 24 \\
\end{array}\]

One can avoid the process of reduction of Quotient stepwise until one gets a positive ID, if the negative Quotient or the remainder is used directly in the calculations.
Example 4: $6974 + 7$ (Single digit divisor) this divisor needs to be converted to Vinculum to facilitate straight Division

Current Method

7) $6974 (996$

$\begin{array}{c}
996 \\
67 \\
63 \\
44 \\
42 \\
2 \\
\end{array}$

Quotient = 996, Remainder = 2

Vedic Method

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 = 13</td>
<td>6974</td>
</tr>
</tbody>
</table>

D $\begin{array}{c}3 \\
6 \quad 9 \quad 7 : 4 \\
\end{array}$

Pd $\begin{array}{c}1 \\
0 \quad 0 : 0 \\
\end{array}$

$\begin{array}{c}
R_1 \\
R_2 \\
R_3 \\
\end{array}$

$\begin{array}{c}
6 \\
27 \\
88 : 268 \\
\end{array}$

$Q_1, Q_2, Q_3$

Q = Quotient = 958
R = Remainder

$\frac{3}{88} = 4 + 264 = 268$

R = 268 > 7 (Divisor)
Hence further division by 13 is continued treating, the final reminder R as the dividend.

Various steps for the division of R

(1) $\begin{array}{c}3 \\
2 \quad 6 : 8 \\
\end{array}$

$\begin{array}{c}0 \\
0 : 0 \\
\end{array}$

$\begin{array}{c}
R_1' \\
R_2' \\
\end{array}$

$\begin{array}{c}
2 \\
12 : \rightarrow Q' \\
Q_1', Q_2', Q_3' \\
\end{array}$

Quotient, $Q' = 32$

Remainder, $R' = 8 - \frac{3}{12} = 44$

again the reminder $R' = 44 > 7$ (Divisor)

(2) $\begin{array}{c}3 \\
4 : 4 \\
\end{array}$

$\begin{array}{c}1 \\
0 : 0 \\
\end{array}$

$\begin{array}{c}
R_1'' \\
\end{array}$

$\begin{array}{c}
4 : \rightarrow Q'' \\
Q_1'', Q_2'' = Quotient = 4 \\
\end{array}$
\[ R'' = \text{Remainder} \]

\[
\begin{array}{c}
3 \\
\uparrow 4
\end{array}
= 4 \rightarrow \frac{3}{4} = 4 + 12 = 16
\]

The reminder \( R'' = 16 > 7 \) (Divisor)

\[
\begin{array}{c|c}
3 & 1 : 6 \\
\hline
1 & 0 \\
\hline
1 & R'''
\end{array}
\]

\[ Q''' = \text{Quotient} = 1 \]
\[ R''' = \text{Remainder} = 9 \]

\[
\begin{array}{c}
3 \\
\uparrow 1
\end{array}
= 6 \rightarrow \frac{3}{1} = 6 + 3 = 9
\]

The reminder \( R''' = 9 > 7 \) (Divisor)

The dividend 9 is converted into vinculum to facilitate partition

\[
\begin{array}{c|c}
3 & 1 : 1 \\
\hline
1 & 0 \\
\hline
1 & R''''
\end{array}
\]

\[ Q'''' = \text{Quotient} = 1 \]
\[ R'''' = \text{Remainder} = 2 \]

\[ = i = \left\{ \begin{array}{c} 3 \\
1 \end{array} \right\} = -1 + 3 = 2 \]

By adding \( Q'''' \) to the quotients obtained in the above steps we get the final quotient

\[ : \text{Quotient} = 958 + 32 + 4 + 1 + 1 = 996 \]

\[ Q'' \quad Q''' \quad Q'''' \]

\[ Quotient = Q + Q' + Q'' + Q''' + Q'''' \]
\[ (Q = Q_1 + Q_2 + Q_3) + (Q' = Q_1' + Q_2') \]
\[ + (Q'' = Q_1'' ) + (Q''' = Q_1''') \]
\[ + (Q'''' = Q_1'''' ) = 996 \]

Remainder = 2
Example 5: 7652 ÷ 23
The answer is represented as quotient and remainder and continued for decimals in the quotient.)

Current Method
3) 7 6 5 2 (3 3 2
   6 9
   7 5
   6 9
   6 2
   4 6
   1 6

Quotient = 332
Remainder = 16

Vedic Method

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>7652</td>
</tr>
</tbody>
</table>

\[ a) \]
\[
\begin{array}{ccc}
3 & 7 & 6 \ 5 \\
2 & 1 & 1 \\
R_1 & R_2 & R_3 (m)
\end{array}
\]

Quotient = 332
Remainder = 22

If one wants to work the problem of division to a specific number say 4 decimal places, in the quotient then one has to include as many zeros at the end of the dividend as to accommodate the number of decimal places and continue for decimals.

Current Method
3) 7 6 5 2 (3 3 2, 6 9 5 6 5 2
6 9
7 5
6 2
4 6
1 6 0
1 3 8
2 2 0
2 0 7
1 3 0
1 1 5
1 5 0
1 3 8
1 2 0
1 1 5
5 0
4 6
4

Vedic Method

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>7652</td>
</tr>
</tbody>
</table>

\[
\begin{array}{ccc}
3 & 7 & 6 \ 5 \\
2 & 1 & 1 \\
R_1 & R_3 & R_4 & R_5 & R_6 & R_7 \\
(m) & (m) & (m) & (m) & (m) & (m)
\end{array}
\]

Quotient = 332.6956
Vedic Method Steps:

Step 1:
\[ 2 \times 7 (3 \times Q_1) \]
\[ \frac{6}{3} \]
\[ \underline{(R_1)} \]
\[ Q_1 = 3 \]

Step 2:
\[ \frac{D_1}{3} \]
\[ \underline{(ID)} 16 - \frac{3}{3} = 16 - 9 = 7 \text{ (ND)} \]
\[ Q_1 \]
\[ 2 \times 7 (3 \times Q_2) \]
\[ \frac{6}{3} \]
\[ \underline{(R_2)} \]
\[ Q_2 = 3 \]

Step 3:
\[ \frac{D_1}{3} \]
\[ \underline{(ID)} 15 - \frac{3}{3} = 15 - 9 = 6 \text{ (ND)} \]
\[ Q_3 \]

2) 6 (3 \times Q_3)
\[ \frac{6}{3} \]
\[ 0 \text{ (R_3)} \]

Remainder Part:

Step 4:
\[ \frac{D_1}{3} \]
\[ 2 - \frac{3}{3} = 2 - 9 = -7 \text{ (negative value)} \]
\[ Q_3 \]

\[ \therefore \text{We reduce the quotient } Q_3 \text{ by 1.} \]

Step 4:
Replacement by vinculum
\[ ID = 02 \text{ ND} = 02 - 9 = -7 \]

2) \( \frac{7}{6} \times \frac{3}{3} \text{ (Q_4)} \)
\[ \frac{6}{1} \]
\[ (R_4) \]

2) 6 (2 \times [Q_3 \text{ (m)}])
\[ \frac{4}{2} \times [R_3 \text{ (m)}] \]
\[ Q_3 \text{ (m)} = 2 \]

(Intermediate Remainder) \[ 22 - \frac{3}{2} = 22 - 6 = 16 \text{ (Remainder)} \]

Quotient = 332, Remainder = 16

If decimal points in the quotient are needed then continue the process by treating the remainder 16 as new dividend.
Vedic Mathematics

2) 16 (8 (Q₄)

\[
\begin{array}{c|c}
\hline
16 & \\
\hline
0 & (R₄) \\
\hline
\end{array}
\]

Step 5:

\[
\begin{array}{c|c}
D₁ & \\
\hline
(3) & 0 - 24 = -24 \text{ (negative value)} \\
\hline
Q₄ & \\
\end{array}
\]

\[
\begin{array}{c|c}
D₁ & \\
\hline
(3) & 10 - 9 = \bar{1} \text{ (ND)} \\
\hline
Q₄ & \\
\end{array}
\]

\[
2) \bar{1} (\bar{1} (Q₃)
\]

\[
\begin{array}{c|c}
\hline
2 & \bar{1} (0 (Q₃) \\
\hline
\end{array}
\]

:: We reduce the quotient Q₄ by 1

\[
\begin{array}{c|c}
2 & 16 (7 [Q₄(m)] \\
\hline
14 & \bar{2} \cdot R₄ (m) \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
D₁ & \\
\hline
(3) & 20 - 21 = -1 \text{ (negative value)} \\
\hline
Q₄(m) & \\
\end{array}
\]

:: We reduce the modified quotient Q₄ further by 1

\[
\begin{array}{c|c}
2 & 16 \text{ (6 [Q₄(m)]} \\
\hline
12 & \bar{4} [R₄(m)] \\
\hline
Q₄(m) = 6 \\
\end{array}
\]

\[
\begin{array}{c|c}
(40 - \overline{6} & 40 - 18 = 22 \text{ (ND)} \\
\hline
Q₄(m) & \\
\end{array}
\]

2) 22 (11 (Q₅)

\[
\begin{array}{c|c}
22 & \\
\hline
0 & (R₅) \\
\end{array}
\]
Vedic Mathematics

Step 6:

\[
\frac{D_1}{Q_s} \quad \text{(ID) } 0 - \left(\frac{3}{11}\right) = 0 - 33 = -33 \text{ (negative value)}
\]

\[
\frac{D_1}{Q_s} \quad \text{(ID) } 10 - \left(\frac{3}{0}\right) = 10 - 0 = 10
\]

\[\because \text{ We reduce the quotient } Q_s \text{ by 1} \]

\[2 \) 22 (10 [Q_s(m)])
\]

\[
\frac{20}{2} \quad \frac{10}{0} \quad [R_s(m)]
\]

\[
\frac{D_1}{Q_s} \quad \text{(ID) } 20 - \left(\frac{3}{10}\right) = 20 - 30 = -10 \text{ (negative value)}
\]

\[
\frac{D_1}{Q_s(m)} \quad \text{(ID) } 40 - \left(\frac{3}{9}\right) = 40 - 27 = 13 \text{ (ND)}
\]

\[\because \text{ We reduce the quotient } Q_s \text{ further by 1} \]

\[2 \) 22 (9 [Q_s(m)])
\]

\[
\frac{18}{4} \quad \frac{10}{0} \quad [R_s(m)]
\]

\[
\frac{D_1}{Q_s(m)} \quad \text{(ID) } 40 - \left(\frac{3}{9}\right) = 40 - 27 = 13 \text{ (ND)}
\]

\[\because \text{ We reduce the quotient } Q_s \text{ further by 1} \]

\[2 \) 13 (6 [Q_s(m)])
\]

\[
\frac{12}{1} \quad \frac{10}{0} \quad (R_s)
\]

\[\text{Step 7:} \]

\[
\frac{D_1}{Q_s} \quad \text{(ID) } 10 - \left(\frac{6}{3}\right) = 10 - 18 = -8 \text{ (negative value)}
\]

\[
\frac{D_1}{Q_s} \quad \text{(ID) } 0 - \left(\frac{5}{3}\right) = 0 - 15 = -15 \text{ (ND)}
\]

\[\because \text{ We reduce the quotient } Q_s \text{ by 1.} \]

\[2 \) 13 (5 [Q_s(m)])
\]

\[
\frac{10}{3} \quad \frac{14}{1} \quad (R_s)
\]

\[2 \) 15 (7 [Q_s(m)])
\]

\[
\frac{14}{1} \quad \frac{15}{1} \quad (R_s)
\]
\[ D_1 \]

\((\text{ID})\ 30 - \left( \frac{3}{6} \right) = 30 - 15 = 15 \ (\text{ND}) \]

\[ Q_6(m) \]

2) 15 (Q_7)

\[
\begin{array}{c}
14 \\
_1 (R_7)
\end{array}
\]

Step 8:

\((\text{ID})\ 10 - \left| \frac{7}{7} \right| = 10 - 21 = -9 \ (-\text{ve value}) \]

\[ Q_7 \]

\[ \therefore \text{We reduce quotient } Q_7 \text{ by } 1 \]

2) 15 (6 [Q_7(m)])

\[
\begin{array}{c}
12 \\
3 [R_7(m)]
\end{array}
\]

2) II (5 (Q_8))

\[
\begin{array}{c}
\frac{10}{1} (R_8)
\end{array}
\]

\[ D_1 \]

\((\text{ID})\ 30 - \left( \frac{3}{6} \right) = 30 - 18 = 12 \ (\text{ND}) \]

\[ Q_7(m) \]

\[ \therefore \text{Quotient } = 332.6956 \]

Vinculum:

\[
\begin{array}{c|ccccccc}
3 & 7 & 6 & 5 & 2 & 0 & 0 & 0 \\
2 & 1 & 1 & : & 0 & 1 & 0 & 1 \\
\hline
3 & 3 & 3 & . & 3 & 0 & 5 & 7 & 5 \\
3 & 3 & 2 & 6 & 9 & 5 & 6 & 5
\end{array}
\]

\[ \text{Ans} \]

we can see the ease with which the problem is worked out using Vinculum
Vedic Mathematics

Proof:

\[ (2x+3)(7x^2 + 6x^2 + 5x + 2) = (3x^2 + 3x + 2) \]

\[ x^2 + 6x \]

\[ -9x^2 \]

\[ 9x^2 \]

\[ 3x^2 \]

\[ = x^2(x + 6) - 9x^2 \]

\[ \cdot 16x^2 - 9x^2 \] Here \( x = 10 \)

\[ = 7x^2 \]

\[ 7x^2 + 5x \]

\[ 6x^2 + 9x \]

\[ x^2 + 5x \]

\[ -9x \]

\[ = x(x + 5) - 9x \]

\[ = 15x - 9x \]

\[ = 6x \]

\[ 6x + 2 \]

\[ 4x + 6 \]

\[ 2x + 2 \]

\[ -6 \]

\[ = 22 - 6 \]

\[ = 16 \]

Remainder = 16

If decimal points are needed then one has to continue the procedure as given below:

The remainder is again converted into polynomial and one has to consider it only after multiplying with 10 in order to proceed into the decimal working. Hence the remainder 16 becomes 16x and is divided by 2x + 3 (Here \( x = 10 \))

Division of the remainder

\[ (2x+3)16x \]

\[ = (0.6) \]

\[ 9 \]

\[ 5 \]

\[ 6 \]

\[ 5 \]

\[ D_1 \]

\[ 3 \]

\[ = 18 \]

\[ Q_4 \]

\[ 6 \]

\[ = 27 \]

\[ Q_5 \]

\[ 9 \]

\[ = 15 \]

\[ Q_6 \]

\[ 5 \]

\[ = 18 \]

\[ Q_7 \]

\[ 6 \]
Example 6: 8954 ÷ 89 (Division resulting in remainder and the answer is represented as quotient and remainder. The work is continued for decimals in the quotient) up to 5 places of decimals.

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>89) 8954 (100 60674</td>
<td>Divisor 89</td>
</tr>
<tr>
<td>8900</td>
<td>9</td>
</tr>
<tr>
<td>540</td>
<td>8</td>
</tr>
<tr>
<td>534</td>
<td>R₁ R₂ R₃ R₄ R₅ R₆ R₇ R₈</td>
</tr>
<tr>
<td>600</td>
<td>1 0 0 . 6 0 6 7 4</td>
</tr>
<tr>
<td>523</td>
<td>Q₁ Q₂ Q₃ Q₄ Q₅ Q₆ Q₇ Q₈</td>
</tr>
<tr>
<td>370</td>
<td></td>
</tr>
<tr>
<td>356</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Quotient = 100

D₁

Remainder = 54 - \[\begin{pmatrix} 9 \\ 0 \end{pmatrix} = 54\]

Q₃

Quotient in decimals = 100.60674

One can keep more than one digit in the Dhwajanka, if the divisor has more than two digits. In such a case the procedure is as follows.
Example 7: \(897356 \div 721\) (Division where the Dhvajanka has two digits and the answer is represented as quotient and remainder and continued up to 5 decimals in the quotient)

**Current Method**

\[
\begin{align*}
721 & \mid 897356 \hspace{1cm} (1244.599) \\
721 & \\
1763 & \\
1442 & \\
3215 & \\
2884 & \\
3316 & \\
2884 & \\
4320 & \\
3602 & \\
7150 & \\
6489 & \\
6610 & \\
6489 & \\
121 &
\end{align*}
\]

**Vedic Method**

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>721</td>
<td>897356</td>
</tr>
</tbody>
</table>

Dhvajanka

\[
\begin{array}{cccccccc}
D_1 & D_2 & \hspace{1cm} & I_D s & & & & \hspace{1cm} & \text{Remain} \text{d} \\
2 & 1 & & 8 & 9 & 7 & 3 : & 5 & 6 & 0 & 0 & 0 & 0 \\
7 & & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & (m) & (m) \\
1 & 2 & 4 & 6 & 5 & 9 & 9 & 1 & 6 & & & \\
\hline
\text{Part} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & (m) & (m) \\
\end{array}
\]

Quotient = 1244, Remainder = 432
Quotient in decimals = 1244.59916

**Vedic Method Steps:**
The first step is same as in previous cases.

**Step 1:**

\[
\begin{align*}
7) 8 & (Q_1) \\
7 & \\
1 & (R_1)
\end{align*}
\]

This gives rise to the intermediate dividend 19

**Step 2:**

After the first step is over, while the multiplication part is taken up, as there is only one quotient digit, one has to multiply the quotient digit (Q_1 with the first digit of the Dhvajanka. The product is subtracted from the intermediate dividend, 19, to give rise to the new dividend.

\[
(ID) 19 - \left[ \begin{array}{c}
2 \\
1
\end{array} \right] = 17 \text{ (ND)}
\]

Divide this new dividend 17 by 7, then 2 is the quotient and the remainder is 3, giving an intermediate dividend 37
7) 17 (Q₂)
   14
   3 (R₃)

Step 3:

Consider the multiplication of the Q₁, Q₂ so far obtained with the Dhwajanka digits D₁, D₂ by Tiryak multiplication. The new dividend is obtained by subtracting the result of this Tiryak-multiplication from the intermediate dividend, 37.

\[
(ID)\ 37 - \begin{array}{c}
  2 \\
  \times \\
  1 \\
  + \\
  2 \\
  \hline \\
  32 \\
\end{array} = 32 \text{ (ND)}
\]

Divide the new dividend, 32 by 7 to get the next intermediate dividend, 43.

7) 32 (Q₃)
   28
   4 (R₄)

Step 4:

As the Dhwajanka contains two digits, we now consider two quotient digits Q₂ and Q₃ for Tiryak-multiplication with the Dhwajanka digits D₁D₂. The Tiryak-multiplication result is subtracted from the corresponding intermediate dividend, 43 to get the corresponding new dividend, 33 as follows:

\[
(ID)\ 43 - \begin{array}{c}
  2 \\
  \times \\
  1 \\
  + \\
  4 \\
  \hline \\
  33 \\
\end{array} = 33 \text{ (ND)}
\]

7) 33 (Q₄)
   28
   5 (R₅)

The working has now entered into the remainder part.

In order to get the remainder, one has to stop at the stage of entering into the remainder part after part of the dividend is considered as intermediate remainder (556).

One has to compute from this value the actual remainder by subtracting the result of Tiryak and Urdhva multiplication as follows.

\[
\text{ Remainder is } 556 - \begin{array}{c}
  2 \\
  \times \\
  1 \\
  + \\
  4 \\
  \hline \\
  5 \\
\end{array} \times 10 - \begin{array}{c}
  1 \\
  \times \\
  4 \\
  \hline \\
  Q \\
\end{array} \text{ (D₃ and Q₃ are in tens place)}
\]
Vedic Mathematics

Division

\[ 556 - 120 = 432 \]
Quotient = 1244, Remainder = 432.
Similar procedure is continued to get the other dividends in the decimal region.

Step 5:

\[
\begin{array}{cccc}
D_1 & D_2 \\
2 & 1 \\
4 & 3 \\
Q_3 & Q_4 \\
\end{array}
\]

(ID) 55 - \(\frac{21}{43} = 43\) (ND)

7) 43 (Q₅)

[42]

[1] (R₅)

Step 6:

\[
\begin{array}{cccc}
D_1 & D_2 \\
2 & 1 \\
4 & 6 \\
Q_4 & Q_3 \\
\end{array}
\]

(ID) 16 - \(\frac{21}{0} = 0\) (ND)

7) 0 (Q₆)

[0]

0 (R₆)

Step 7:

\[
\begin{array}{cccc}
D_1 & D_2 \\
2 & 0 \\
6 & 6 \\
Q_5 \\
\end{array}
\]

(ID) 0 - \(\frac{20}{0} = 0 - 6 = -6\) (negative value)

\[\frac{\overline{6}}{7} = 0 - 6 = 6\]

But the quotient is zero, so if we reduce this value it becomes negative. Therefore, we reduce previous quotient 6 (obtained in step 5) as 5.

7) 43 (Q₅(m))

\[\frac{25}{8} = 5\] (Q₅(m))

7) 6 (\(\overline{7}\) [Q₇])

\[\frac{7}{1} = 1\] (R₇)

(ID) 86 - \(\frac{21}{5} = 86 - 14 = 72\) (ND)

7) 72 (Q₆(m))

\[\frac{70}{2} = 35\] (Q₆(m))

7) 6 (\(\overline{1}\) [Q₇]}
\[ \begin{array}{c}
20 - \binom{2}{5} \times \binom{1}{10} = 20 - 25 = -5 \text{ (negative value)} \\
Q_5 \quad Q_6 \\
\text{(m) (m)}
\end{array} \]

Ve reduce quotient 10 by 1.

\[ \begin{array}{c}
72 \ (9 \ [Q_6(m)]) \\
52 \\
9 \ [R_4 \ (m)]
\end{array} \]

\[ Q_6 = (m) = 9 \]

\[ \begin{array}{c}
90 - \binom{2}{5} \times \binom{1}{9} = 90 - 23 = 67 \text{ (ND)} \\
Q_5(m) \quad Q_6(m)
\end{array} \]

\[ \begin{array}{c}
57 \ (9 \ (Q_7)) \\
52 \\
4 \ (R_7)
\end{array} \]

\[ Q_7 = 9 \]

\[ \begin{array}{ccc}
\text{p 8:} & \text{Step 8:} & \text{p 9:} \\
\begin{array}{c}
40 - \binom{2}{9} \times \binom{1}{9} = 40 - 27 = 13 \text{ (ND)} \\
Q_6(m) \quad Q_7
\end{array} & (\text{ID}) \ 10 - \binom{2}{0} \times \binom{1}{1} = 12 \text{ (ND)} & \begin{array}{c}
60 - \binom{2}{9} \times \binom{1}{1} = 60 - 11 = 49 \text{ (ND)} \\
Q_7 \quad Q_8
\end{array} \\
& \begin{array}{c}
7 \ (1 \ Q_9) \\
7 \ (R_4)
\end{array} & \begin{array}{c}
49 \ (7 \ (Q_9)) \\
42 \\
0 \ (R_9)
\end{array}
\end{array} \]
Step 10:
\[
\begin{bmatrix}
2 & 1 \\
1 & 7
\end{bmatrix} = 0 - 15 = -15 \text{ (negative value)}
\]
\[
\begin{bmatrix}
2 & 1 \\
1 & 7
\end{bmatrix} = -15
\]

\[
7 \div 2 = \frac{15}{1} Q_{10}
\]

\[
\frac{14}{1} R_{10}
\]

\[
\therefore \text{We reduce the quotient 7 by 1.}
\]

\[
7) 49 \left[ Q_{9}(m) \right] \quad Q_{9}(m) = 6
\]

\[
\begin{array}{c}
42 \\
7 \left[ R_{9}(m) \right]
\end{array}
\]

\[
\begin{aligned}
\text{(ID) } & \quad 70 - \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix} = 70 - 13 = 57 \text{ (ND)} \\
Q_{8} & \quad Q_{8}(m)
\end{aligned}
\]

\[
\text{Vinculum: (Direct)}
\]

\[
\begin{array}{cccccccccccc}
& 21 & 8 & 9 & 7 & 3 & : & 5 & 6 & 0 & 0 & 0 & 0 \\
\hline
& 1 & 3 & 4 & 5 & 1 & 0 & 1 & 5 & 0 & 1 \\
\hline
R_{1} & R_{2} & R_{3} & R_{4} & R_{5} & R_{6} & R_{7} & R_{8} & R_{9} & R_{10}
\end{array}
\]

\[
\begin{aligned}
1 & \quad 2 & \quad 4 & \quad 6 & \quad 0 & \quad 1 & \quad 1 & \quad 7 & \quad 2 \\
Q_{1} & Q_{2} & Q_{3} & Q_{4} & Q_{5} & Q_{6} & Q_{7} & Q_{8} & Q_{9} & Q_{10}
\end{aligned}
\]

\[
= 1244.599168
\]

\[
\therefore \text{Quotient} = 1244.59916
\]

In case of Dhwajanka having two digits the procedure is as described in the diagrams and the difference term ('-' term) can be identified with the Urdhva or cross multiplication as the case may be. It is exemplified in the following steps. The proof is given below:
Vedic Mathematics

1897356 + 721

Division

Proof:

\[ \begin{array}{cccccccc}
D_1 & D_2 & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\
7x^3 + 2x + 1 & 8x^4 + 9x^3 + 7x^2 + 3x^2 + 5x + 6 & 5 & 9 & 9 & 1 & 6 \\
\end{array} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
\downarrow & & & & & & & \\
1 & 7x^4 & 17x^3 & 14x^4 + 4x^3 + x^2 \\
& & & & & & & \\
& & & & & & & \\
= x^2(3x + 7) - 5x^3 \\
= 37x^2 - 5x^3 \\
= 32x^2 + 3x^2 \\
& & & & & & & \\
& & & & & & & \\
= 33x^3 + 5x \\
& & & & & & & \\
= x(5x + 5) - 12x \\
= 55x - 12x \\
= 43x + 6 \\
\end{array} \]

* \[ \frac{1}{43x + 6} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
= 43x + 6 \\
& & & & & & & \\
= 35x^2 + 10x + 4x^* \\
& & & & & & & \\
= x(8x + 6) - 14x \\
= 86x - 14x = 72x \\
\end{array} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
62x^2 & 63x^2 + 18x + 5x^* \\
9x^2 - 18x - 5x \\
\end{array} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
= x(9x) - 23x \\
= 90x - 23x = 67x \\
\end{array} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
67x^2 & 63x^2 + 18x + 9x^* \\
4x^2 - 18x - 9x \\
\end{array} \]

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
= 40x - 27x = 13x \\
\end{array} \]

*Intermediate Step to get there remainder or to extend to decimals

*\( x = 10 \) is introduced through conversion of dividend under decimal working and is denoted with \( (\cdot) \) notation and is shown as \( \overline{Q_6} \ldots \overline{Q_6} \).
Vedic Mathematics

\[
\begin{align*}
13x^2 & \quad D_1 \quad D_2 \\
7x^2 + 2x + 9x^2 & \quad 2x \quad 1 \\
6x^2 - 2x - 9x & \\
\quad = x(6x) - 11x & \quad *9x \quad 1 \\
\quad = 60x - 11x = 49x & \quad Q_1 \quad Q_8 \\
49x^2 & \quad D_1 \quad D_2 \\
42x^2 + 12x + x^2 & \quad 2x \quad 1 \\
7x^2 - 12x - & \\
\quad = x(7x) - 13x & \quad *x \quad 6 \\
\quad = 70x - 13x = 57x & \quad Q_4 \quad Q_9 \\
\end{align*}
\]

In the proof at the stage when we get 43x + 6, one should decide whether one has to stop at this stage to work out the remainder or one has to go still further to get the decimals in the quotients. In the first case, the completion of the step is arrived by subtracting \(\frac{1}{4}\) from 43x + 6, i.e., 43x + 6 - 4 = 43x + 2. Now the quotient = \(x^3 + 2x^2 + 4x + 4\) = 1244. Remainder = 43x + 2 = 432. But if one wants to go in for the decimals in the quotient, then one has to continue from the step 43x + 6.

We can continue like this to as many digits as per specifications. One can work out for the remainder by getting down all the remaining dividend parts and then to work out the corresponding quotient and proceeding further. This is clearly shown in the working given below.

\[
\begin{align*}
\uparrow & \quad \uparrow & \quad \uparrow & \quad \uparrow & \quad \uparrow & \quad \uparrow \\
D_1 & \quad D_2 & \quad Q_1 & \quad Q_2 & \quad Q_3 & \quad Q_4 \\
7x^2 + 2x + 1 & \quad 8x^3 + 9x^4 + 7x^3 + 3x^2 + 5x + 6 & \quad x^2 + 2x^2 + 4x + 4 \\
7x^2 + 2x^4 & \quad x^3 + 9x^4 - 2x^4 \\
\quad = x^4(x + 9) - 2x^4 & \quad = 19x^4 - 2x^4 = 17x^4 \\
\end{align*}
\]

\[
\begin{align*}
17x^4 + 7x^3 & \\
14x^4 + 4x^3 + x^1 & \quad 3x^4 + 7x^3 - 4x^3 - x^3 \\
\quad = x^4(3x + 7) - 5x^3 & \quad = 37x^3 - 5x^3 = 32x^3 \\
\end{align*}
\]

\[
\begin{align*}
32x^3 + 3x^2 & \\
28x^3 + 8x^2 + 2x^2 & \quad 4x^2 + 3x^2 - 8x^2 - 2x^2 \\
\quad = x^2(4x + 3) - 10x^2 & \quad = 43x^2 - 10x^2 = 33x^2 \\
33x^2 + 5x + 6 & \quad \rightarrow \quad \text{Remaining Dividend Part}
\end{align*}
\]
Vedic Mathematics

\[
\begin{align*}
33x^2 + 5x + 6 \\
28x^2 + 8x + 4 + 4
\end{align*}
\]

\[
\text{Remainder region of the dividend}
\]

Remainder = \[
\frac{5x^2 + 5x + 6 - (8x + 4x) - 4}{5x^2 + 5x + 6 - (8x + 4x) - 4}
\]

\[
= \frac{556 - 120 - 4}{556 - 120 - 4}
\]

\[
= 432.
\]

\[
D_1 \quad D_2 \quad D_3
\]

\[
\begin{bmatrix}
2x & 1 & 1 \\
4x & 4 & 4
\end{bmatrix}
\]

\[
= -(8x + 4x) + 4
\]

\[
Q_3 \quad Q_2
\]

If one wants decimal points in the quotient, then start the division by taking the remainder as the dividend and the same procedure explained above is followed.

The remainder is converted into polynomial and one has to consider it only after multiplying with 10 in order to proceed into the decimal working and hence it becomes:

\[
43x^2 + 2x \text{ (remainder)}
\]

The actual division repeats from this stage onwards.(Refer page for the reminder as \(43x + 2\))

\[
7x^2 + 2x + 1) 43x^2 + 2x (6
\]

\[
\begin{bmatrix}
42x^2 + 12x \\
\frac{x^2 + 2x - 12x}{x(x + 2) - 12x} \quad \cdot \quad (1)
\end{bmatrix}
\]

\[
2x - 12x
\]

\[
x(12) - 12x
\]

\[
= 0
\]

\[
7x^2 + 2x + 1) 0 \quad (0
\]

\[
\begin{bmatrix}
0 + \frac{6}{-6}
\end{bmatrix}
\]

\[
= \frac{6}{6}
\]

As negative dividend is not acceptable, one has to consider previous quotient as 5 but not as 6

\[
7x^2 + 2x + 1) 43x^2 + 2x (5
\]

\[
\begin{bmatrix}
35x^2 + 10x \\
8x^2 + 2x \quad -10x \\
x(8x + 2)
\end{bmatrix} \quad \cdot \quad (3)
\]

\[
= x(82) - 10x
\]

\[
= 72x
\]

\[
D_1 \quad D_2
\]

\[
\begin{bmatrix}
2x & 1 \\
6 & 0
\end{bmatrix}
\]

\[
= 10x
\]

\[
Q_3 \quad Q_6
\]

Now multiplying new dividend also by 10, we get \(72x^2\)

Again dividing by the divisor;

\*Intermediate remainder
Vedic Mathematics

Division

\[ 7x^2 + 2x + 1 \hspace{1cm} 1 \hspace{1cm} 72x^2 \]

\[ \frac{70x^2 + 20x + 5x}{2x^2 - 20x - 5x} \]

\[ = x(2x) - 20x - 5x \hspace{1cm} (4) \]

\[ = 20x - 25x \]

\[ = -5x \]

This remainder is also discarded. Hence the quotient should be 9. Proceeding further with the quotient:

\[ 7x^2 + 2x + 1 \hspace{1cm} 1 \hspace{1cm} 72x^2 \]

\[ \frac{63x^2 + 18x + 5x}{9x^2 - 18x - 5x} \]

\[ = x(9x) - 23x \hspace{1cm} (5) \]

\[ = 90x - 23x \]

\[ = 67x \hspace{1cm} (R_4) \]

Further proceeding we get 9 as next quotient:

\[ 7x^2 + 2x + 1 \hspace{1cm} 1 \hspace{1cm} 67x^2 \]

\[ \frac{63x^2 + 18x + 9x}{4x^2 - 18x - 9x} \]

\[ = x(4x) - 18x - 9x \hspace{1cm} (6) \]

\[ = 40x - 27x \]

\[ = 13x \hspace{1cm} (R_4) \]

\[ 7x^2 + 2x + 1 \hspace{1cm} 1 \hspace{1cm} 13x^2 \]

\[ \frac{7x^2 + 2x + 9x}{6x^2 - 2x - 9x} \]

\[ = x(6x) - 11x \hspace{1cm} (7) \]

\[ = 60x - 11x = 49x \hspace{1cm} (R_3) \]

\[ 7x^2 + 2x + 1 \hspace{1cm} 1 \hspace{1cm} 49x^2 \]

\[ \frac{42x^2 + 12x + x}{7x^2 - 12x - x} \]

\[ = x(7x) - 13x \hspace{1cm} (8) \]

\[ = 70x - 13x = 57x \]

In this process also one can work out the decimal points of the division as per one’s choice.

One can extend the procedure to any number of digits in Dhawajanka for multiplication or as a matter of fact even to any number of digits, which are considered to be active in the division part. In this method it appears that the entire problem is divided into different working units applying simple division, simple multiplication and also the Urdhva Tiryak multiplication. Each time getting the value of the quotient and the corresponding remainder, an intermediate dividend, new dividend, followed by the corrected dividend, if necessary. With the help of these, the process is continued.
Example 8: $549876 \div 1246$ (Division where the Dhrajanka has two digits and Part divisor has two digits and the answer is represented as quotient and remainder and continued up to 6 decimals in the quotient) (Up to 6 decimal places)

Current Method

1246) 549876 (441.313001

4984
5147
4984
1636
1246
3900
3738
1620
1246
3740
3738
2000
1246
754

Vedic Method

\[
\begin{array}{cccccccccc}
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
\end{array}
\]

Quotient = 441

Remainder =

\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
4 & 6 & 10 - \binom{6}{1} \\
Q_2 & Q_3 \\
\end{array}
\]

$= 676 - (280 + 6)$

$= 390$

Quotient in decimals = 441.313001

\[
\begin{align*}
Q &= 441 \\
R &= 390
\end{align*}
\]

We can consider even more than two digits in the Dhrajanka, in which case some of the steps deal with three-digit multiplications with the quotients. (or even more depending on the digits in the Dhrajanka)

The following example shows the details. In the example given below the fourth step is the remainder step. Here we come across three-digit multiplications together with two-digited and one-digited, giving the final remainder as 154 and quotient 288.
Example 9: 985342 + 4321 (Division where Dhwajanka has three digits and the answer is represented as quotient and remainder.)

Current Method

\[
\begin{array}{c}
4321) 985342 (228 \\
8642 \\
12114 \\
8642 \\
34722 \\
34568 \\
154
\end{array}
\]

Vedic Method

\[
\begin{array}{c|c|c|c|c|c|c}
D_1 & D_2 & D_3 & 9 & 8 & 5 & 3 \\
3 & 2 & 1 & 4 & 4 & 2 \\
\hline
2 & 2 & 8 & 228 & Q_1 & Q_2 & Q_3 \\
\hline
\end{array}
\]

Vedic Method Steps:

Step 1:

4) 9 (2 (Q_1))

\[
\begin{array}{c}
\frac{8}{1} (R_1) \\
Q_1 = 2
\end{array}
\]

Step 2:

(ID) 18 - \[
\begin{array}{c}
D_1 \\
3 \uparrow \\
4 \uparrow \\
2 \\
Q_1
\end{array}
\]

= 18 - 6 = 12 (ND)

4) 12 (3 (Q_2))

\[
\begin{array}{c}
\frac{12}{0} (R_2) \\
Q_2 = 0
\end{array}
\]

Step 3:

(ID) 5 - \[
\begin{array}{c}
D_1 \\
3 \uparrow \\
2 \\
Q_1
\end{array}
\]

\[
\begin{array}{c}
D_2 \\
2 \uparrow \\
3 \\
Q_2
\end{array}
\]

= 5 - 4 - 9 = -8 (negative value)

\[\therefore \text{We reduce quotient } Q_2 \text{ by 1.}\]

4) 12 (2 [Q_2(m)])

\[
\begin{array}{c}
\frac{8}{4} [R_2(m)] \\
Q_2(m) = 2
\end{array}
\]

Step 3:

\[
\begin{array}{c}
4 \uparrow \frac{8}{2} \\
\frac{8}{0}
\end{array}
\]

Quotient = 228

Remainder = 154
(D) 45 - \[
\begin{bmatrix}
\begin{array}{cc}
D_1 & D_2 \\
3 & 2
\end{array}
\end{bmatrix}
\]
\[
\begin{array}{c}
Q_1 \\
Q_2 \\
Q_3(m)
\end{array}
\]

= 45 - 6 = 35 (ND)

\[
\begin{array}{c}
\frac{35}{22} \\
\frac{3}{3}
\end{array}
\]

Q_3 = 8

sp 4: Remainder step

\[
\begin{array}{c}
\begin{array}{c}
D_1 D_2 D_3 \\
3 2 2
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
D_1 D_2 D_3 \\
2 1 2
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
D_1 D_2 D_3 \\
1 2 8
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
D_1 D_2 D_3 \\
1 1 8
\end{array}
\end{array}
\]

Vinculum: (Direct)

\[
\begin{array}{c}
\begin{array}{c}
3 2 1 \\
9 8 5
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
4 \\
1 0 1
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
2 3 \\
2 0
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
3 6 \\
3 6
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
5 0 \\
6 0
\end{array}
\end{array}
\]
\[
\begin{array}{c}
\begin{array}{c}
1 \\
\overline{1}
\end{array}
\end{array}
\]

Q = 228 0356399

Quotient = 228, Remainder = 154

\text{vof is given below:}

\[
\begin{align*}
3 + 3x^2 + 2x + 1 & \mid 9x^3 + 8x^2 + 5x^3 + 3x^2 + 4x + 2 - 2(x^2 + 2x + 8) \\
8x^3 + 6x^4 & \quad x^3 + 8x^3 \quad - 6x^4 \quad \rightarrow (1) \\
= x^3(x + 8) & \quad - 6x^4 \\
= 18x^4 - 6x^4 & \quad 2x^2 \\
12x^4 + 5x^3 & \quad Q_1 \\
6x^4 + 6x^3 + 4x^3 & \quad 3x^2 2x^2 \\
4x^4 + 5x^3 - 10x^3 & \quad Q_1 Q_2 \\
= x^3(4x + 5) & \quad 6x^3 + 4x^3 \quad \rightarrow (2) \\
= 45x^3 - 10x^3 & \quad 2x^4 2x \\
35x^3 + 3x^3 + 4x + 2 & \quad Q_1 Q_2 \\
32x^4 + 24x^3 + 4x^2 + 2x^2 + 16x + 2x + 8 & \quad Q_1 Q_2 \\
(3x^3 + 3x^2 + 4x + 2) & \quad 16x - 2x - 8 \quad \rightarrow (5) \\
= 3342 - 3000 - 180 - 8 & \quad (3) \\
= 3342 - 3188 = 154 & \quad (4) \\
\end{align*}
\]

\[
\begin{align*}
3x^2 2x 1 & \quad 24x^2 + 4x^2 + 2x^2 \quad \rightarrow (4) \\
2x^2 2x 8 & \quad 2x 1 \\
\quad & \quad 2x 8 \\
Q_1 Q_2 Q_3 & \quad Q_2 Q_3 \\
\end{align*}
\]

\text{An example for decimal working where Dhwajanka contains three digits is given below. The proof for this is also given.}

* Intermediate Remainder
**Example 10:** 89124 + 5378 (Division where Dhwanjanka has three digits and the answer is represented as quotient and remainder and is continued to decimals in the quotient.)

**Vedic Method**

<table>
<thead>
<tr>
<th>89124</th>
<th>5378</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccc}
R_1 & R_2 & R_3 & R_4 & R_5 & R_6 \\
\hline
16 & 5 & 7 & 1 & 9 \\
\end{array}
\]

Quotient = 16

Remainder =

\[
\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
\hline
6124 - 2500 - 500 - 48 \\
= 6124 - 3048 = 3076 \\
Quotient in decimals = 16.5719
\end{array}
\]

**Vedic Method Steps:**

**Step 1:**

5) \(8 (Q_1)\)

\[
\begin{array}{c}
5 \div 8 = 0 \text{ remainder } 5 \\
\end{array}
\]

\[Q_1 = 1\]

**Step 2:**

\[
\begin{array}{c}
(100 - 12 + 6) = 36 \text{ (ND)} \\
\end{array}
\]

**Step 3:**

5) \(6 (Q_2)\)

\[
\begin{array}{c}
25 \div 6 = 4 \text{ remainder } 1 \\
\end{array}
\]

\[Q_2 = 4\]

\[
\begin{array}{c}
(100 - 21 - 7) = -17 \text{ (negative value)} \\
\end{array}
\]

We reduce quotient by 1.

**Step 3:**

5) \(17 (Q_3)\)

\[
\begin{array}{c}
15 \div 2 = 7 \\
\end{array}
\]

\[Q_3 = 7\]
5) 36 \(6 [Q_2(m)]\)
\[
\begin{array}{c}
36 \\
30 \\
6 [R_3(m)]
\end{array}
\]
\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
3 & 7 & 8 \\
1 & 6 & 7 \\
Q_1 & Q_2 & Q_3
\end{array}
\]
ID) 61 - 18 = 36 (ND)
\[Q_3(m) = 6\]

5) 36 \(7 [Q_3(m)]\)
\[
\begin{array}{c}
35 \\
25 \\
1 [R_3]
\end{array}
\]
Step 4:
\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
3 & 7 & 8 \\
1 & 6 & 7 \\
Q_1 & Q_2 & Q_3
\end{array}
\]
ID) 12 - 21 = -59 (negative value)
\[
\frac{22}{22} - \\ \frac{48}{8} = 66
\]
\[Q_3(m) = 5\]

\[\therefore\text{We reduce quotient by 1.}\]

5) 36 \(6 [Q_3(m)]\)
\[
\begin{array}{c}
36 \\
30 \\
6 [R_3(m)]
\end{array}
\]
\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
3 & 7 & 8 \\
1 & 6 & 6 \\
Q_1 & Q_2 & Q_3
\end{array}
\]
(ID) 62 - 18 = -6 (negative value)
\[\therefore\text{We reduce the quotient further.}\]

5) 36 \(5 [Q_3(m)]\)
\[
\begin{array}{c}
25 \\
11 [R_3(m)]
\end{array}
\]
\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
3 & 7 & 8 \\
1 & 6 & 5 \\
Q_1 & Q_2 & Q_3
\end{array}
\]
(ID) 112 - 15 = 47 (ND)

5) 47 \(9 [Q_4]\)
\[
\begin{array}{c}
45 \\
2 (R_4)
\end{array}
\]
Step 5:

\[
\begin{align*}
\text{(ID) } 24 - \begin{pmatrix} 3 & 7 & 7 \\ 6 & 6 & 8 \\ Q_2 & Q_3 & Q_4 \end{pmatrix} &= 24 - 27 - 48 - 35 = -86 \\
&= 24 - (1 \times 7) - (1 \times 7) - (1 \times 7) = -86
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 74 - \begin{pmatrix} 3 & 7 & 7 \\ 6 & 6 & 8 \\ Q_2 & Q_3 & Q_4 \end{pmatrix} &= 74 - 24 - 48 - 35 = -33
\end{align*}
\]

\[\text{We reduce quotient by 1.}\]

5) 47 (8 [Q₄(m)])

\[
\begin{align*}
40 & \\
7 & R_4 (m)
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 74 - \begin{pmatrix} 3 & 7 & 7 \\ 6 & 6 & 8 \\ Q_2 & Q_3 & Q_4 \end{pmatrix} &= 74 - 24 - 48 - 35 = -33
\end{align*}
\]

(We reduce the quotient further.)

5) 47 (7 [Q₄(m)])

\[
\begin{align*}
35 & \\
12 & R_4 (m)
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 124 - \begin{pmatrix} 3 & 7 & 7 \\ 6 & 6 & 7 \\ Q_2 & Q_3 & Q_4 \end{pmatrix} &= 124 - 21 - 48 - 35 = 20 \text{ (ND)}
\end{align*}
\]

5) 20 (4 [Q₃])

\[
\begin{align*}
20 & \\
0 & R_3
\end{align*}
\]

Step 6:

\[
\begin{align*}
\text{(ID) } 0 - \begin{pmatrix} 3 & 7 & 9 \\ 5 & 4 & 4 \\ Q_3 & Q_4 & Q_5 \end{pmatrix} &= 0 - 12 - 40 - 49 = -101
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 20 - \begin{pmatrix} 3 & 7 & 8 \\ 3 & 11 & 9 \\ Q_3 & Q_4 & Q_5 \end{pmatrix} &= 20 - (1 \times 7) - (1 \times 7) = -101
\end{align*}
\]
Vedic Mathematics

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Division

we reduce the quotient by 1.

5) 20 (3 \{Q_3 (m)\})

\[
\begin{array}{c}
15 \\
5 \{R_5 (m)\}
\end{array}
\]

5) 95 (19 \{Q_6\})

\[
\begin{array}{c}
95 \\
0 \{R_6\}
\end{array}
\]

D) 50 - \[
\begin{array}{c}
D_1 \\
5 \\
Q_3 \\
(m)
\end{array}
\]

D) 98 - \[
\begin{array}{c}
D_1 \\
95 \\
Q_5 \\
(m)
\end{array}
\]

= 50 - 9 - 40 - 49 = -48 (negative value)

5) 20 (2 \{Q_3(m)\})

\[
\begin{array}{c}
10 \\
10 \{R_5 (m)\}
\end{array}
\]

D) 100 - \[
\begin{array}{c}
D_1 \\
5 \\
Q_5 \\
(m)
\end{array}
\]

D) 100 - \[
\begin{array}{c}
D_1 \\
5 \\
Q_5 \\
(m)
\end{array}
\]

= 100 - 6 - 40 - 49 = 5 (ND)

5) 5 (1 \{Q_6\})

\[
\begin{array}{c}
5 \\
0 \{R_6\}
\end{array}
\]

p 7:

\[
\begin{array}{c}
D_1 \\
3 \\
Q_4 \\
(m)
\end{array}
\]

\[
\begin{array}{c}
D_1 \\
7 \\
Q_6 \\
(m)
\end{array}
\]

ID) 0 - \[
\begin{array}{c}
D_1 \\
7 \\
Q_6 \\
(m)
\end{array}
\]

= 0 - 3 - 14 = -56 = -73 (negative value)

\[
\begin{array}{c}
D_1 \\
3 \\
Q_4 \\
(m)
\end{array}
\]

\[
\begin{array}{c}
D_1 \\
7 \\
Q_6 \\
(m)
\end{array}
\]

\[
\begin{array}{c}
D_1 \\
3 \\
Q_4 \\
(m)
\end{array}
\]

We reduce quotient by 1.

5) 5 (0 \{Q_6(m)\})

\[
\begin{array}{c}
5 \\
0 \{R_6 (m)\}
\end{array}
\]

ID) 50 - \[
\begin{array}{c}
D_1 \\
3 \\
Q_4 \\
(m)
\end{array}
\]

ID) 50 - \[
\begin{array}{c}
D_1 \\
7 \\
Q_6 \\
(m)
\end{array}
\]

= 50 - 0 - 14 - 56 = -20 (negative value)
Q₆ₐ(m) in this procedure of reduction, cannot be further reduced because as the reduced quotient leads to -'ve value. Hence we have to go back to immediate quotient Q₅ and reduce it by 1, i.e., Q₅(m) = 1

:. We reduce the Q₅ value 2 by 1.

5) 20 [Q₅(m)]

\[
\begin{array}{c}
5 \\
15 \text{[R₅(m)]}
\end{array}
\]

\[
\begin{array}{ccc}
D₁ & D₂ & D₃ \\
3 & 7 & 8 \\
5 & 7 & 1
\end{array}
\]

(ID) 150 - \[
\begin{array}{c}
3 \\
7 \\
8
\end{array}
\] = 150 - 3 - 49 - 40 = 58 (ND)

We continue the calculations to get the value of Q₆ also.

5) 58(11) [Q₆(m)]

\[
\begin{array}{c}
55 \text{[R₆(m)]}
\end{array}
\]

\[
\begin{array}{ccc}
D₁ & D₂ & D₃ \\
3 & 7 & 8 \\
7 & 1 & 1
\end{array}
\]

30 - \[
\begin{array}{c}
3 \\
7 \\
8
\end{array}
\] = 30 - 33 - 56 - 78 = -66

Q₆ₐ(m) is further reduced by 1.

5) 58(10)

\[
\begin{array}{c}
50 \\
8
\end{array}
\]

Step 8:

\[
\begin{array}{ccc}
D₁ & D₂ & D₃ \\
3 & 7 & 8 \\
7 & 1 & 10
\end{array}
\]

\[
\begin{array}{c}
Q₄ \quad Q₅ \quad Q₆ \\
(m) \quad (m) \quad (m)
\end{array}
\]
\[
\begin{align*}
\text{Vedic Mathematics} & \quad \text{Division} \\
\text{We reduce the value of } Q_6 	ext{ by } 1 \text{ i.e. } Q_6 &= 9 \\
508(9) & \quad Q_6(m) \\
\text{Quotient} &= 16.5719 \\
\text{Proof:} & \\
5x^4 + 3x^2 + 7x + 8 & \quad 8x^4 + 9x^2 + x^2 + 2x + 4 \quad (x + 6) \quad 5 \quad 7 \quad 1 \\
\text{First Division:} & \\
5x^4 + 3x^2 & \quad 3x^2 \\
3x^4 + 9x^2 & \quad 3x^2 \quad (1) \\
& \quad = x^2(3x + 9) - 3x^2 \\
& \quad = 36x^2 \\
& \quad 36x^2 + x^2 \\
20x^3 + 18x^2 + 7x & \quad 6x^2 + x^2 - 18x^2 - 7x^2 \quad (2) \\
& \quad 18x^2 + 7x^2 \\
= x^2(6x + 1) - 25x^2 & \quad 61x^2 - 25x^2 \\
& \quad = 36x^2 \\
= 36x^2 + 2x & \quad \text{Remainder region} \\
36x^2 + 2x & \quad 3x^2 \quad 7x \quad 8 \\
& \quad 25x^3 + 15x^2 + 42x^2 + 8x^2 \\
& \quad 11x^4 + 2x^2 - 15x^2 - 42x^2 - 8x^2 \quad (3) \\
& \quad = x^2(11x + 2) - 65x^2 \\
& \quad = 112x^2 - 65x^2 \\
& \quad = 47x^2 \\
& \quad 47x^2 + 4x \\
& \quad 47x^2 + 4x^2 \\
& \quad 35x^2 + 21x^2 + 25x^2 + 48x^2 \\
& \quad 12x^4 + 4x^2 - 21x^2 - 35x^2 - 48x^2 \quad (4) \\
= x^2(12 + 4) - 104x^2 \\
& \quad = 124x^2 - 104x^2 \\
& \quad = 20x^2 \\
& \quad 5x^2 + 3x^2 + 49x^2 + 40x^2 \\
& \quad 15x^2 - 3x^2 - 49x^2 - 40x^2 \quad (5) \\
& \quad = x^2(15x) - 92x^2 \\
& \quad = 150x^2 - 92x^2 = 58x^2 \\
& \quad 36x^2 + 2x + 4 \\
& \quad + 42x + 8x + 48 \\
& \quad 36x^2 + 2x + 4 - 42x - 8x - 48 = 36x^2 - 48x - 44
\end{align*}
\]
Vedic Mathematics

Division

For remainder:

\[
\begin{align*}
D_1 & \quad D_2 D_3 & \quad Q_1 & \quad Q_2 & \quad \frac{5x^3 + 3x^2 + 7x + 8}{5x^4 + 3x^3} \\
& & & & = x^2(3x + 9) - 3x^3 \\
& & & & \text{Remainder} \quad = 6x^3 + x^2 + 2x + 4 - 18x^2 - 7x^2 - 42x - 8x - 48 \\
& & & & \text{Remainder} \quad = 6124 - 2500 - 500 - 48 \\
& & & & \text{Remainder} \quad = 6124 - 3048 = 3076
\end{align*}
\]

\[
\begin{align*}
D_1 & \quad D_2 & \quad D_3 & \quad Q_1 & \quad Q_2 & \quad \frac{5x^3 + 3x^2 + 7x + 8}{3x^2 7x 8} \\
& & & & = 18x^2 + 7x^2 \\
& & & & \text{Remainder} \quad = 48 \\
\end{align*}
\]

Decimal points from the remainder: 

\[
\begin{align*}
D_1 & \quad D_2 D_3 & \quad Q_3 & \quad Q_4 & \quad Q_5 & \quad \frac{5x^3 + 3x^2 + 7x + 8}{30x^2 + 7x^2 + 6x} \\
& & & & = 15x^2 \\
& & & & \text{Remainder} \quad = 42x^2 \\
& & & & \text{Remainder} \quad = 21x^2 + 35x^2 \\
& & & & \text{Remainder} \quad = 76x^2 - 56x^2 \\
& & & & \text{Remainder} \quad = 20x^2
\end{align*}
\]
**Example 11:** 789421 + 10321 (Division where the Dhwajanka has three digits and the part divisor has two digits and the answer is represented as quotient and remainder.)

**Current Method**

<table>
<thead>
<tr>
<th>10321</th>
<th>789421 (76)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72247</td>
<td></td>
</tr>
<tr>
<td>66951</td>
<td></td>
</tr>
<tr>
<td>61926</td>
<td></td>
</tr>
<tr>
<td>5025</td>
<td></td>
</tr>
</tbody>
</table>

**Vedic Method**

<table>
<thead>
<tr>
<th>321</th>
<th>78 9 : 4 2 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8 : 8</td>
</tr>
<tr>
<td></td>
<td>R1 R2</td>
</tr>
<tr>
<td>7  6 :</td>
<td>Q1 Q2</td>
</tr>
</tbody>
</table>

Quotient = 76

Remainder =

\[
8421 - \left( \frac{3 \ 2 \ 1}{0 \ 7 \ 6} \right) 100 - \left( \frac{2 \ 1}{7 \ 6} \right) 10 - \frac{1}{Q1 Q2 Q3}
\]

\[
R = 5025
\]

**Example 12:** 6543 + 89798 (Division where the Dhwajanka has four digits.)

**Current Method**

<table>
<thead>
<tr>
<th>89798) 654300 (0.0728</th>
</tr>
</thead>
<tbody>
<tr>
<td>628586</td>
</tr>
<tr>
<td>237140</td>
</tr>
<tr>
<td>172596</td>
</tr>
<tr>
<td>775440</td>
</tr>
<tr>
<td>718384</td>
</tr>
<tr>
<td>570556</td>
</tr>
</tbody>
</table>

**Vedic Method**

<table>
<thead>
<tr>
<th>9798</th>
<th>6 5 4 3 0 → Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>6 9 15 22 3 → Remainders</td>
</tr>
<tr>
<td></td>
<td>R1 R2 R3 R4 R5 (m) (m)</td>
</tr>
<tr>
<td></td>
<td>Q1 Q2 Q3 Q4 Q5 (m) (m)</td>
</tr>
</tbody>
</table>

Quotient = 0.07286

Refer 4 (2) - Straight Division Page: 12

**Vedic Method Steps:**

Step 1:
8) 6 (0 (Q1))

5 (Ri)
Vedic Mathematics

Step 2:

\[
\begin{align*}
&\begin{array}{c}
9 \\
0
\end{array} = 65 \text{ (ND)} \\
&\begin{array}{c}
65 \\
1
\end{array} \quad (Q_1)
\end{align*}
\]

8) 65 (Q_2)

\[
\begin{array}{c}
64 \\
1
\end{array} \quad (R_3)
\]

Step 3:

\[
\begin{align*}
&\begin{array}{c}
9 \\
0
\end{array} = 14 - 72 = -58 \text{ (negative value)} \\
&\begin{array}{c}
Q_1 \\
Q_2
\end{array} \quad (Q_3)
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
56 \\
9
\end{array} \quad (R_3)
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
94 \\
0
\end{array} = 94 - 63 = 31 \text{ (ND)} \\
&\begin{array}{c}
Q_1 \\
Q_2
\end{array} \quad (m)
\end{align*}
\]

8) 31 (Q_3)

\[
\begin{array}{c}
24 \\
7
\end{array} \quad (R_3)
\]

Step 4:

\[
\begin{align*}
&\begin{array}{c}
73 \\
0
\end{array} = 73 - 76 = -3 \\
&\begin{array}{c}
Q_1 \\
Q_2 \quad Q_3
\end{array} \quad \text{ (negative value)} \quad (m)
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
23 \\
0
\end{array} = 23 - 10 = 13 \\
&\begin{array}{c}
Q_1 \\
Q_2 \quad Q_3
\end{array} \quad (m)
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
16 \\
15
\end{array} \quad Q_3 \text{ (m)} \quad 2 \quad (R_4)
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
153 \\
0
\end{array} = 153 - 67 = 86 \text{ (ND)} \\
&\begin{array}{c}
Q_1 \\
Q_2 \quad Q_3
\end{array} \quad (m)
\end{align*}
\]
8) 86 (10 (Q₄)
80
6 (R₄)
Step 5:

\[
\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
9 & 7 & 9 & 8 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
(m) & (m) & (m) & (m)
\end{array}
\]

(ID) \ 60 - \ \begin{array}{cccc}
0 & 7 & 2 & 10 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
\end{array}
\]
\[= 60 - 167 = -107 \quad \begin{array}{cccc}
D_1 & D_2D_3 & D_4 \\
9 & 7 & 9 & 8 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
\end{array}
\]

\[\text{Step 5:} \quad 20 - \begin{array}{cccc}
0 & 8 & 7 & 1 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
\end{array}
\]

\[\text{\text{negative value}} \quad \cdot \frac{34}{32} \quad \frac{8}{2} (Q₃)
\]

\[\text{We reduce the quotient by 1.} \]

8) 86 (9 [Q₄(m)]
72
14 R₄ (m)

\[\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
9 & 7 & 9 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
(m) & (m) & (m) & (m)
\end{array}
\]

(ID) \ 140 - \ \begin{array}{cccc}
0 & 7 & 2 & 9 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
\end{array}
\]
\[= 140 - 158 = -18 \quad \text{\text{negative value}}
\]

\[\text{\text{We reduce the quotient further.}}
\]

8) 86 (8 [Q₄(m)]
64
22 R₄ (m)

\[\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
9 & 7 & 9 & 8 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
(m) & (m) & (m) & (m)
\end{array}
\]

(ID) \ 220 - \ \begin{array}{cccc}
0 & 7 & 2 & 8 \\
Q_1 & Q_2 & Q_3 & Q_4 \\
\end{array}
\]
\[= 220 - 149 = 71 \quad \text{\text{ND}}
\]

8) 71 (8 (Q₃)
64
7 (R₃)

\[Q₃ = 8
\]

\[\text{\text{Quotient = 0.0728}}
\]

\[\begin{array}{cccc}
9 & 7 & 9 & 8 \\
\end{array}
\]

\[Vinculum:
\]
\[\begin{array}{cccc}
. & 6 & 5 & 4 \\
\end{array}
\]
\[\begin{array}{cccc}
3 & 0 & 0 & \\
\end{array}
\]
\[\begin{array}{cccc}
8 & 6 & 1 & 2 \\
\end{array}
\]
\[\begin{array}{cc}
2 & \end{array}
\]
\[\begin{array}{cccc}
0 & 8 & 7 & 1 \\
\end{array}
\]
\[\begin{array}{cccc}
4 & \end{array}
\]
\[Q = .08 \overline{71} \frac{4}{4}
\]
\[=.07286
\]

\[\text{We can do the above problem by using Vinculum in the divisor also}
\]
Vedic Mathematics

Current Method

6543 + 89798

89798 \( \rightarrow \) 657300 (.07286
628586
257140
179596
775440
718384
570560
538788
31772

Vedic Method using Vinculum in the Divisor

\[ \begin{array}{c}
89798 = 9 \overline{0} 2 \overline{0} 2 \\
\begin{array}{ccccccc}
0 & \overline{2} & 0 & \overline{2} & 6 & 5 & 4 & 3 & 0 & 0 \\
\end{array} \\
\begin{array}{ccccccc}
6 & 2 & 6 & 5 & 0 \\
\end{array} \\
\begin{array}{ccccccc}
R_1 & R_3 & R_3 & R_4 & R_5 \\
\end{array} \\
\begin{array}{ccccccc}
1 & 0 & 7 & 2 & 8 & 6 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \\
\end{array}
\end{array} \]

Quotient = 0.07286

Vedic Method Steps:

Step 1:
9) 6 (0 (Q_1)
0
6 (R_1)

Step 2:

(ID) 65 \( \rightarrow \uparrow \) = 65 (ND)

9) 65 (7 (Q_2)
63
2 (R_2)

Step 3:

(ID) 24 \( \rightarrow \) = 24 (ND)

9) 24 (2 (Q_3)
18
6 (R_3)

Step 4:

(ID) 63 \( \rightarrow \) = 63 \( \rightarrow \) (-14) = 77 (ND)
Vedic Mathematics

Step 5:

\[
\begin{align*}
\text{(ID) } 50 & - (\text{Q}_3 \cdot 4) = 54 \text{ (ND)} \\
& \text{Q}_1 \quad \text{Q}_2 \quad \text{Q}_3 \quad \text{Q}_4
\end{align*}
\]

9) 54 (Q_3)

\[
\begin{array}{c}
54 \\
0 \text{ (R}_3)
\end{array}
\]

Case (a): if the dividend has less number of digits than the Dhujajanka

Example 13: \[78 + 21345 \text{ (Up to 7 decimal places)}\]

Refer. 4 (2) – Straight Division for partition rules. Page:

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>21345) 78000 (0.0036542</td>
<td>1345</td>
</tr>
<tr>
<td>64035</td>
<td>7 8 0 0 0 0 0</td>
</tr>
<tr>
<td>139650</td>
<td>/ / / / / /</td>
</tr>
<tr>
<td>128070</td>
<td>1 3 5 7 8</td>
</tr>
<tr>
<td>115800</td>
<td>. 0 0 3 6 5 4 2</td>
</tr>
<tr>
<td>108725</td>
<td>Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7</td>
</tr>
<tr>
<td>90750</td>
<td>(m) (m) (m) (m)</td>
</tr>
<tr>
<td>85380</td>
<td></td>
</tr>
<tr>
<td>53700</td>
<td></td>
</tr>
<tr>
<td>42690</td>
<td></td>
</tr>
<tr>
<td>11910</td>
<td></td>
</tr>
</tbody>
</table>
Vedic Method Steps:

Step 1:
2) $7 \cdot (Q_3)$

\[ \frac{\text{6}}{1} \quad \text{(R3)} \]

Step 2:

\[
\begin{align*}
\text{(ID) } & 18 - \left\lfloor \frac{1}{3} \right\rfloor = 18 - 3 = 15 \quad \text{(ND)}
\end{align*}
\]

2) $15 \cdot (Q_4)$

\[ \frac{\text{14}}{1} \quad \text{(R4)} \]

Step 3:

\[
\begin{align*}
\text{ID} & 10 - \left\lfloor \frac{1}{3} \right\rfloor = 10 - 16 = -6 \quad \text{(negative value)}
\end{align*}
\]

\[\vdots \text{We reduce the quotient } Q_4 \text{ by 1.}\]

2) $15 \cdot (Q_4(m))$

\[ \frac{\text{12}}{3} \quad \text{(R4)} \]

\[
\begin{align*}
\text{(ID) } & 30 - \left\lfloor \frac{1}{3} \right\rfloor = 30 - 15 = 15 \quad \text{(ND)}
\end{align*}
\]

(or) 30 -

\[
\begin{align*}
\frac{1}{3} & \quad \text{(m)}
\end{align*}
\]

2) $15 \cdot (Q_5)$

\[ \frac{\text{14}}{1} \quad \text{(R5)} \]
Vedic Mathematics

Step 4:

\[
\begin{align*}
\text{(ID) } 10 - \begin{bmatrix}
D_1 & D_2 & D_3 \\
1 & 3 & 4 \\
3 & 6 & 7 \\
\end{bmatrix} &= 10 - 37 = -27 \text{ (negative value)} (\text{or}) 10 - \\
\text{Q}_3 & \text{Q}_4 & \text{Q}_5 \\
(m) & & \\
\end{align*}
\]

\[Q_3 \text{ by } 1.\]

2) 15 (6 [Q_3(m)]

\[
\begin{align*}
12 & \\
3 & (R_3) \\
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 30 - \begin{bmatrix}
D_1 & D_2 & D_3 \\
1 & 3 & 4 \\
3 & 6 & 6 \\
\end{bmatrix} &= 30 - 36 = -6 \text{ (negative value) (or) } 30 - \\
\text{Q}_3 & \text{Q}_4 & \text{Q}_5 \\
(m)(m) & & \\
\end{align*}
\]

\[Q_3 \text{ further by } 1.\]

2) 15 (5 [Q_3(m)]

\[
\begin{align*}
10 & \\
5 & [R_3 (m)] \\
\end{align*}
\]

\[
\begin{align*}
\text{(ID) } 50 - \begin{bmatrix}
D_1 & D_2 & D_3 \\
1 & 3 & 4 \\
3 & 6 & 5 \\
\end{bmatrix} &= 50 - 35 = 15 \text{ (ND)} \\
\text{Q}_3 & \text{Q}_4 & \text{Q}_5 \\
(m)(m) & & \\
\end{align*}
\]

2) 15 (7 (Q_3)

\[
\begin{align*}
14 & \\
1 & (R_4) \\
\end{align*}
\]

Step 5:

\[
\begin{align*}
\text{(ID) } 10 - \begin{bmatrix}
D_1 & D_2 & D_3 & D_4 \\
1 & 3 & 4 & 5 \\
3 & 6 & 5 & 7 \\
\end{bmatrix} &= 10 - 61 = -51 \text{ (negative value)} \\
\text{Q}_3 & \text{Q}_4 & \text{Q}_5 & \text{Q}_6 \\
(m) & (m) & & \\
\end{align*}
\]

\[Q_6 \text{ by } 1.\]
2) 15 (6 \([Q_6(m)]\))

\[
\begin{array}{c}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \\
\begin{array}{c}
1 \ 3 \ 4 \ 5 \\
3 \ 6 \ 5 \ 6 \\
Q_3 \ Q_4 \ Q_5 \ Q_6 \\
(m)(m)(m)(m)
\end{array}
\end{array}
\end{array}
\]

\((ID)\ 30 - 60 = -30\) (negative value)

\[\therefore \text{We reduce the quotient } Q_6 \text{ further by 1.}\]

2) 15 (5 \([Q_6(m)]\))

\[
\begin{array}{c}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \\
\begin{array}{c}
1 \ 3 \ 4 \ 5 \\
3 \ 6 \ 5 \ 5 \\
Q_3 \ Q_4 \ Q_5 \ Q_6 \\
(m)(m)(m)(m)
\end{array}
\end{array}
\end{array}
\]

\((ID)\ 50 - 59 = -9\) (negative value)

\[\therefore \text{We reduce the quotient } Q_6 \text{ still further by 1}\]

2) 15 (4 \([Q_6(m)]\))

\[
\begin{array}{c}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \\
\begin{array}{c}
1 \ 3 \ 4 \ 5 \\
3 \ 6 \ 3 \ 4 \\
Q_3 \ Q_4 \ Q_5 \ Q_6 \\
(m)(m)(m)(m)
\end{array}
\end{array}
\end{array}
\]

\((ID)\ 70 - 70 + 24 + 15 + 4 = 70 - 58 = 12\) (ND)

2) 12 (6 \([Q_7]\))

\[
\begin{array}{c}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \\
\begin{array}{c}
1 \ 3 \ 4 \ 5 \\
6 \ 5 \ 4 \ 6 \\
Q_4 \ Q_5 \ Q_6 \ Q_7 \\
(m)(m)(m)(m)
\end{array}
\end{array}
\end{array}
\]

\((ID)\ 0 - 0 = 0 \text{ (negative value)}\)

\[\text{Step 6:}\]
.: We reduce the quotient \( Q_7 \) by 1

2) \( 12 \) (5 [\( Q_7 \) (m)]

\[
\begin{array}{c}
10 \\
2 \\
\end{array}
\]

\[
[R_7 \text{ (m)}]
\]

\[
\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
1 & 3 & 4 & 5 \\
6 & 4 & 5 & 6 \\
\end{array}
\]

(ID) \( 20 - \left( 30 + 20 + 12 + 5 \right) = 20 - 67 = -47 \) (negative value)

\[
Q_4 \ Q_3 \ Q_2 \ Q_7 \\
(m) (m) (m) (m)
\]

.: We reduce the quotient \( Q_7 \) further by 1.

2) \( 12 \) (4 [\( Q_7 \) (m)]

\[
\begin{array}{c}
8 \\
4 \\
\end{array}
\]

\[
[R_7 \text{ (m)}]
\]

\[
\begin{array}{cccc}
D_2 & D_3 & D_4 \\
3 & 4 & 5 \\
6 & 4 & 5 \\
\end{array}
\]

(ID) \( 40 - \left( 30 + 20 + 12 + 4 \right) = 40 - 66 = -26 \) (ND)

\[
Q_4 \ Q_3 \ Q_2 \ Q_7 \\
(m)(m) (m) (m)
\]

.: We reduce the quotient \( Q_7 \) further still by 1

2) \( 12 \) (3 [\( Q_7 \) (m)]

\[
\begin{array}{c}
6 \\
6 \\
\end{array}
\]

\[
[R_7 \text{ (m)}]
\]

\[
\begin{array}{cccc}
D_1 & D_2 & D_3 & D_4 \\
1 & 3 & 4 & 5 \\
6 & 5 & 4 & 3 \\
\end{array}
\]

(ID) \( 60 - \left( 30 + 20 + 12 + 3 \right) = 60 - 65 = -5 \) (negative value)

\[
Q_4 \ Q_3 \ Q_2 \ Q_7 \\
(m)(m)(m)(m)
\]

.: We reduce the quotient \( Q_7 \) further still more by 1.

2) \( 12 \) (2 [\( Q_7 \) (m)]

\[
\begin{array}{c}
4 \\
8 \\
\end{array}
\]

\[
[R_7 \text{ (m)}]
\]
Vedic Mathematics

Division

Vinculum

\[
\begin{align*}
\text{ID} & \quad 80 - 1 \cdot 3 \cdot 4 \cdot 5 = 80 - 64 = 16 (\text{ND}) \quad 1 \ 3 \ 4 \ 5 \\
Q_4 \ Q_3 \ Q_2 \ Q_1 & \quad 7 \ 8 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\
(\text{m}) \ (\text{m}) \ (\text{m}) \ (\text{m}) & \\
\end{align*}
\]

Case (b)(i): Where the dividend only has decimals
Refer Example: 14 Page No.
for Vinculum working details

Example 14: 89.69 + 243
(Up to 4 decimal places)

Current Method

\[
\begin{align*}
24300) \quad & 89690 \ (0.3690946 \\
& 72900 \\
& 167900 \\
& 145800 \\
& 221000 \\
& 218700 \\
& 230000 \\
& 218700 \\
& 113000 \\
& 97200 \\
& 158000 \\
& 145800 \\
& 122000 \\
\end{align*}
\]

Vedic Method

\[
\begin{align*}
43 \quad & 8 \ 9 \ 6 \ 9 \ 0 \ 0 \\
& / \ / \ / \ / \ / \\
2 \quad & 2 \ 5 \ 5 \ 5 \ 5 \\
R_1 \ R_2 \ R_3 \ R_4 \ R_5 & \\
(\text{m}) (\text{m}) (\text{m}) (\text{m}) (\text{m}) \\
& .3 \ 6 \ 9 \ 0 \ 0 \\
Q_1 \ Q_2 \ Q_3 \ Q_4 \ Q_5 & \\
\end{align*}
\]

Quotient = 0.36909

Vedic Method Steps:

Step 1:

2) 8 (Q_1)
8
0 (R_1)
Vedic Mathematics

Division

Step 2:

\[ D_1 \]

(ID) \( 9 - \left\lfloor \frac{4}{3} \right\rfloor = 9 - 16 = -7 \) (negative value)

2) \( \frac{7}{3} (Q_2) \)

\( \frac{1}{R_2} \)

.: We reduce the quotient by 1.

2) \( 8 (Q_3) \)

\( \frac{6}{2} R_4 (m) \)

\[ D_1 \]

(ID) \( 29 - \left\lfloor \frac{4}{3} \right\rfloor = 29 - 12 = 17 \) (ND)

\( Q_1(m) \)

2) \( 17 (Q_3) \)

\( \frac{16}{1} \) (R_3)

Step 3:

\[ D_1 \quad D_2 = 16 - [9 + 32] \]

(ID) \( 16 - \left\lfloor \frac{4}{3} \right\rfloor = 16 - 41 = -25 \) (negative value)

\( Q_1 \quad Q_2 \)

\( (m) \)

.: We reduce the quotient by 1.

2) \( 17 (Q_3) \)

\( \frac{14}{3} R_5(m) \)

2) \( \frac{4}{2} (Q_5) \)

\( \frac{4}{0} \) (R_5)
Vedic Mathematics

\[
\begin{align*}
(D) \ 36 - \left( \begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
3 & 7 \\
\end{array} \right) &= 36 - [9 + 28] \\
Q_1 & Q_2 \\
(m) & (m)
\end{align*}
\]

\[
36 - 37 = -1 \text{ (negative value)}
\]

\[\therefore \text{ We reduce the quotient further.}\]

2) \(17 \ [\text{Q}_2(m)]\)

\[
\begin{array}{c}
12 \\
\underline{5} \ R_2 \ (m)
\end{array}
\]

\[
\begin{align*}
(D) \ 56 - \left( \begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
3 & 6 \\
\end{array} \right) &= 56 - [9 + 24] \\
Q_1 & Q_2 \\
(m) & (m)
\end{align*}
\]

\[
56 - 33 = 23 \text{ (ND)}
\]

2) \(23 \ [\text{Q}_3]\)

\[
\begin{array}{c}
22 \\
\underline{1} \ (R_3)
\end{array}
\]

\[
\begin{align*}
\text{Step 4:} & \quad \text{Step 4}
\end{align*}
\]

\[
\begin{align*}
(D) \ 19 - \left( \begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
6 & 11 \\
\end{array} \right) &= 19 - [18 + 44] \\
Q_3 & Q_1 \\
(m) & (m)
\end{align*}
\]

\[
19 - 62 = 43 \text{ (negative value)}
\]

\[
\therefore \text{ We reduce quotient by 1.}
\]

2) \(23 \ [\text{Q}_3(m)]\)

\[
\begin{array}{c}
20 \\
\underline{3} \ R_3 \ (m)
\end{array}
\]

\[
\begin{align*}
\text{Step 4:} & \quad \text{Step 4}
\end{align*}
\]

\[
\begin{align*}
(D) \ 39 - \left( \begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
6 & 10 \\
\end{array} \right) &= 39 - [18 + 40] \\
Q_3 & Q_1 \\
(m) & (m)
\end{align*}
\]

\[
39 - 58 = -19 \text{ (negative value)}
\]

\[
\therefore \text{ We reduce the quotient further}
\]
2) 23 \( [Q_3(m)] \)
\[
\begin{array}{c}
18 \\
5 \quad \text{R}_3 \quad (m)
\end{array}
\]
\[Q_3 \quad (m) = 9\]

(ID) \[59 - \begin{array}{c}
4 \\
3 \\
9 \\
6 \\
\end{array}
\] = 59 - [18 + 36]
= 59 - 54 = 5 (ND)

2) 5 \( [Q_4(m)] \)
\[
\begin{array}{c}
4 \\
1 \quad \text{R}_4
\end{array}
\]
\[Q_4 = 2\]

Step 5:

(ID) \[10 - \begin{array}{c}
4 \\
3 \\
9 \\
2 \\
\end{array}
\] = 10 - [27 + 8]
= 10 - 35 = -25 (negative value)

\[\therefore \text{We reduce the quotient by 1}\]

2) 5 \( [Q_4(m)] \)
\[
\begin{array}{c}
2 \\
3 \quad \text{R}_4 \quad (m)
\end{array}
\]

(ID) \[30 - \begin{array}{c}
4 \\
3 \\
9 \\
1 \\
\end{array}
\] = 30 - [27 + 4]
= 30 - 31 = -1 (negative value)

\[\therefore \text{We reduce quotient further}\]

2) 5 \( [Q_4(m)] \)
\[
\begin{array}{c}
0 \\
5 \quad \text{R}_4 \quad (m)
\end{array}
\]

(ID) \[50 - \begin{array}{c}
4 \\
3 \\
9 \\
0 \\
\end{array}
\] = 50 - [27 + 0]
= 50 - 27 = 23 (ND)
2) 23 \( 11 \) \( Q_5 \)
\[
\begin{array}{c}
22 \\
1 \ (R_5)
\end{array}
\]

Step 6:
\[
\begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
0 & 11 \\
Q_4 & Q_3
\end{array}
\]

(ID) \( 10 - \) \( \begin{array}{cc} \hline 4 & 3 \\ 0 & 11 \hline \end{array} \) = 10 - \( [0 + 44] \) = 10 - 44 = -33 (negative value)

\[
\begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
13 & 23 \\
Q_4 & Q_3
\end{array}
\]

(ID) 0 - \( \frac{39 + 92}{33} \) = 53

\[ \therefore \text{We reduce the quotient by } 1. \]

2) 23 \( 10 \) \( [Q_5(m)] \)
\[
\begin{array}{c}
20 \\
3 \ R_5 \ (m)
\end{array}
\]

\[
\begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
0 & 10 \\
Q_4 & Q_3
\end{array}
\]

(ID) \( 30 - \) \( \begin{array}{cc} \hline 4 & 3 \\ 0 & 10 \hline \end{array} \) = 30 - 40 = -10 (negative value)

\[ \therefore \text{We reduce the quotient further.} \]

2) 23 \( 9 \) \( [Q_5(m)] \)
\[
\begin{array}{c}
18 \\
5 \ R_5 \ (m)
\end{array}
\]

\[
\begin{array}{cc}
D_1 & D_2 \\
4 & 3 \\
0 & 9 \\
Q_4 & Q_3
\end{array}
\]

(ID) \( 50 - \) \( \begin{array}{cc} \hline 4 & 3 \\ 0 & 9 \hline \end{array} \) = 50 - 36 = 14

Direct Vinculum

\[
\begin{array}{cccccccccccc}
4 & 3 & 8 & 9 & . & . & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
4 & 3 & 2 & 13 & 23 & 26 & 12 & 20 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 \\
\end{array}
\]

\[ \therefore \text{Quotient} = 0.36909 \]
Vedic Mathematics

Example 15: \(0.8927124 \times 96218734\) (Dividing with decimal and Dhawanaka has seven digits)

Current Method

\[
\frac{0.8927124}{96218734} = \frac{8927124}{962187340000000}
\]

\[
9621873400000000 \div 8927124000000000 \approx \frac{96218734}{89271240}
\]

\[
96218734 \div 89271240 = 0.0000000927794
\]

\[
865968606000000
\]
\[
267437940000000
\]
\[
1924374680000000
\]
\[
75004720000000000
\]
\[
673511138000000
\]
\[
764735820000000
\]
\[
67351138000000
\]
\[
912046820000000
\]
\[
865968606000000
\]
\[
460782140000000
\]
\[
384874936000000
\]
\[
759072040000000
\]

Division

Vedic Method

\[
\begin{array}{c}
\text{6218734} \\
\hline
9 \quad : \\
\hline
8 \quad 9 \quad 2 \quad 7 \quad 1 \quad 2 \quad 4 \quad 0 \\
\hline
9 \\
\hline
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 9 \quad 2 \quad 7 \quad 7 \quad 9 \quad 4 \\
\hline
Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7 Q_8 Q_9 Q_{10} Q_{11} Q_{12} Q_{13} Q_{14}
\end{array}
\]

Quotient = 0.0000000927794

Vinculum:

\[
\begin{array}{c}
\text{6218734} \\
\hline
9 \\
\hline
8 \quad 8 \quad 1 \quad 3 \quad 3 \quad 4 \quad 0 \\
\hline
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 9 \quad 3 \quad 2 \quad 1 \quad 10 \quad 5 \quad 2 \quad 2 \\
\hline
9 \quad 3 \quad 2 \quad 1 \quad 105 \quad 3
\end{array}
\]

Quotient = 0.000000092779478
Vedic Method Steps:

Step 1: \[ Q_1 \rightarrow Q_7 \text{(Seven zero)} \]

\[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(ID) 8 - \[
\begin{array}{cccccc}
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

= 8 (ND)

Step 2:

9) 8 (0 (Q_8))

\[
\begin{array}{c}
\scriptstyle 0 \\
\scriptstyle 8 \\
\end{array}
\]

(R_8)

Step 3:

\[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(ID) 89 - \[
\begin{array}{cccccc}
Q_1 & Q_2 & Q_3 & Q_6 & Q_4 & Q_5 & Q_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

= 89 (ND)

9) 89 (9 (Q_9)) \| Q_9 = 9

\[
\begin{array}{c}
\scriptstyle 81 \\
\scriptstyle 8 \text{ (R_9)} \\
\end{array}
\]

Step 4:

\[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 9 \\
\end{array}
\]

(ID) 82 - \[
\begin{array}{cccccc}
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

= 82 - 54 = 28 (ND)

9) 28 (3 (Q_{10})) \boxed{Q_{10} = 3}

\[
\begin{array}{c}
\scriptstyle 27 \\
\scriptstyle 1 \text{ (R_{10})} \\
\end{array}
\]
Vedic Mathematics

Step 5:

\[ \frac{D_1 D_2 D_3 D_4 D_5 D_6 D_7}{6 \quad 2 \quad 1 \quad 8 \quad 7 \quad 3 \quad 4} \]

\[
\text{(ID) } 17 - \quad \begin{array}{ccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 3 \\
Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10}
\end{array}
\]

\[ -17 \div 16 = 19 \text{ (negative value)} \]

\[
\frac{\text{We reduce the quotient by } 1}{1}
\]

9) \[ 28 \quad [Q_{10(m)}] \]

\[ \frac{18}{10} \quad [R_{10(m)}] \]

\[
\text{(ID) } 107 - \quad \begin{array}{ccccccc}
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10(m)}
\end{array}
\]

\[ = 107 - 30 = 77 \text{ (ND)} \]

9) \[ 77 \quad [Q_{11}] \]

\[ \frac{72}{5} \quad [R_{11}] \]

Step 6:

\[
\text{(ID) } 51 - \quad \begin{array}{ccccccc}
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10,11(m)}
\end{array}
\]

\[ = 51 - 61 = -10 \text{ (negative value)} \]

\[
\text{We reduce the quotient by } 1.
\]

9) \[ 77 \quad [Q_{11(m)}] \]

\[ \frac{63}{14} \quad [R_{11(m)}] \]

\[
\text{ID } 141 - \quad \begin{array}{ccccccc}
6 & 2 & 1 & 8 & 7 & 3 & 4 \\
Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10,11(m)}
\end{array}
\]

\[ = 141 - 55 = 86 \text{ (ND)} \]

\[ \frac{\text{Vinculum Continued.} \quad D_1 D_2 D_3 D_4 D_5 D_6 D_7}{6,} \]

\[ \begin{array}{ccccccc}
0 & 0 & 0 & 0 & 9 & 3 & 2/ \\
Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11}
\end{array}
\]

\[ = \frac{11 - [2 + 6 + 9]}{1} \]

\[ = 11 - [3] = 12 \]
Vedic Mathematics

9) $86 \ (Q_{12}) \ \ \ \ \ Q_{12} = 9$

\[
\begin{array}{c}
81 \\
5 \\
\end{array}
\begin{array}{c}
\text{(R}_{12})
\end{array}
\]

Step 7:

\[
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \ D_7 \\
0 \ 0 \ 0 \ 9 \ 2 \ 7 \ 9 \\
\text{(m) (m)}
\end{array}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \\
6 \ 2 \ 1 \ 8 \ 7 \ 3 \\
\text{(negative value)}
\end{array}
\begin{array}{c}
6 \\
0 \\
\text{(m) (m)}
\end{array}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \ D_7 \\
0 \ 0 \ 0 \ 9 \ 3 \ 2 \ 1 \\
\end{array}
\begin{array}{c}
(\overline{Q}_{12}) \\
\text{(R}_{12})
\end{array}
\]

\[
\begin{align*}
&= 52 - 142 = -90 \text{ (or)} \ 3 \ 2 \\
&= 32 - [6 + 4 + 3 + 72] \\
&= 32 - 65 = \overline{93}
\end{align*}
\]

\[\therefore \text{ We reduce the quotient by 1} \]

9) $86 \ (Q_{12(m)}) \ \ \ \ Q_{12(m)} = 8$

\[
\begin{array}{c}
72 \\
14 \\
\text{[R}_{12 (m)}]
\end{array}
\]

Step 8:

\[
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \ D_7 \\
0 \ 0 \ 9 \ 2 \ 7 \ 8 \ 0 \\
\text{(m)(m)(m)}
\end{array}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \\
6 \ 2 \ 1 \ 8 \ 7 \ 3 \\
\text{(negative value)}
\end{array}
\begin{array}{c}
0 \\
\text{(m)(m)(m)}
\end{array}
\begin{array}{c}
D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \\
6 \ 2 \ 1 \ 8 \ 7 \ 3 \\
\text{(m)(m)(m)}
\end{array}
\]

\[
\begin{align*}
&= 64 - 102 = -38 \\
&= 34 - (60 + 2 + 2 + 24 + 63) \\
&= 34 - 23 = 11 = 49
\end{align*}
\]

9) $86 \ (Q_{13}) \ \ \ \ Q_{13} = 0$

\[
\begin{array}{c}
0 \\
\text{[R}_{13}]
\end{array}
\]

9) $93 \ (Q_{13}) \ \ \ \ Q_{13} = 0$

\[
\begin{array}{c}
90 \\
\text{[R}_{13}]
\end{array}
\]
Vedic Mathematics

Division

If we reduce the quotient, 0, by 1, then it becomes negative
Therefore, we reduce previous quotient, 8, by 1

9) \( \frac{86}{63} (7) \) \[Q_{12}(m) = 7\]

\[ \frac{63}{23} \] \[R_{12}(m)\]

\( \text{ID} \ 232 - \left[ \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 0 & 2 & 1 & 8 \\ Q_6 & Q_7 & Q_8 & Q_9 \\ 9 & 2 & 7 & 7 \end{array} \right] = 232 - 130 = 102 \) (ND)

9) \( \frac{102}{99} (11) \) \[Q_{13}(m) = 11\]

\[ \frac{99}{3} \] \[R_{13}(m)\]

\( \text{(ID)} \ 34 - \left[ \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 0 & 2 & 1 & 8 \\ Q_7 & Q_8 & Q_9 & Q_{10} \\ 2 & 7 & 7 & 7 \end{array} \right] = 34 - 166 = -132 \) (–ve value)

\[ Q_{13}(m) = 11 \]

\[ Q_{13}(m) = 10 \] \[R_{13}(m)\]

\( \text{ID} \ 124 - \left[ \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 0 & 2 & 1 & 8 \\ Q_7 & Q_8 & Q_9 & Q_{10} \\ 9 & 2 & 7 & 7 \end{array} \right] = 124 - 160 = -36 \) (–ve value)

\[ Q_{13}(m) = 10 \] \[R_{13}(m)\]

\[ Q_{13}(m) = 9 \] \[R_{13}(m)\]

\[ Q_{13}(m) = 9 \] \[R_{13}(m)\]
Vedic Mathematics

ID 214 - \[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
0 & 6 & 2 & 1 & 8 & 7 & 3 \\
Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} & Q_{12} & Q_{13} \\
(m)(m)(m)(m)(m)(m)
\end{array}
\]
\[= 214 - 154 = 60 \text{ (ND)}\]

9) 60 \(6 (Q_{14})\)
\[\begin{array}{c}
Q_{14} = 6
\end{array}\]

Step 9:

ID 60 - \[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
0 & 6 & 2 & 1 & 8 & 7 & 3 \\
Q_8 & Q_9 & Q_{10} & Q_{11} & Q_{12} & Q_{13} & Q_{14} \\
(m)(m)(m)(m)(m)(m)
\end{array}
\]
\[= 60 - 158 = -98 \text{ (negative value)}\]

\[\frac{40 - [19]}{3} = \frac{3}{1} = \frac{2}{1}\]

9) \[\overline{2} \overline{1} \overline{2} \overline{1} \overline{8} (Q_{15})\]

\[\overline{1} \overline{8} \overline{3}\]

\[\overline{3}\]

\[\therefore \text{We reduce the quotient by 1.}\]

9) 60 \(5 [Q_{14(m)}]\)
\[\begin{array}{c}
Q_{14(m)} = 5
\end{array}\]

ID 150 - \[
\begin{array}{cccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
0 & 6 & 2 & 1 & 9 & 8 & 3 \\
Q_8 & Q_9 & Q_{10} & Q_{11} & Q_{12} & Q_{13} & Q_{14} \\
(m)(m)(m)(m)(m)(m)
\end{array}
\]
\[= 150 - 152 = 2 \text{ (negative value)}\]

\[\frac{30 - [1]}{2} = \frac{18}{2}\]

9) \[\overline{2} \overline{0} \overline{2} \overline{1} \overline{8} \overline{3} (Q_{16})\]

\[\overline{1} \overline{8} \overline{3}\]

\[\overline{1} \overline{8} \overline{3}\]

\[\overline{2} (R_{16})\]

\[\therefore \text{We reduce the quotient further}\]

9) 60 \(4 [Q_{14(m)}]\)
\[\begin{array}{c}
Q_{14(m)} = 4
\end{array}\]

\[\overline{3} \overline{6} \overline{2} \overline{4}\]

\[\overline{2} (R_{16})\]
Vedic Mathematics

Division

\[
\begin{array}{cccccc}
\text{D}_1 & \text{D}_2 & \text{D}_3 & \text{D}_4 & \text{D}_5 & \text{D}_6 \\
240 & 2 & 1 & 8 & 7 & 3 & 4 \\
\end{array}
\]

\(240 - 146 = 94 \text{ (ND)}\)

\[
\begin{array}{cccccccc}
\text{Q}_8 & \text{Q}_9 & \text{Q}_{10} & \text{Q}_{11} & \text{Q}_{12} & \text{Q}_{13} & \text{Q}_{14} \\
(\text{m}) & (\text{m}) & (\text{m}) & (\text{m}) & (\text{m}) & (\text{m}) & (\text{m}) \\
\end{array}
\]

\[\therefore \text{Quotient} = 0.000000009277948\]

Case (b)(ii): Where the divisor only has decimals.

The division is carried out in the usual way not taking cognisence of the decimal while dividing. After the division is over, the decimal point is shifted towards right side in the quotient through the same number of digits that are existing in the divisor.

**Example 16:** \[15628 + 23.4\]

**Current Method**

<table>
<thead>
<tr>
<th>15628</th>
<th>156280</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.4</td>
<td>234</td>
</tr>
</tbody>
</table>

\[2.3.4) 1 5 6 2 8 0 (6 6 7 8 6 3 2 4\]

\[
\begin{array}{cccccc}
\text{R}_1 & \text{R}_2 & \text{R}_3 & \text{R}_4 & \text{R}_5 & \text{R}_6 & \text{R}_7 & \text{R}_8 \\
3 & 6 & 6 & 7 & 6 & 4 & 3 & 4 \\
\text{(m)} & \text{(m)} & \text{(m)} & \text{(m)} & \text{(m)} & \text{(m)} & \text{(m)} & \text{(m)} \\
\end{array}
\]

\[Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6 \quad Q_7 \quad Q_8 \\
\text{(m)} \quad \text{(m)} \quad \text{(m)} \quad \text{(m)} \quad \text{(m)} \quad \text{(m)} \quad \text{(m)} \quad \text{(m)}
\]

Quotient = 667.86324

\[Q = 667.86324\]

**Vedic Method Steps:**

**Step 1:**

2) 15 \((Q_1)\)

\[
\begin{array}{ccc}
14 & \text{(R}_1) \\
\end{array}
\]
Step 2:

\[ \begin{array}{c}
\text{(ID) } 16 - \uparrow 7 = 16 - 21 = -5 \text{ (negative value)} \\
\downarrow 7 \\
Q_1
\end{array} \]

\[ \begin{array}{c}
\therefore \text{We reduce the quotient by 1}
\end{array} \]

\[ \begin{array}{c}
2) 15 (6 [Q_1(m)] \\
12 \\
3 R_1(m)
\end{array} \]

\[ \begin{array}{c}
D_1 \\
\text{(ID) } 36 - \left\{ \begin{array}{c} 3 \\
6 \\
Q_1(m) \end{array} \right\} = 36 - 18 = 18 \text{ (ND)}
\end{array} \]

2) 18 (9 [Q_2])

\[ \begin{array}{c}
18 \\
0 (R_2)
\end{array} \]

Step 3:

\[ \begin{array}{c}
\left\{ \begin{array}{c} 3 \\
4 \\
6 \ 9 \end{array} \right\} - 2 \text{ (27 + 24) \\
\text{(ID) } 2 - \left\{ \begin{array}{c} 3 \\
4 \\
6 \ 9 \end{array} \right\} = 2 - 51 - \cdot 49 \text{ (negative value)} \\
Q_1(m) \ Q_2
\end{array} \]

\[ \begin{array}{c}
\therefore \text{We reduce the quotient}
\end{array} \]

\[ \begin{array}{c}
2) 18 (8 [Q_2(m)] \\
16 \\
2 R_2 (m)
\end{array} \]

Step 3:

\[ \begin{array}{c}
\left\{ \begin{array}{c} 3 \\
4 \\
7 \ 2 \end{array} \right\} - \left\{ \begin{array}{c} 3 \\
4 \\
7 \ 2 \end{array} \right\} = 12 - (5 + 28) = -12 - (22) = -12 + \overline{22} \\
\overline{3} \ 0
\end{array} \]

2 )\overline{30} ( \overline{15} \ (Q_3)

\[ \begin{array}{c}
\frac{30}{0} (R_3)
\end{array} \]
Vedic Mathematics

Division

(ID) 22 \[ \begin{array}{cc}
3 & 4 \\
6 & 8 \\
\end{array} \] = 22 - 48 = -26 (negative value)
\[ \begin{array}{cc}
Q_1 & Q_2 \\
(m) & (m) \\
\end{array} \]

\[ \therefore \text{We reduce the quotient further} \]

2) 18 \[ \begin{array}{c}
7 \end{array} \] \[ \begin{array}{c}
Q_2(m) \\
\end{array} \]
\[ \begin{array}{c}
14 \\
4 \end{array} \] \[ \begin{array}{c}
R_2(m) \\
\end{array} \]

(ID) 42 \[ \begin{array}{cc}
3 & 4 \\
6 & 7 \\
\end{array} \] = 42 - 45 = -3 (negative value)
\[ \begin{array}{cc}
Q_1 & Q_2 \\
(m) & (m) \\
\end{array} \]

\[ \therefore \text{We reduce the quotient further} \]

2) 18 \[ \begin{array}{c}
6 \end{array} \] \[ \begin{array}{c}
Q_2(m) \\
\end{array} \]
\[ \begin{array}{c}
12 \\
6 \end{array} \] \[ \begin{array}{c}
R_2(m) \\
\end{array} \]

(ID) 62 \[ \begin{array}{cc}
3 & 4 \\
6 & 6 \\
\end{array} \] = 62 - 42 = 20
\[ \begin{array}{cc}
Q_1 & Q_2 \\
(m) & (m) \\
\end{array} \]

2) 20 \[ \begin{array}{c}
10 \end{array} \] \[ \begin{array}{c}
Q_3 \\
\end{array} \]
\[ \begin{array}{c}
20 \\
0 \end{array} \] \[ \begin{array}{c}
R_3 \\
\end{array} \]

Step 4:

(ID) 8 \[ \begin{array}{cc}
3 & 4 \\
6 & 10 \\
\end{array} \] = 8 - 54 = -46 (negative value)
\[ \begin{array}{cc}
Q_2 & Q_1 \\
(m) & (m) \\
\end{array} \]

\[ \text{We reduce the quotient by 1} \]
Vedic Mathematics

2) 20 \( \div 9 \) \( \text{[Q}_3\text{(m)}] \)

\[
\begin{array}{c}
\text{18} \\
\text{2 R}_3\text{(m)}
\end{array}
\]

\( \text{D}_1 \quad \text{D}_2 \)

\[
\begin{array}{c}
\text{3} \\
\text{6}
\end{array}
\quad \begin{array}{c}
\text{4} \\
\text{9}
\end{array}
\]

\( \text{(ID) } 28 - \left( \frac{3}{6} \right) = 28 - 51 = -23 \) (negative value)

\( \text{Q}_2 \quad \text{Q}_3 \)

\( \text{m} \quad \text{m} \)

\[
\text{We reduce the quotient further}
\]

2) 20 \( \div 8 \) \( \text{[Q}_3\text{(m)}] \)

\[
\begin{array}{c}
\text{16} \\
\text{4 R}_1\text{(m)}
\end{array}
\]

\( \text{D}_1 \quad \text{D}_2 \)

\[
\begin{array}{c}
\text{3} \\
\text{6}
\end{array}
\quad \begin{array}{c}
\text{4} \\
\text{8}
\end{array}
\]

\( \text{(ID) } 48 - \left( \frac{3}{6} \right) = 48 - 48 = 0 \) (ND)

\( \text{Q}_2 \quad \text{Q}_3 \)

\( \text{m} \quad \text{m} \)

2) 0 \( \div 0 \) \( \text{[Q}_4\text{]} \)

\[
\begin{array}{c}
\text{0} \\
\text{0 R}_4
\end{array}
\]

\[
\text{Step 5:}
\]

\( \text{D}_1 \quad \text{D}_2 \)

\[
\begin{array}{c}
\text{3} \\
\text{8}
\end{array}
\quad \begin{array}{c}
\text{4} \\
\text{8}
\end{array}
\]

\( \text{(ID) } 0 - \left( \frac{3}{8} \right) = 0 - 32 = -32 \) (negative value)

\( \text{Q}_3 \quad \text{Q} \)

\( \text{m} \quad \text{m} \)

\[
\text{Step 5:}
\]

\( \text{D}_1 \quad \text{D}_2 \)

\[
\begin{array}{c}
\text{3} \\
\text{15}
\end{array}
\quad \begin{array}{c}
\text{4} \\
\text{30}
\end{array}
\]

\( 10 - \left( \frac{3}{15} \right) = 10 - [90 + 60] \)

\( \text{Q}_3 \quad \text{Q}_4 = \frac{20}{20} \)

\[
\text{If we reduce the quotient, 0, it becomes negative Therefore, we reduce the previous quotient, 8, by 1.}
\]

2) 20 \( \div 7 \) \( \text{[Q}_3\text{(m)}] \)

\[
\begin{array}{c}
\text{14} \\
\text{6 \text{[R}_3\text{(m)]}}
\end{array}
\]

\[
\text{} \quad \boxed{\text{Q}_3\text{(m)} = 7}
\]

\[
\begin{array}{c}
\text{20} \\
\text{20}
\end{array}
\]

\[
\text{20} \\
\text{0 \text{[R}_3\text{(m)]}}
\]
Vedic Mathematics

\[
\begin{align*}
\text{(ID) } 68 - \begin{pmatrix} 3 & 4 \\ 6 & 7 \end{pmatrix} &= 68 - 45 = 23 \text{ (ND)} \\
Q_2 & \quad Q_3 \\
(m) & \quad (m)
\end{align*}
\]

2) 23 \(\{11 [Q_4 (m)]\)  

\[
\begin{aligned}
22 & \\
1 & \{R_4 (m)\}
\end{aligned}
\]

\[
\begin{align*}
\text{(ID) } 10 - \begin{pmatrix} 3 & 4 \\ 7 & 11 \end{pmatrix} &= 10 - 61 = -51 \text{ (negative value)} \\
Q_3 & \quad Q_4 \\
(m) & \quad (m)
\end{align*}
\]

\[\therefore \text{ We reduce the quotient by 1.}\]

2) 23 \(\{10 [Q_4 (m)]\) \hfill \text{\(Q_4 (m) = 10\)}

\[
\begin{aligned}
20 & \\
3 & \{R_4 (m)\}
\end{aligned}
\]

\[
\begin{align*}
\text{(ID) } 30 - \begin{pmatrix} 3 & 4 \\ 7 & 10 \end{pmatrix} &= 30 - 58 = -28 \text{ (negative value)} \\
Q_3 & \quad Q_4 \\
(m) & \quad (m)
\end{align*}
\]

\[\therefore \text{ We reduce quotient further}\]

2) 23 \(\{9 [Q_4 (m)]\)

\[
\begin{aligned}
18 & \\
5 & \{R_4 (m)\}
\end{aligned}
\]

\[
\begin{align*}
\text{(ID) } 50 - \begin{pmatrix} 3 & 4 \\ 7 & 9 \end{pmatrix} &= 50 - 55 = -5 \text{ (negative value)} \\
Q_3 & \quad Q_4 \\
(m) & \quad (m)
\end{align*}
\]

\[\therefore \text{ We reduce quotient further}\]
2) 23 (8 [Q₄(m)])

\[
\begin{array}{c}
16 \\
7 \ [R₄ \ (m)]
\end{array}
\]

\[
\begin{array}{c}
D₁ \ D₂ \\
3 \ 4 \\
7 \ 8 \\
Q₃ \ Q₄ \ (m) \ (m)
\end{array}
\]

(ID) \ 70 - \ \begin{array}{c} D₁D₂ \\
3 \ 4 \\
7 \ 8 \\
Q₃ \ Q₄ \ (m) \ (m)
\end{array} = 70 - 52 = 18 \ (ND)

2) 18 (9 [Q₃])

\[
\begin{array}{c}
18 \\
0 \ (R₃)
\end{array}
\]

Step 6:

\[
\begin{array}{c}
D₁ \ D₂ \\
3 \ 4 \\
8 \ 9 \\
Q₄ \ Q₅ \ (m)
\end{array}
\]

(ID) \ 0 - \ \begin{array}{c} D₁D₂ \\
3 \ 4 \\
8 \ 9 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array} = 0 - 59 = -59 \ (negative \ value)

\[
\begin{array}{c}
D₁ \ D₂ \\
3 \ 4 \\
30 \ 10 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array}
\]

0 - \ \begin{array}{c} D₁D₂ \\
3 \ 4 \\
30 \ 10 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array} = 0 - [30 + 120] = 90

\[\text{\therefore \ We \ reduce \ the \ quotient \ by \ 1.}\]

2) 18(8 [Q₅(m)])

\[
\begin{array}{c}
16 \\
2 \ [R₅ \ (m)]
\end{array}
\]

\[
\begin{array}{c}
D₁ \ D₂ \\
3 \ 4 \\
8 \ 8 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array}
\]

(ID) \ 20 - \ \begin{array}{c} D₁D₂ \\
3 \ 4 \\
8 \ 8 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array} = 20 - 56 = -36 \ (negative \ value)

\[\text{\therefore \ We \ reduce \ the \ quotient \ further.}\]

2) 18 (7 [Q₆(m)])

\[
\begin{array}{c}
14 \\
4 \ [R₆ \ (m)]
\end{array}
\]

\[
\begin{array}{c}
D₁ \ D₂ \\
3 \ 4 \\
8 \ 7 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array}
\]

(ID) \ 40 - \ \begin{array}{c} D₁D₂ \\
3 \ 4 \\
8 \ 7 \\
Q₄ \ Q₅ \ (m) \ (m)
\end{array} = 40 - 53 = -13 \ (negative \ value)
Vedic Mathematics

2) 18 (6 [Q₅(m)])

\[
\begin{array}{c}
12 \\
6 [R₅ (m)]
\end{array}
\]

\[
D_1 D_2 \\
3 4 \\
\begin{array}{c}
8 \\
6
\end{array}
\]

(ID) 60 \(-\)

\[
\begin{array}{c}
Q_4 Q_5 \\
(m) (m)
\end{array}
\]

\[= 60 - 50 = 10 \text{ (ND)}\]

\[Q_5 \text{ (m)} = 6\]

2) 10 (5 [Q₆(m)])

\[
\begin{array}{c}
10 \\
0 \text{ (R₆)}
\end{array}
\]

Step 7:

\[
\begin{array}{c}
D_1 D_2 \\
3 4 \\
\begin{array}{c}
6 \\
5
\end{array}
\]

(ID) 0 \(-\)

\[
\begin{array}{c}
Q_5 Q_6 \\
(m) (m)
\end{array}
\]

\[= -39 \text{ (negative value)}\]

\[0 - \left[\frac{36}{10} \cdot \frac{15}{45}\right] = 175\]

\[
\begin{array}{c}
D_1 D_2 \\
3 4 \\
\begin{array}{c}
10 \\
45
\end{array}
\]

\[
\begin{array}{c}
Q_5 Q_6 \\
(m) (m)
\end{array}
\]

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\]
2) $7 \div (3 \cdot Q_7)$

\[
\begin{array}{c}
\text{Step 8:} \\
D_1 \quad D_2 \\
\begin{array}{c}
3 \\
3 \\
4 \\
Q_6 \\
Q_7 \\
(m) \\
\end{array}
\end{array}
\]

\[
10 - 21 = -11 \text{ (negative value)}
\]

\[Q_7 \text{ (m)} = 2\]

\[
\begin{array}{c}
\text{Step 8:} \\
10 - \left( \frac{3 \times 4}{45 \times 87} \right) = 10 - (261 + 180)
\end{array}
\]

\[
= 10 - 121 = 10 + 121 = 131 = 71
\]

\[
\begin{array}{c}
2) \quad 71 \div (35)
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
6 \\
11 \\
10 \\
i \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\text{Quotient} = 667.8632
\end{array}
\]

\[
\begin{array}{c}
\text{Vinculum:}
\end{array}
\]

\[
\begin{array}{c|cccccccc}
34 & 15 & 6: & 2 & 8 & 0 & 0 & 0 & 0 & 0 \\
\hline
2 & 1 & \cdot & 0 & 1 & 0 & 0 & 1 & 1 \\
\hline
7 & \overline{2} & : & 15 & .30 & 10 & 45 & 87 & 35 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 \\
\end{array}
\]

As the divisor has one digit after decimal the decimal point is shifted by one digit towards right.

\[
\begin{array}{c}
\text{Quotient} = 732.14345 \\
\quad = 667.86335
\end{array}
\]

Case (b)(iii): In case the decimal point exists both in the divisor as well as in the dividend, then both Case (b)(i) and Case (b)(ii) are applied in order.
### Vedic Mathematics

#### Example 17: \[ 134289 + 276 \]

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
<th>[as per b(i)]</th>
</tr>
</thead>
</table>
| \[
\begin{array}{c}
134.289 \\
276
\end{array} = \frac{134289}{2760}
\] | \[\begin{array}{c|cccccccc}
76 & 1 & 3 & 4 & 2 & 8 & 9 & 0 & 0 \\
\hline
2 & 1 & 5 & 10 & 10 & 8 & 8 & 7 \\
0 & 4 & 8 & 6 & 5 & 5 & 4
\end{array}\] | Quotient = 486554 (as per b[ii]) |
| \[
2760 \) 134289 (486554 \\
11040
\] | \[
\begin{array}{c}
23889 \\
22080 \\
18090 \\
16560 \\
15300 \\
13800 \\
15000 \\
13800 \\
12000 \\
11040 \\
\hline
960
\end{array}
\] |

#### Example 18: \[ 2.1367 + 0.312 \]

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
<th>(as per b[ii])</th>
</tr>
</thead>
</table>
| \[
\begin{array}{c}
2.1387 \\
0.312
\end{array} = \frac{21387}{3120}
\] | \[\begin{array}{c|cccccccc}
12 & 2 & 3 & 8 & 7 & 0 & 0 & 0
\hline
3 & 2 & 3 & 3 & 3 & 4 & 2 & 4 & 3
\hline
0 & 0 & 6 & 8 & 5 & 4 & 8 & 0 & 7
\end{array}\] | Quotient = 6.85480769 (as per b[ii]) |
| \[
3120 \) 21387 (685480769 \\
18720
\] | \[
\begin{array}{c}
26670 \\
24960 \\
17100 \\
15600 \\
15000 \\
12480 \\
25200 \\
24960 \\
24000 \\
21840 \\
21600 \\
18720 \\
28800 \\
28080 \\
\hline
720
\end{array}
\] |
**Example 19:** \[ 0.461397 ÷ 123.4 \]

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method (as per b [i])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 461397</td>
<td>34</td>
</tr>
<tr>
<td>123 4</td>
<td>.4</td>
</tr>
<tr>
<td>123400000</td>
<td>6</td>
</tr>
<tr>
<td>61397000</td>
<td>13</td>
</tr>
<tr>
<td>3702000000</td>
<td>9</td>
</tr>
<tr>
<td>911970000</td>
<td>7</td>
</tr>
<tr>
<td>863800000</td>
<td>0</td>
</tr>
<tr>
<td>481700000</td>
<td>3</td>
</tr>
<tr>
<td>3702000000</td>
<td>7</td>
</tr>
<tr>
<td>1115000000</td>
<td>3</td>
</tr>
<tr>
<td>1110600000</td>
<td>9</td>
</tr>
<tr>
<td>440000000</td>
<td>0</td>
</tr>
<tr>
<td>3702000000</td>
<td>0</td>
</tr>
<tr>
<td>698000000</td>
<td>3</td>
</tr>
</tbody>
</table>

Quotient = 0.00373903 (as per b [ii])

Working details of Vinculum Method and reduction method are shown in page No.
(b) **Reduction Method (Simplified) for Straight Division:**

While working out a division problem using straight division, some of intermediate dividends give rise to negative values, on subtraction of the Urdhva – Tiryak multiplication value of the Dhvajanka (D₁D₂…..etc) and quotients (Q₁Q₂… etc). Reduction of this negative value, by modifying the previous quotients or by considering the negative value itself as vinculum, one can carry out the problem. These methods are already explained earlier.

There is one more method for reduction and the procedure is as follows:

1. The partition rules of dividend and divisor are same as explained earlier.
2. The reduction starts when one arrives at a negative value, in the process of subtraction of Urdhva – Tiryak multiplication result from the intermediate dividend.

At this Stage:

(a) One has to reduce the previous quotient by ‘1’

(b) Add the part divisor to the current intermediate remainder

(c) Carry out the process of subtraction, with the new quotient and remainder

(d) If one again gets a negative value, repeat steps (a),(b),(c) until the negative value vanishes. This procedure is elaborately explained in the following examples.

In the straight division, we come across the modifications of dividend and reduction of quotients.
**Example 1:** 7896456 + 34 (Example 3 Page No.)

\[
\begin{array}{c|cccccc|c}
D & 7 & 8 & 9 & 6 & 4 & 5 & 6 \\
\hline
4 & / & / & / & / & / & / \\
1 & 1 & 1 & 2 & 4 & & 5 \\
(P.D)3 & & & & & & & \\
2 & 3 & 2 & 2 & 5 & 4 & 9 \\
| Q_1 & Q_2 & Q_1 & Q_4 & Q_5 & Q_6 |
\end{array}
\]

The first five steps do not involve any modification, so they are the same.

**Step 1:** \(3 \) \( 7 \) (2 \( Q_1 \))

\[
\begin{array}{c}
D \\
6 \\
\hline
1 \ (R_1)
\end{array}
\]

**Step 2:** \(18 - \left( \begin{array}{c} 4 \\ 2 \end{array} \right) = 10 \)

\[
\begin{array}{c}
D \\
3 \ (Q_2) \\
9 \\
\hline
2 \ (R_2)
\end{array}
\]

**Step 3:** \(19 - \left( \begin{array}{c} 3 \\ 3 \end{array} \right) = 7 \)

\[
\begin{array}{c}
D \\
3 \ (Q_3) \\
6 \\
\hline
1 \ (R_3)
\end{array}
\]

**Step 4:** \(16 - \left( \begin{array}{c} 4 \\ 2 \end{array} \right) = 8 \)

\[
\begin{array}{c}
D \\
3 \ (Q_3) \\
6 \\
\hline
2 \ (R_4)
\end{array}
\]

**Step 5:** \(24 - \left( \begin{array}{c} 4 \\ 2 \end{array} \right) = 16 \)

\[
\begin{array}{c}
D \\
3 \ (Q_3) \\
15 \\
\hline
1 \ (R_5)
\end{array}
\]
Vedic Mathematics

Step 6: Here the Urdhva multiplication and its subtraction from the intermediate dividend gives a negative value

\[
\begin{align*}
D & \left( \begin{array}{c} 4 \\ 5 \end{array} \right) \\
i.e. \quad 15 - \left( \begin{array}{c} 4 \\ 5 \end{array} \right) & = 15 - 20 = -5.
\end{align*}
\]

So a reduction of ‘1’ is made in the quotient \(Q_5\) and the part divisor (PD) is added to the remainder \(R_5\). So \(Q_5(m) = 4\), \(R_5(m) = 1 + 3 = 4\).

Now the new intermediate dividend (ID) is 45

\[
\begin{align*}
D & \left( \begin{array}{c} 4 \\ \end{array} \right) \\
45 - & 29
\end{align*}
\]

\(Q_5(m)\)

Thus the negative value is eliminated. Now following the usual procedure, the problem is carried out

\[
\begin{align*}
3 & \left( \begin{array}{c} 9 \\ 27 \\ 2 \end{array} \right) 29(9(Q_6)) \\
& \underline{27} \\
& \underline{2}(R_6)
\end{align*}
\]

Step 7: We have entered into the remainder region

\[
D
\]

Again, 26 - 26 - 36 = -10, a negative value is obtained

\(R_6\)

So a reduction of ‘1’ in \(Q_6 = 9\) and addition of (PD) to \(R_6 = 2\) are made to obtain modified quotient \(Q_6(m)\) as 8 and modified remainder \(R_6(m)\) as 3.

Now the Urdhva multiplication and subtraction gives a positive value, 24

\[
\begin{align*}
i.e., \quad 56 - & = 56 - 32 = 24 \\
\left( \begin{array}{c} 8 \\ \end{array} \right) & Q_6(m)
\end{align*}
\]

This is considered as the final remainder.

\[
\therefore \quad \text{Final Quotient} = 232248 \\
\text{Final Remainder} = 24
\]
Vedic Mathematics

If one wishes to get decimals, the procedure can be continued as explained before. Some more examples are illustrated below and their working details are also included.

Example 2:  

\[ 7652 \div 23 \text{ (Example 5, Page No: )} \]

Worked out for four decimals

\[
\begin{array}{c|ccc|cccccccc}
D & 7 & 6 & 5 & 2 & 0 & 0 & 0 & 0 & 0 \\
(PD) & 3 & 6 & 1 & 2 & 4 & 4 & 3 & 3 & 2 \\
\hline
3 & 3 & 3 & 3 & 8 & 11 & 6 & 7 & 6 & 2 \\
Q_1 & Q_2 & Q_3 & 7 & 10 & 5 & 6 & 5 & Q_4(m)Q_5(m)Q_6(m) \\
\hline
Q_3(m) & Q_4(m) & Q_5(m) & 2 & 6 & 9 \\
\end{array}
\]

Step 1:

\[
2 \quad 7 \quad (3 \quad (Q_1)) \\
6 \quad 1 \quad (R_1)
\]

Step 2:

\[
16 - \left( \begin{array}{c}
3 \\
3 \\
\end{array} \right) = 7 \\
Q_1 \\
2 \quad 7 \quad (3 \quad (Q_2)) \\
\therefore \text{ Modified } Q_2 = 2 \\
6 \quad 1 \quad (R_2)
\]

Step 3:

\[
15 - \left( \begin{array}{c}
3 \\
3 \\
\end{array} \right) = 6 \\
Q_3 \\
2 \quad 6 \quad (3 \quad (Q_3)) \\
6 \quad 0 \quad (R_3)
\]

Step 4:

\[
02 - \left( \begin{array}{c}
3 \\
3 \\
\end{array} \right) = -7, \text{ a negative value} \\
\therefore \text{ Modified } R_3 = 0 + 2 = 2 \\
\text{ Now the new ID is 22} \\
\begin{array}{c}
D \\
(3) \\
\end{array} \\
\quad \text{ie, } 22 - \left( \begin{array}{c}
2 \\
\end{array} \right) = 16 \\
\quad \begin{array}{c}
Q_3(m) \\
\end{array} \\
\begin{array}{c}
2 \quad 16 \quad (8 \quad (Q_4)) \\
16 \quad 0 \quad (R_4)
\end{array}
\]

Division
Step 5: \[ 0 - \binom{3}{8} = -24, \text{ a negative value.} \]

Hence \( Q_4 \) value is reduced by 1

\[ Q_4(m) = 7, \ R_4(m) = 2 \]

\[ 20 - \binom{3}{7} = -1 \text{ again a negative value} \]

So reduce \( Q_4(m) \) further to 6 and raise \( R_4(m) \) to \( 2 + 2 = 4 \)

Now the ID is 40

\[ 40 - \binom{3}{6} = 22 \]

\[ 2 \ ) 22 \ (11 \ (Q_3) \]

\[ 22 \frac{0}{0} \ (R_3) \]

Step 6: \[ 0 - \binom{3}{11} = -33 \]

So reduce \( Q_3 \) to 10 and increase \( R_3 \) to 2.

\[ 20 - \binom{3}{10} = -10 \]

Again a negative a value.

So reduce \( Q_3 \) further to 9 and raise \( R_4 \) to 4.
\[ \begin{align*}
D & \quad \begin{array}{c} 3 \uparrow \\ 9 \end{array} \quad = \quad 13 \\
\therefore \quad 40 \quad \begin{array}{c} 9 \uparrow \\ \end{array} \quad = \quad 13 \\
Q_2(m) & \\

2) \quad 13 \quad (Q_6) \\
\quad \underline{12} \\
\quad (R_5) \\

\begin{array}{c}
\text{Step 7:} \\
10 \quad \begin{array}{c} 6 \uparrow \\ \end{array} \quad = \quad -8 \\
Q_6 \\
\therefore \quad Q_6 \text{ is reduced to 5 and the remainder is raised to 3} \\
\begin{array}{c}
D \\
3 \uparrow \\
\end{array} \\
30 \quad \begin{array}{c} 5 \uparrow \\ \end{array} \quad = \quad 15 \\
Q_6(m) \\
\quad \underline{14} \\
\quad (R_7) \\
\quad 2) \quad 15 \quad (Q_7) \\
\end{array}

\begin{array}{c}
\text{Step 8:} \\
10 \quad \begin{array}{c} 7 \uparrow \\ \end{array} \quad = \quad -11 \\
Q_7 \\
\therefore \quad Q_7 \text{ is reduced to 6 and } R_7 \text{ is raised to 3.} \\
\begin{array}{c}
D \\
3 \uparrow \\
30 \quad \begin{array}{c} 6 \uparrow \\ \end{array} \quad = \quad 12 \\
Q_7(m) \\
\quad \underline{12} \\
\quad (R_8) \\
\quad 2) \quad 12 \quad (Q_8) \\
\quad \underline{0} \\
\quad (R_8) \\
\end{array}

\text{Quotient in Decimal form is } 332.69562 \]
Vedic Mathematics

Note

One has to stop the calculation after seeing to the correctness of the last quotient i.e., if one wants 4 decimal places, then calculate for the fifth decimal and see that there is no negative value. If the negativity continues reduce the previous quotient until the negativity vanishes and so on.

Example 3: 123456789 + 4321 (Example: 9, Page No. decimal calculation)

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>4321 123456789</td>
<td>D1D2D3</td>
</tr>
<tr>
<td>8642</td>
<td>28571.346</td>
</tr>
<tr>
<td>37036</td>
<td>1 2 3 4 5 6</td>
</tr>
<tr>
<td>34568</td>
<td>7 8 9 0 0</td>
</tr>
<tr>
<td>24687</td>
<td>4</td>
</tr>
<tr>
<td>21605</td>
<td>4 3 6</td>
</tr>
<tr>
<td>30828</td>
<td>3 9 6 8 1</td>
</tr>
<tr>
<td>30247</td>
<td>3 5 7 7</td>
</tr>
<tr>
<td>5819</td>
<td>2 8 3 7</td>
</tr>
<tr>
<td>4321</td>
<td>4 6</td>
</tr>
</tbody>
</table>

Quotient = 28571.346

Indicates modification

If the part divisor cannot divide the first digit then one can consider first two digits or three digits as the case may be with reference to the number of digits in (PD) For example, in this problem PD (4) cannot divide the first digit, hence the first two digits 1.2 are considered for division by 4 giving a quotient 3 and remainder zero.

1. 12 + 4 = 3, (0)  Q1 = 3, R1 = 0

2. 03 – = –6(ve)

So reduce Q1 = 3 by 1 and so R1(m) = 4
Vedic Mathematics

\[
\begin{align*}
43 - \left( \begin{array}{c} 3 \\ 2 \end{array} \right) &= 43 - 6 = 37 + 4 = 9 \quad (1) \\
Q_3 \quad (R_2)
\end{align*}
\]

3. \[
14 - \left( \begin{array}{c} 3 \\ 2 \end{array} \right) = 14 - 31 = 17 \text{ (neg)}
\]

\(Q_2\) is reduced to 8 and \(R_2\) is increased to 5.

\[
\begin{align*}
54 - \left( \begin{array}{c} 3 \\ 2 \end{array} \right) &= 54 - 28 = 26 + 4 = 6 \quad (2) \\
Q_3 \quad (R_3)
\end{align*}
\]

4. \[
25 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 2 \ 8 \ 6 \end{array} \right) = 25 - 36 = -11 \text{ (neg)}
\]

\(Q_3\) is reduced to 5 and \(R_3\) raised to 6.

\[
\begin{align*}
65 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 2 \ 8 \ 5 \end{array} \right) &= 65 - 33 = 32 + 4 = 8 \quad (0) \\
Q_4 \quad (R_4)
\end{align*}
\]

5. \[
06 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 8 \ 5 \ 8 \end{array} \right) = 6 - 42 = -36 \text{ (neg)}
\]

\(Q_4\) is reduced to 7 and \(R_4\) is raised to 4.

\[
\begin{align*}
46 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 8 \ 5 \ 7 \end{array} \right) &= 46 - 39 = 7 + 4 = 1 \quad (3) \\
Q_5 \quad (R_3)
\end{align*}
\]

1^st \ decimal \ calculation:

6. \[
37 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 5 \ 7 \ 1 \end{array} \right) = 37 - 22 = 15 + 4 = 3 \quad (3)
\]
Vedic Mathematics

Division

2nd decimal:

\[
\begin{array}{c}
3 & 2 & 1 \\
7 & 38 & - & \frac{\underline{\phantom{38}}}{713} \\
& 38 - 18 & = & 20 - 4 = 5 (0)
\end{array}
\]

3rd decimal

\[
\begin{array}{c}
3 & 2 & 1 \\
8 & 09 & - & \frac{\underline{\phantom{09}}}{321} \\
& 09 - 22 & = & -13 (-ve)
\end{array}
\]

Hence 5 is reduced to 4 and remainder is increased to 4

\[
\begin{array}{c}
3 & 2 & 1 \\
49 & - & \frac{\underline{\phantom{49}}}{134} \\
& 49 - 19 & = & 30 - 4 = 7 (2)
\end{array}
\]

\[
\begin{array}{c}
3 & 2 & 1 \\
9 & 20 & - & \frac{\underline{\phantom{20}}}{347} \\
& 20 - 32 & = & -12 (-ve)
\end{array}
\]

Hence 7 is reduced to 6 and 2 is increased to 6.

\[
\begin{array}{c}
3 & 2 & 1 \\
60 & - & \frac{\underline{\phantom{60}}}{346} \\
& 60 - 29 & = & 31 - 4 = 7 (3)
\end{array}
\]

Remainders:

At step 5 (before entering into decimals) one can calculate the remainder (absolute).

\[
3789 - \frac{\underline{\phantom{3789}}}{571} \times 100 - \left(2 \frac{\underline{\phantom{1}}}{1} \right) \times 10 - \left(1 \frac{\underline{\phantom{1}}}{1} \right) \times 1
\]

\[
3789 - 2200 - 90 - 1 = 1498 (R_1)
\]

At step 6 (After first decimal)
Vedic Mathematics

\[
\begin{align*}
3890 - & \quad \frac{21}{100} - \frac{1}{10} - \frac{1}{3} \\
713 & \\
= 3890 - 1800 - 70 - 3 & = 2017 (R_2) \\
\text{At step 7 (After second decimal)} & \\
4900 - & \quad \frac{21}{100} - \frac{1}{10} - \frac{1}{3} \\
134 & \\
= 4900 - 1900 - 110 - 4 & = 2886 (R_3) \\
\text{At step 8 (After third decimal)} & \\
6000 - & \quad \frac{21}{100} - \frac{1}{10} - \frac{1}{3} \\
346 & \\
= 6000 - 2000 - 160 - 6. = 6000 - 3066 & = 2934 \\
\text{These remainders are well comparable with those obtained from Current Method} & \\
\end{align*}
\]

Example 4: \( 98765 + 1321 \)

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
</table>
| 1321 \( \overline{| 98765 \phantom{0} | 74.765} \) 9247 | \( \begin{array}{c}
321 \\
\text{9} \\
\text{8} \\
\text{7} \\
\text{6} \\
\text{5} \\
\text{0}
\end{array} \) | \( \begin{array}{c}
9 \\
7 \\
6 \\
5 \\
0 \\
0 \\
0
\end{array} \) |
| 6295 5284 10110 R_1 9247 | 1 1 1 1 1 | 1 1 1 1 |
| 8630 R_3 7926 | 2 2 2 2 2 | 2 2 2 2 |
| 7040 R_3 6605 | 3 3 3 3 3 | 3 3 3 3 |
| 435 R_4 | 4 4 4 4 4 | 4 4 4 4 |
| 321 | 9 7 11 10 9 6 | 9 7 10 9 8 6 |
| \( Q_{1m} \) 4 | \( Q_{2m} \) 5 | \( Q_{3m} \) 6 | \( Q_{4m} \) 5 | \( Q_{5m} \) 6 |
1. \[ 9 \div 1 = 9 \ , \ 0 \]
   \[ \text{Q}_1 \ 	ext{R}_1 \]

2. \[ 08 - \left( \begin{array}{c} 3 \\ 9 \end{array} \right) = 08 - 27 = -19 \ 	ext{(ve)} \]
   \[ \text{Q}_1 \ 	ext{is to be modified.} \]

   \[ 18 - \left( \begin{array}{c} 3 \\ 8 \end{array} \right) = 18 - 24 = -6 \ 	ext{(ve)} \]

   \[ 28 - \left( \begin{array}{c} 3 \\ 7 \end{array} \right) = 28 - 21 = 7 \]
   \[ \text{Q}_1(\text{m}) = 7 \]
   \[ 7 + 1 = 7 \ , \ 0 \]
   \[ \text{Q}_2 \ 	ext{R}_2 \]

3. \[ 1^{\text{st}} \text{Decimal} \]

   \[ 07 - \left( \begin{array}{c} 3 \\ 7 \times 7 \end{array} \right) = 07 - 35 = -19 \ 	ext{(ve)} \]

   \[ 17 - \left( \begin{array}{c} 3 \\ 7 \times 6 \end{array} \right) = 17 - 32 = -15 \ 	ext{(ve)} \]

   \[ 27 - \left( \begin{array}{c} 3 \\ 7 \times 5 \end{array} \right) = 27 - 29 = -2 \ 	ext{(ve)} \]

   \[ 37 - \left( \begin{array}{c} 3 \\ 7 \times 4 \end{array} \right) = 37 - 26 = 11 \]
   \[ 11 + 1 = 11 \ , \ 0 \]
   \[ \text{Q}_1 \ 	ext{R}_1 \]
   \[ \therefore \text{Q}_2(\text{m}) = 4 \ 	ext{and so on} \]

Before entering into decimal points the remainder is

\[ 3765 - \left( \begin{array}{c} 3 \ 2 \ 1 \\ 0 \ 7 \ 4 \end{array} \right) \times 100 - \left( \begin{array}{c} 2 \ 1 \\ 7 \ 4 \end{array} \right) \times 10 - \left( \begin{array}{c} 1 \\ 4 \end{array} \right) \times 1 \]

\[ = 3765 - 2600 - 150 - 4 = 1011 \ 	ext{(R}_4\text{)} \]
Vedic Mathematics

4. 2\textsuperscript{nd} Decimal

\[
\begin{array}{c}
06 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = - \text{ve} \\
\begin{pmatrix} 7 & 4 & 11 \end{pmatrix}
\end{array}
\]

\[
\begin{array}{c}
16 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = - \text{ve} \\
\begin{pmatrix} 7 & 4 & 10 \end{pmatrix}
\end{array}
\]

\[
\begin{array}{c}
26 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = - \text{ve} \\
\begin{pmatrix} 7 & 4 & 9 \end{pmatrix}
\end{array}
\]

\[
\begin{array}{c}
36 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = - \text{ve} \\
\begin{pmatrix} 7 & 4 & 8 \end{pmatrix}
\end{array}
\]

\[
\begin{array}{c}
46 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = 46 - 36 = 10 \\
\begin{pmatrix} 7 & 4 & 7 \end{pmatrix}
\end{array}
\]

\[
\therefore Q_3(m) = 7
\]

\[
10 - 1 = 10, \quad 1
\]

\[
Q_4, \quad R_4
\]

Remainder after second decimal point is

\[
4650 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \end{pmatrix} \times 10 - \begin{pmatrix} 1 & \uparrow \end{pmatrix} \times 1
\]

\[
\begin{pmatrix} 0 & 7 & 4 \end{pmatrix} \times 47
\]

\[
= 4650 - 2600 - 180 - 7 = 863 \; (R_3)
\]
5. 3rd decimal

\[
\begin{align*}
05 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 10 \end{pmatrix} &= -ve \\
15 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 9 \end{pmatrix} &= -ve \\
25 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 8 \end{pmatrix} &= -ve \\
35 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 7 \end{pmatrix} &= -ve \\
45 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 6 \end{pmatrix} &= 45 - 36 = 9 \\
\end{align*}
\]

\[\therefore Q_3(m) = 6\]

\[9 - 1 = 9, 0\]

\[Q_3 R_3\]

Remainder after third decimal point is

\[
\begin{align*}
4500 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 6 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix} \times 10 - \begin{pmatrix} 1 \\ 6 \end{pmatrix} \times 1 \\
= 4500 - 3600 - 190 - 6 = 704 (R_3)
\end{align*}
\]
Vedic Mathematics

6. 4th decimal

\[
\begin{align*}
0 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} & = -ve \\
10 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} & = -ve \\
20 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} & = -ve \\
30 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} & = -ve \\
40 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} & = 40 - 34 = 6 \\
\end{align*}
\]

\[ \therefore Q_5(m) = 5 \]

\[ 6 + 1 = 6, \ 0 \]

\[ Q_6 \ R_6 \]

Remainder after fourth decimal point is

\[
4000 - \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \end{pmatrix} \times 10 \begin{pmatrix} 1 \end{pmatrix} \times 1 = 4000 - 3400 - 160 - 5 = 435 \ R_4 \ \text{and so on}
\]

Final Quotient = 74.7656
**Vedic Mathematics**

**Example 5:** \( 123456789 + 10321 \)

**Vedic Method:**

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<tr>
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<td>4</td>
</tr>
</tbody>
</table>

**Current Method:**

\[
10321 \big| 123456789 \big| 11961.70807
\]

\[
\begin{align*}
10321 & ) 123456789 \\
20246 & \quad 0
\end{align*}
\]

\[
\begin{align*}
10321 & ) 99257 \\
92889 & \quad 63688
\end{align*}
\]

\[
\begin{align*}
10321 & ) 61926 \\
17629 & \quad 41397
\end{align*}
\]

\[
\begin{align*}
10321 & ) 17629 \\
10321 & \quad 73080
\end{align*}
\]

\[
\begin{align*}
10321 & ) 72247 \\
72247 & \quad 0
\end{align*}
\]

\[
\begin{align*}
10321 & ) 83300 \\
82568 & \quad 73200
\end{align*}
\]

\[
\begin{align*}
10321 & ) 72147 \\
1153 & \quad 1
\end{align*}
\]

\[
\begin{align*}
\text{Quotient} = & 11961.70807
\end{align*}
\]

---

**C) WORKING DETAILS OF A FEW OF EXAMPLES ALREADY GIVEN IN THIS LECTURE NOTES**

1) These consist of details of Vinculum Method for some examples already given in the text.
2) Include decimal working.
3) Details of reduction method.
4) Include working details of 16 decimals using vinculum method for one problem.

**Problem 1:**

a) Consider \( 98765 + 399 \)

\[
D_1, D_2
\]

\[
\begin{array}{cccc}
9 & 9 & 9 & 8 \\
\hline
0 & 7 & 6 & 5
\end{array}
\]

\[
\begin{array}{cccc}
3 & 6 & 8 & 1 \\
Q_1 & Q_2 & Q_3 & -1 \\
& & & 212
\end{array}
\]

Quotient = \( 3 \overline{67} = 247 \)
Vedic Mathematics

\[
\begin{array}{ccc}
D_1 & D_2 & D_3 \\
\end{array}
\]

\[
\text{Remainder } = 65 - \left( \begin{array}{c}
5 \\
9 \\
9 \\
\end{array} \right) 10 - \left( \begin{array}{c}
8 \\
9 \\
8 \\
\end{array} \right) = 65 - (220) - 72 = 65 + 220 + 72
\]

\[
\begin{array}{ccc}
Q_2 & Q_3 & Q_4 \\
\end{array}
\]

\[
213 + 399 = 212
\]

When the remainder is negative one has to add \( n \) times the divisor where \( n = 1, 2, 3, \ldots \) to get positive value and finally one has to deduct \( n \) from the quotient.

\[
Q = 247; \quad R = 212
\]

(b) Using vinculum in both divisor and dividend

\[
\begin{array}{c}
\text{Divisor} = 399 = 40\overline{1} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Dividend} = 98765 = 101\overline{235} \\
\end{array}
\]

\[
\begin{array}{c|cccc|c}
D_1 & 10 & 1 & 2 & 3 & 5 \\
\hline
1 & 1 & 10 & 19 & 30 & 21 \\
\hline
0 & 0 & 2 & 4 & 7 & 212 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \\
\end{array}
\]

\[
Q = 247; \quad R = 212
\]

**Problem 2:**

\[
124 + 2122
\]

\[
\begin{array}{c|cccc}
D_1D_2 & 1 : 2 & 4 & 0 & 0 \\
\hline
22 & 0 : 1 & 12 & 19 & 4 & 3 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\
\end{array}
\]

\[
Q = 0.05843
\]

1. \( 1 + 21 = 0 \)

\[
\begin{array}{c|c}
Q_1 & R_1 \\
\end{array}
\]
Vedic Mathematics

(2) \[ 12 - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = 12 - 0 = 12 + 21 = 0, 12 \]

\[ Q_2 \quad R_2 \]

\[ D_1 \]

\[ D_1 \quad D_2 \]

(3) \[ 124 - \begin{pmatrix} 2 \\ 0 \end{pmatrix} = 124 \]

\[ Q_1 \quad Q_2 \]

\[ 124 + 21 = 5, 19 \]

\[ Q_3 \quad R_3 \]

\[ D_1 \quad D_3 \]

(4) \[ 190 - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 190 - 10 = 180 \]

\[ Q_3 \quad Q_4 \]

\[ 180 + 21 = 8, 12 \]

\[ Q_4 \quad R_4 \]

\[ D_1 \quad D_3 \]

(5) \[ 120 - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = 120 - 26 = 94 \]

\[ Q_3 \quad Q_4 \]

\[ 94 + 21 = 4, 10 \]

\[ Q_5 \quad R_5 \]

\[ D_1 \quad D_3 \]

(6) \[ 100 - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 100 - 24 = 76 \]

\[ Q_4 \quad Q_5 \]

\[ 76 + 21 = 3, 13 \]

\[ Q_6 \quad R_6 \]

Quotient = 0.05843

Problem 3(a) : \[ 12.4 + 2122 \]

a) Decimal in dividend considered

Current Method

\[
\begin{array}{c}
\frac{12.4}{2122} = \frac{124}{21220} \\
21220 ) 124000 ( 0.005843 \\
106100 \\
179000 \\
169760 \\
92400 \\
84860 \\
75200 \\
63660 \\
11540
\end{array}
\]

Vedic Method

\[
\begin{array}{c|cccccc}
22 & 1 & 2 & . & 4 & 0 & 0 & 0 & 0 \\
\hline
& / & / & / & / & / & / \\
21 & 1 & 12 & 19 & 12 & 10 & 13 \\
0 & 0 & 5 & 8 & 4 & 3 & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6
\end{array}
\]

Quotient = 0.005843

Answer is same as that of Problem 2 but with one decimal shifted to left
Vedic Mathematics

Division

(1) \[1 + 21 = 0, \quad 1 \]
\[Q_1 \quad R_1\]

(2) \[12 - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 12 - 0 = 12, \quad 12 + 21 = 0, \quad 12 \]
\[Q_2 \quad R_3 \]
\[Q_1\]

(3) \[124 - \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix} = 124, \quad 124 + 21 = 5, \quad 19 \]
\[Q_3 \quad Q_3 \]

(4) \[190 - \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} = 180, \quad 180 + 21 = 8, \quad 12 \]
\[Q_4 \quad Q_4 \]

(5) \[120 - \begin{pmatrix} 2 \\ 2 \\ 8 \\ 5 \end{pmatrix} = 120 - 26 = 94, \quad 94 + 21 = 4, \quad 10 \]
\[Q_5 \quad R_5 \]
\[Q_4 \]

(6) \[100 - \begin{pmatrix} 2 \\ 2 \\ 8 \\ 0 \end{pmatrix} = 100 - 24 = 76 + 21 = 3, \quad 13 \]
\[Q_6 \quad R_6 \]
\[Q_5 \]

The answer is 0.0 0 5 8 4 3
Problem 3 (b) \[ 1.24 + 2122 = 124 + 212200 \]

The answer is same as that in Problem 2 but with the decimal shifted to left by two more digit.

i.e., Quotient = 0 0 0 0 0 0 5 8 4 3

Problem 3 (c) \[ 0.124 + 2122 \] answer is same as that in Problem 2 but with decimal shifted to left by three more digits i.e. quotient = 0.00005843

Problem 3 (d) \[ 0.0124 + 2122 = 124 + 2122000 \]

Vedic Method

\[
\begin{array}{cccccccc}
2 & 2 & 0 & 1 & 2 & 4 & 0 & 0 & 0 & 0 \\
\hline
2 & 1 & 1 & 2 & 12 & 19 & 12 & 10 & 13 \\
: & 0 & 0 & 0 & 0 & 5 & 8 & 4 & 3 \\
\end{array}
\]

(Answer is same as that in problem 2 but with decimal shifted to left by four digits)

Answer is 0.000005843

Problem 4 (Decimal in the divisor)

Consider \[ 124 + 2122 \]

Vedic Method

\[
\begin{array}{cccccccc}
D_1 & D_2 \\
2 & 2 \\
\hline
1 & 2 & 4 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 2 & 12 & 19 & 12 & 10 & 13 & 11 \\
\hline
0 & 5 & 8 & 4 & 3 & 5 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
\end{array}
\]

Current Method

\[
\begin{array}{c}
1240 + 2122 \\
2122)12400 (584344 \\
10610 \\
1061 \times 3 \\
17900 \\
16976 \\
9240 \\
8488 \times 8 \\
7520 \\
6366 \\
11540 \\
10610 \\
9300 \\
8488 \\
9120 \\
\end{array}
\]
Vedic Mathematics

Division

(1) \[ 1 + 21 = 0.1 \]
   \[ Q_1 R_1 \]

(2) \[ 12 + 21 = 0.12 \]
   \[ Q_2 R_2 \]

(3) \[ 124 + 21 = 5, 19 \]
   \[ Q_3 R_3 \]
   \[ D_1 D_2 \]
   \[ 190 - \begin{pmatrix} 2 \\ 0 \\ 2 \\ 5 \end{pmatrix} = 180, 180 + 21 = 8, 12 \]
   \[ Q_4 R_4 \]
   \[ Q_2 Q_3 \]
   \[ D_1 D_2 \]

(4) \[ 120 - \begin{pmatrix} 2 \\ 5 \\ 2 \\ 8 \end{pmatrix} = 120 - 26 = 94 + 21 = 4, 10 \]
   \[ Q_5 R_5 \]
   \[ Q_3 Q_4 \]
   \[ D_1 D_2 \]

(5) \[ 100 - \begin{pmatrix} 2 \\ 8 \\ 2 \\ 4 \end{pmatrix} = 100 - 24 = 76 + 21 = 3, 13 \]
   \[ Q_6 R_6 \]
   \[ Q_4 Q_5 \]
   \[ D_1 D_2 \]

(6) \[ 130 - \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = 116 + 21 = 5, 11 \]
   \[ Q_7 R_7 \]
   \[ Q_5 Q_6 \]

Ans = 0.05843

Due to the decimal in the divisor (21.2) the decimal is shifted by one digit to its right to get final Quotient as 0.5843
**Problem 5:** (Decimals in both divisor and dividend)

**Vedic Method**

\[
124 \div 2.122
\]

\[
\begin{array}{cccccccc}
D_1 & D_2 \\
2 & 2 & 1 & 2 & \cdot & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
21 & 1 & 12 & 19 & 12 & 10 & 13 & 11 \\
0 & 0 & 5 & 8 & 4 & 3 & 5 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
\end{array}
\]

\[2122 \) 12400(5.84354 \]

\[
\begin{array}{cccccccc}
124 & 124 & 12400 & 124000 \\
2.122 & 21.22 & 2122 & 2122 \\
\end{array}
\]

\[
10610 \\
17900 \\
16976 \\
9240 \\
8488 \\
7520 \\
6366 \\
11540 \\
10610 \\
930 \\
\]

Final answer 5.8435 as the divisor has 3 decimal points.

\[1 \div 21 = 0.1 \\
Q_1 R_1 \]

\[12 \div 12, 13 \div 21 = 0.12 \\
Q_2 R_2 \]

(3) \[124 \div 2.0 \] 
\[124, 124 + 21 = 5, 19 \\
Q_1 R_3 \]

(4) \[190 \div 2.5 \] 
\[180, 180 + 21 = 8, 12 \\
Q_4 R_4 \]

(5) \[120 \div 2.8 \] 
\[120 - 26 = 94, 94 + 21 = 4, 10 \\
Q_5 R_5 \]
Vedic Mathematics

\[
\begin{align*}
(6) & \quad 100 - \binom{2}{2} = 76, \ 76 + 21 = 3, 13 \\
& \quad Q_6 R_6 \\
& \quad Q_5 \\
(7) & \quad 130 - \binom{2}{3} = 116 + 21 = 5, 11 \\
& \quad Q_7 R_7 \\
(8) & \quad 110 - \binom{2}{3} = 94 + 21 = 4, 10 \\
& \quad Q_8 R_8 \\
\text{Ans} & \quad .00584354
\end{align*}
\]

As there is a decimal in the divisor with three digits after the decimal, the decimal in the answer is to be shifted by three digits towards right

Quotient = 5 84354

Problem 6: (Decimals in both divisor and dividend)

\[
\begin{align*}
\text{Vedic Method} \\
1.24 + 21.22 \\
D_1 D_2 \\
\begin{array}{cccccccc}
2 & 2 & : & 1 & 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
21 & : & 1 & 1 & 12 & 19 & 12 & 10 & 13 & 11 \\
\end{array} \\
\begin{array}{cccccccc}
\cdot & 0 & 0 & 0 & 5 & 8 & 4 & 3 & 5 \\
Q_1 Q_2 Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 \\
\end{array} \\
(1) & 1 + 21 = 0, 1 \\
& Q_2 R_2 \\
(2) & 12 + 21 = 0, 12 \\
& Q_3 R_3 \\
(3) & 124 + 21 = 5, 19 \\
& Q_4 R_4 \\
D_1 & D_2 \\
(5) & 190 - .180, 180 + 21 = 8, 12 \\
& Q_5 R_5 \\
& Q_3 Q_4 \\
\end{align*}
\]

\[
\begin{align*}
\text{Current Method} \\
\begin{array}{cccccccc}
1 & 2 & 4 & & & & & \\
2 & 1 & 2 & 2 & : & 1 & 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 2 & 2 & ) & 1 & 2 & 4 & 0 & 0 & ( & .0 & 5 & 8 & 4 & 3 & 5 \\
\end{array} \\
\begin{array}{cccccccc}
1 & 0 & 6 & 1 & 0 \\
\end{array} \\
\begin{array}{cccccccc}
1 & 7 & 9 & 0 & 0 \\
\end{array} \\
\begin{array}{cccccccc}
1 & 6 & 9 & 7 & 6 \\
\end{array} \\
\begin{array}{cccccccc}
9 & 2 & 4 & 0 \\
8 & 4 & 8 & 8 \\
7 & 5 & 2 & 0 \\
6 & 3 & 6 & 6 \\
1 & 1 & 5 & 4 & 0 \\
1 & 0 & 6 & 1 & 0 \\
\end{array}
\]
Vedic Mathematics

\[
\begin{array}{c}
D_1 & D_2 \\
6 & 120 - \begin{pmatrix} 2 & 2 \\
8 & 5 \\
\end{pmatrix} = 94, 94 + 21 = 4, 10 \\
\hline
Q_4 & Q_3 \\
D_1 & D_2 \\
76, 76 + 21 = 3, 13 \\
\hline
Q_6 & Q_5 \\
D_1 & D_2 \\
116, 116 + 21 = 5, 11 \\
\hline
Q_8 & Q_7
\end{array}
\]

Ans = 0.00058435

Due to two decimals in the divisor, the decimal in the answer is shifted by two digits.

The final answer = 0.058435

**Problem 7:** (Decimal in both divisor and dividend consider)

<table>
<thead>
<tr>
<th>Vedic Method</th>
<th>Current Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.124 + 0.2122</td>
<td>$\frac{0.124}{0.2122}$ - 1240</td>
</tr>
<tr>
<td>$\frac{2122}{2122}$</td>
<td>$\frac{10610}{17900}$</td>
</tr>
</tbody>
</table>

122 . 1 2 4 0 0 0 0 0 0 0

\[
\begin{array}{c}
. 0 0 0 6 1 5 7 4 15 4 \\
Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7 Q_8 Q_9 Q_{10} Q_{11}
\end{array}
\]

Answer is 0.584355

In view of the decimal in the divisor, it is shifted by three digits to the right. (Refer partition rules page: )
Vedic Mathematics

\[ Q_1 = Q_2 = Q_3 = 0 \]

(1) \( 1 + 2 = 0, \quad 1 \)

\[ Q_4 \quad R_4 \]

(2) \( 12 + 2 = 6, \quad 0 \)

\[ D_1 \quad Q_5 \quad R_5 \]

\( \left( 1 \right) \)

(3) \( 4 - \quad 6 = -2, \quad 2 + 2 = 1, \quad 0 \)

\[ Q_6 \quad R_6 \]

\[ Q_3 \]

\[ D_1 \quad D_2 \]

(4) \( 0 - \left( \begin{array}{c} 1 \\ 6 \\ 2 \\ 1 \end{array} \right) \quad -11, \quad 11 + 2 = 5, \quad 1 \)

\[ Q_7 \quad R_7 \]

\[ Q_3 \quad Q_6 \]

\[ D_1 \quad D_2 \quad D_3 \]

(5) \( 10 - \left( \begin{array}{c} 1 \\ 6 \\ 2 \\ 5 \end{array} \right) \quad 10 - 5 = 15, \quad 15 + 2 = 7, \quad 1 \)

\[ Q_8 \quad R_8 \]

\[ Q_9 \quad Q_6 \quad Q_7 \]

\[ D_1 \quad D_2 \quad D_3 \]

(6) \( 10 - \left( \begin{array}{c} 1 \\ 1 \\ 2 \\ 5 \end{array} \right) = 10 - 19 = 10 + 19 = 9 + 2 = 4, \quad 1 \)

\[ Q_9 \quad R_9 \]

\[ Q_6 \quad Q_7 \quad Q_8 \]

\[ D_1 \quad D_2 \quad D_3 \]

(7) \( 10 - \left( \begin{array}{c} 1 \\ 5 \\ 2 \\ 4 \end{array} \right) = 10 - 20 = 10 + 20 = 30 + 2 = 15, \quad 0 \)

\[ Q_{10} \quad R_{10} \]

\[ Q_7 \quad Q_8 \quad Q_9 \]

\[ \text{Ans} = 0.00006 \bar{1} \bar{5} \bar{7} \bar{4} 15 \bar{4} \]

\[ 0.00006 \bar{1} \bar{5} \bar{7} 546 \]

\[ 0.0000584 354 \]

As there are four digits after the decimal in the divisor the decimal has to be shifted by 4 digits.

\[ \therefore \quad \text{Final Quotient} = 0.584354 \]
Problem 8: 7896456 ÷ 34 (Vinculum details for example 3 in page: )

\[
\begin{array}{ccccccc}
4 & 7 & 8 & 9 & 6 & 4 & 5 \\
| & 1 & 1 & 1 & 2 & 1 & : 2 \\
\hline
2 & 3 & 2 & 2 & 5 & \tilde{1} & : 24 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & \text{Remainder}
\end{array}
\]

Step 1: 3) 7 (2 (Q_1)

\[
\begin{array}{c}
\frac{6}{3} \\
\frac{1}{\tilde{1}} (R_1)
\end{array}
\]

Step 2: \[18 - \frac{4\uparrow}{2} = 18 - 8 = 10\]

\[
\begin{array}{c}
\frac{3}{2} (Q_2) \\
\frac{1}{\tilde{1}} (R_2)
\end{array}
\]

Step 3: \[19 - \frac{4\uparrow}{3} = 19 - 12 = 7\]

\[
\begin{array}{c}
\frac{3}{2} (Q_3) \\
\frac{1}{\tilde{1}} (R_3)
\end{array}
\]

Step 4: \[16 - \frac{4\uparrow}{2} = 6 - 8 = 8\]

\[
\begin{array}{c}
\frac{3}{2} (Q_4) \\
\frac{1}{2} (R_4)
\end{array}
\]

Step 5: \[24 - \frac{4\uparrow}{2} = 24 - 8 = 16\]

\[
\begin{array}{c}
\frac{3}{15} (Q_5) \\
\frac{1}{\tilde{1}} (R_5)
\end{array}
\]

Step 6: \[15 - \frac{4\uparrow}{5} = 15 - 20 = \tilde{5}\]

\[
\begin{array}{c}
\frac{3}{\tilde{5}} (Q_6) \\
\frac{2}{2} (R_6)
\end{array}
\]
Step 7: \( D_1 \) Remainder

\[
\begin{align*}
26 - & = 26 - 4 = 26 + 4 = 10 \\
\end{align*}
\]

\(\bar{10} + 34 = 24\) (\(\because\) \(\bar{10}\) is negative, one has to add \((n - 1)\) times the divisor to it until it becomes positive) \(n = 1\) and hence \(1\) is to be subtracted from the quotient.

\(\because\) Quotient = 23225\(\bar{1}\) - 1 = 23225\(\bar{2}\) = 232248

Remainder = 24

**Problem 9:** 897356 + 721 (Vinculum details of example 7 in page)

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccccccccc}
& & 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
1 & 3 & 4 & 5 & 1 & 0 & 1 & 5 & 0 & 1 & 1 & 2
\end{array}
\]

<table>
<thead>
<tr>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
<th>R6</th>
<th>R7</th>
<th>R8</th>
<th>R9</th>
<th>R10</th>
<th>R11</th>
<th>R12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>(\bar{1})</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
<td>Q5</td>
<td>Q6</td>
<td>Q7</td>
<td>Q8</td>
<td>Q9</td>
<td>Q10</td>
<td>Q11</td>
<td>Q12</td>
</tr>
</tbody>
</table>

**Step 1:** \(7 \) \(\bar{8}\) (1 (Q1)

\[
\begin{align*}
7 \\
\bar{1} \quad (R_1)
\end{align*}
\]

\[
D_1
\]

\[
\begin{array}{c}
7 \\
14 \\
\bar{3}
\end{array}
\] \(\Rightarrow\) \(R_2\)

**Step 2:** 19 - \(\uparrow\)\(\underline{1}\) = 17

\[
\begin{align*}
& (2) \\
& 14 \\
& \bar{3}
\end{align*}
\]

\[
Q_1
\]

\[
D_1 \quad D_2
\]

\[
\begin{array}{c}
7 \\
28 \\
\bar{4}
\end{array}
\] \(\Rightarrow\) \(R_3\)

**Step 3:** 37 - \(\uparrow\)\(\underline{1}\)\(\underline{2}\) = 32

\[
\begin{align*}
& (2) \\
& \bar{4}
\end{align*}
\]

\[
Q_1 \quad Q_2
\]
Vedic Mathematics

Division

Step 4:
\[ D_1 \quad D_2 \]
\[ Q_2 \quad Q_3 \]
\[ D_1 \quad D_2 \]
\[ 55 - \begin{pmatrix} 2 & 1 \\ 4 & 4 \end{pmatrix} = 43 \]
\[ Q_5 \quad Q_4 \]
\[ 7 \) 33 \( 4 \) (Q_4) \]
\[ 28 \]
\[ \bar{5} \quad (R_4) \]

Step 5:
\[ 7 \) 43 \( 6 \) (Q_5) \]
\[ 42 \]
\[ \bar{1} \quad (R_5) \]

Step 6:
\[ D_1 \quad D_2 \]
\[ 16 - \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix} = 0 \]
\[ Q_4 \quad Q_3 \]
\[ 7 \) 0 \( 0 \) (Q_6) \]
\[ 0 \]
\[ \bar{0} \quad (R_6) \]

Step 7:
\[ D_1 \quad D_2 \]
\[ 00 - \begin{pmatrix} 2 & 1 \\ 6 & 0 \end{pmatrix} = 0 \]
\[ Q_5 \quad Q_6 \]
\[ 7 \) 6 \( 1 \) (Q_7) \]
\[ 7 \]
\[ \bar{1} \quad (R_7) \]

Step 8:
\[ D_1 \quad D_2 \]
\[ 10 - \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} = 12 \]
\[ Q_6 \quad Q_7 \]
\[ 7 \) 12 \( 1 \) (Q_8) \]
\[ 7 \]
\[ \bar{5} \quad (R_8) \]

Step 9:
\[ D_1 \quad D_2 \]
\[ 50 - \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = 49 \]
\[ Q_7 \quad Q_8 \]
\[ 7 \) 49 \( 7 \) (Q_9) \]
\[ 49 \]
\[ \bar{0} \quad R_9 \]

Step 10:
\[ D_1 \quad D_2 \]
\[ 0 - \begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix} = 15 \]
\[ Q_8 \quad Q_9 \]
\[ 7 \) 15 \( 2 \) (Q_{10}) \]
\[ 14 \]
\[ \bar{1} \quad (R_{10}) \]
Step 11:  
\[ \begin{array}{c} 7)13 \underbrace{\overline{2}}_{14} (Q_{11}) \\
14 \\
1 (R_{11}) \end{array} \]

\[ \frac{10}{-} \]

Step 12:  
\[ \begin{array}{c} 7)16(2(Q_{12}) \\
2 \\
(R_{12}) \end{array} \]

Quotient = \[1244.60\|1722 = 1244.5991682\]

**Problem 10:** 7652 + 23 (Vinculum details of example • 5 in page )

\[
\begin{array}{cccccccccccc}
D_1 \\
3 & 7 & 6 & 5 : 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 \\
3 & 3 & 3 : 3 & 0 & 5 & 7 & 5 & 2 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 \\
\end{array}
\]

**Step 1:**  
\[ \begin{array}{c} 2)7(3 \; (Q_1) \\
\frac{6}{-} \; (R_1) \end{array} \]

\[ \begin{array}{c} \begin{array}{c} D_1 \\
\left( \begin{array}{c} 3 \\
3 \end{array} \right) \end{array} \\
\begin{array}{c} \begin{array}{c} 3 \\
3 \end{array} \end{array} \\
\begin{array}{c} 16 - \left( \begin{array}{c} 3 \\
3 \end{array} \right) \end{array} = 16 - 9 = 7 \\
2)7(3 \; (Q_2) \\
\frac{6}{-} \; (R_2) \end{array} \]

\[ \begin{array}{c} 15 - = 15 - 9 = 6 \\
\left( \begin{array}{c} 3 \\
3 \end{array} \right) \end{array} \]

**Step 3:**  
\[ \begin{array}{c} 2)6(3 \; (Q_3) \\
\frac{6}{-} \; (R_3) \end{array} \]

\[ \begin{array}{c} \left( \begin{array}{c} 3 \\
3 \end{array} \right) \end{array} \]

\[ \begin{array}{c} Q_2 \end{array} \]
Vedic Mathematics

Division

\[
\begin{array}{c}
D_1 \\
\rightarrow \backslash \\
\text{Step 4 : } & 02 - \quad 2 + 9 = 7 & 2 \cdot 7 (Q_4) \\
& & \frac{\bar{3}}{6} (R_4)
\end{array}
\]

\[
\begin{array}{c}
\text{Step 5 : } & 10 - \quad 10 - \bar{9} = 19 = 1 & 2 \cdot 1 (0) (Q_5) \\
& & \frac{0}{1} (R_5)
\end{array}
\]

\[
\begin{array}{c}
D_1 \\
\rightarrow \backslash \\
\text{Step 6 : } & 10 - \quad 10 - 0 = 10 & 2 \cdot 10 (5) (Q_6) \\
& & \frac{\bar{1}0}{0} (R_6)
\end{array}
\]

\[
\begin{array}{c}
\text{Step 7 : } & 0 - \quad 0 - 15 = 15 & 2 \cdot 15 (7) (Q_7) \\
& & \frac{14}{1} (R_7)
\end{array}
\]

\[
\begin{array}{c}
\text{Step 8 : } & 10 - \quad 10 - 21 = 11 & 2 \cdot \bar{1}1 (5) (Q_8) \\
& & \frac{\bar{1}0}{1} (R_8)
\end{array}
\]

\[
\begin{array}{c}
Q_7 \\
D_1 \\
\left( \begin{array}{c} 3 \end{array} \right) \\
\text{Step 9 : } & \bar{1}0 - \quad \uparrow = \bar{1}0 - \bar{1}5 = 5 & 2 \cdot 5 (2) (Q_9) \\
& & \frac{4}{1} (R_9)
\end{array}
\]

\[
\text{Quotient } = 333.\overline{3}05752 = 332.695652
\]
**Problem 11:** $8954 \div 89$ (Vinculum details for example: 6 in Page )

$$
\begin{array}{c|cccccccccccc}
 & 8 & 9 & 5 & : & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
8 & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} & R_{11} \\
\hline
1 & 0 & 1 & : & 4 & 1 & 4 & 8 & 6 & 1 & 6 & 3 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} \\
\end{array}
$$

**Step 1:**

\[
8 \bigg) \begin{array}{c}
8 \\
\underline{8} \\
0 \\
\underline{0}
\end{array} (Q_1)
\]

\[
\begin{array}{c}
9 \\
\underline{9}
\end{array} (R_1)
\]

**Step 2:**

\[
9 - \begin{array}{c}
1 \\
\underline{1}
\end{array} = 0
\]

\[
8 \bigg) \begin{array}{c}
0 \\
\underline{0}
\end{array} (Q_2)
\]

\[
0 \\
\underline{0} (R_2)
\]

**Step 3:**

\[
5 - \begin{array}{c}
9 \\
\underline{0}
\end{array} = 5
\]

\[
8 \bigg) \begin{array}{c}
5 \\
\underline{5}
\end{array} (Q_3)
\]

\[
\begin{array}{c}
8 \\
\underline{3}
\end{array} (R_3)
\]

**Step 4:**

\[
\begin{array}{c}
34 \\
\underline{35}
\end{array} = \begin{array}{c}
9 \\
\underline{1}
\end{array}
\]

\[
8 \bigg) \begin{array}{c}
35 \\
\underline{32}
\end{array} (Q_4)
\]

\[
\begin{array}{c}
3 \\
\underline{3} (R_4)
\end{array}
\]

**Step 5:**

\[
\begin{array}{c}
30 \\
\underline{24}
\end{array} = \begin{array}{c}
9 \\
\underline{4}
\end{array}
\]

\[
8 \bigg) \begin{array}{c}
6 \\
\underline{6}
\end{array} (Q_5)
\]

\[
\begin{array}{c}
8 \\
\underline{2} (R_5)
\end{array}
\]
Vedic Mathematics

Step 6: \[ \overline{20} \div \overline{9} = \overline{29} \]

\[ \overline{Q_5} \]

\[ D_1 \]

Step 7: \[ \overline{30} \div \overline{4} = \overline{66} \]

\[ \overline{Q_6} \]

\[ D_1 \]

Step 8: \[ \overline{20} \div \overline{8} = \overline{52} \]

\[ \overline{Q_7} \]

\[ D_1 \]

Step 9: \[ \overline{40} \div \overline{6} = \overline{40 - 54 = 14} \]

\[ \overline{Q_8} \]

\[ D_1 \]

Step 10: \[ \overline{60} \div \overline{51} = \overline{51} \]

\[ \overline{Q_9} \]

Step 11: \[ 30 \div 54 = \overline{24} \]

\[ 8 \div 24 \overline{3} (Q_{11}) \]

\[ \frac{24}{0} (R_{11}) \]

Quotient = \[ 101 \overline{41486163} = 10060674157 \]
Problem 12: 89124 \div 5378  \hspace{1em} (\text{Vinculum details of Example 10 in Page No.})

\[ \begin{array}{cccccccc}
D_1 & D_2 & D_3 \\
\begin{array}{cccccccc}
8 & 9 & : & 1 & 2 & 4 & 0 & 0 & 0 \\
/ & / & / & / & / & / & / \\
3 & : & 1 & 2 & 1 & 2 & 0 & 2 \\
R_1 & : & R_7 & R_4 & R_1 & R_6 & R_7 \\
1 & 7 & : & 3 & 13 & 0 & 19 & 9 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
\end{array}
\end{array} \]

(1) \hspace{1em} 5 \hspace{1em} 8 \hspace{1em} (1 \hspace{1em} Q_1) \\
\hspace{2em} 2 \\
\hspace{3em} 2 \hspace{1em} (R_1) \\
\hspace{4em} D_1 \\
\hspace{5em} \left( \begin{array}{c}
3 \\
\uparrow \\
1 \\
\end{array} \right) = 36 \\
\hspace{6em} Q_1 \\
\hspace{7em} D_1 \hspace{1em} D_2 \\
5 \hspace{1em} 36 \hspace{1em} (7 \hspace{1em} Q_2) \\
\hspace{2em} 25 \\
\hspace{3em} \underline{1} \hspace{1em} (R_2) \\

(2) \hspace{1em} 39 - \left( \begin{array}{c}
3 \\
1 \\
\end{array} \right) = 36 \\
\hspace{2em} Q_1 \\
\hspace{3em} D_1 \hspace{1em} D_2 \\
5 \hspace{1em} 36 \hspace{1em} (7 \hspace{1em} Q_2) \\
\hspace{2em} 25 \\
\hspace{3em} \underline{1} \hspace{1em} (R_2) \\

(3) \hspace{1em} 11 - \left( \begin{array}{c}
3 \\
1 \\
\end{array} \right) = \bar{1} \bar{7} \\
\hspace{2em} Q_1 \\
\hspace{3em} D_1 \hspace{1em} D_2 \\
5 \hspace{1em} \bar{1} \bar{7} \hspace{1em} (3 \hspace{1em} Q_3) \\
\hspace{2em} \bar{1} \bar{5} \\
\hspace{3em} \underline{2} \hspace{1em} (R_3) \\

(4) \hspace{1em} 22 - \left( \begin{array}{c}
3 \\
1 \\
\end{array} \right) = \bar{6} \bar{6} \\
\hspace{2em} Q_1 \\
\hspace{3em} D_1 \hspace{1em} D_2 \hspace{1em} D_3 \\
5 \hspace{1em} \bar{6} \bar{6} \hspace{1em} (13 \hspace{1em} Q_4) \\
\hspace{2em} \bar{6} \bar{3} \\
\hspace{3em} \underline{1} \hspace{1em} (R_4) \\

(5) \hspace{1em} \bar{1} \bar{4} - \left( \begin{array}{c}
3 \\
1 \\
\end{array} \right) = \bar{0} \bar{2} \\
\hspace{2em} Q_1 \\
\hspace{3em} D_1 \hspace{1em} D_2 \hspace{1em} D_3 \\
5 \hspace{1em} \bar{2} \hspace{1em} (0 \hspace{1em} Q_5) \\
\hspace{2em} \bar{0} \\
\hspace{3em} \underline{2} \hspace{1em} (R_5)
Vedic Mathematics

Division

\[
\begin{align*}
D_1 D_2 D_3 &
\begin{array}{c}
3 \ 7 \ 8 \\
3 \ 1 \ 3 \\
\end{array}
\end{align*}
\]

(6) \ 20 - \left( \begin{array}{c} 3 \ 7 \ 8 \\ 3 \ 1 \ 3 \end{array} \right) = 115 = 95

\[\begin{array}{c} 5 \end{array}\]
\[\begin{array}{c} 95 \\
95 \\
\end{array}\]
\[\begin{array}{c} \text{Q} \text{Q} \text{Q} \\
\text{Q} \text{Q} \\
\end{array}\]
\[\begin{array}{c} (Q_4) \\
0 \quad (R_6) \\
\end{array}\]

\[
\begin{array}{c}
D_1 D_2 D_3 \\
3 \ 7 \ 8 \\
1 \ 3 \ 0 \ 1 \ 9
\end{array}
\]

(7) \ 00 - \left( \begin{array}{c} 3 \ 7 \ 8 \\ 1 \ 3 \ 0 \ 1 \ 9 \end{array} \right) = 153 = 47

\[\begin{array}{c} 5 \end{array}\]
\[\begin{array}{c} 47 \ (Q_3) \\
45 \\
2 \quad (R_7)
\end{array}\]

Quotient = 17.3 \ 13 \ 199 = 17.3 \ 3199 = 16.57199

Problem 13:

6543 + 89798 \quad \text{(Vinculum details of Example 12 in Page No.)}

\[
\begin{array}{c}
D_1 D_2 D_3 D_4 \\
9 \ 7 \ 9 \ 8
\end{array}
\]

\[
\begin{array}{c}
\text{9} \text{7} \text{9} \text{8} \\
\text{8}
\end{array}
\]

\[
\begin{array}{c}
\text{R}_1 \text{R}_2 \text{R}_3 \text{R}_4 \text{R}_5
\end{array}
\]

\[
\begin{array}{c}
0 \ 8 \ 7 \ 1 \ 4
\end{array}
\]

\[
\begin{array}{c}
\text{Q}_1 \text{Q}_2 \text{Q}_3 \text{Q}_4 \text{Q}_5
\end{array}
\]

\[
\begin{array}{c}
\text{!}
\end{array}
\]

(1) \ 8 \) 6 ( 0 (Q_1

\[\begin{array}{c}
\text{0} \\
\text{0}
\end{array}\]

\[
\begin{array}{c}
\text{D}_1
\end{array}
\]

\[
\begin{array}{c}
\text{(9) \\
0}
\end{array}
\]

(2) \ 65 - \left( \begin{array}{c} 9 \\
0 \end{array} \right) = 65

\[\begin{array}{c}
\text{8} \text{) 65 ( 8 \ldots (Q_2) \\
\ldots
\end{array}\]

\[
\begin{array}{c}
\ldots \\
\text{R}_2
\end{array}
\]

\[
\begin{array}{c}
\ldots
\end{array}
\]

\[
\begin{array}{c}
\ldots
\end{array}
\]

\[
\begin{array}{c}
\ldots
\end{array}
\]

\[
\begin{array}{c}
\ldots
\end{array}
\]

\[
\begin{array}{c}
\ldots
\end{array}
\]
Vedic Mathematics

\[ D_1D_2 \]

(3) \[ 14 \begin{array}{c} 7 \\ 0 \end{array} = 58 \]

\[ Q_1Q_2 \]

\[ D_1D_2D_3 \]

(4) \[ 2 \begin{array}{c} 3 \\ 0 \end{array} = 10 \]

\[ Q_1Q_2Q_3 \]

\[ D_1D_2D_3D_4 \]

(5) \[ 2 \begin{array}{c} 0 \end{array} = 46 = 34 \]

\[ Q_1Q_2Q_3Q_4 \]

Quotient = 0.08714

= 0.07286

Problem 14: \[ 78 + 21345 \] (Vinculum details of example 13 Page No up to 6 decimals places)

\[ D_1D_2D_3D_4 \]

1 3 4 5

7 8 0 0 0 0 0 0 0 0 0 0 0 0 0

As the Dhwajanka has 4 digits the quotient starts with two zeros after the decimal (shifting the partition by two digits into the left of the dividend) \(Q_1\) \(Q_2\) can be considered as 00 which are passive in the calculations, One can start writing with the digit 7 in the dividend. But while writing the quotient the decimal computed should be considered

Step 1: 00 includes \(Q_1, Q_2\)
Vedic Mathematics

Step 2: \( 2 \div 7(3) \quad (Q_3) \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 18 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= 15
\end{array} \]

Step 3: \( 18 - \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 15 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= 10 - 16 = 6
\end{array} \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 15 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= \frac{6}{0}
\end{array} \]

Step 4: \( 6(\frac{3}{4}) (Q_4) \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 6 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= \frac{1}{0}
\end{array} \]

Step 5: \( 0(\frac{15}{5}) (Q_6) \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 30 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= \frac{3}{0}
\end{array} \]

Step 6: \( 0(\frac{18}{1}) (Q_7) \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 19 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= \frac{18}{1}
\end{array} \]

Step 7: \( 0(\frac{20}{1}) (Q_8) \)

\[ \begin{array}{c}
D_1D_2D_3 \\
134 \\
003
\end{array} = 21 \quad \begin{array}{c}
Q_1 \quad Q_2 \quad Q_3 \\
= \frac{20}{1}
\end{array} \]
Vedic Mathematics

Step 8: 10 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
3 & 15 & 9 & 10 \\
Q_3 Q_6 Q_7 Q_8 \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 102 \]
\[2 \) 102 \) \(51 \) \(Q_6 \)
\[\frac{102}{0} \) \(R_6 \)

Step 9: 0 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
15 & 9 & 10 & 51 \\
Q_6 Q_7 Q_8 Q_9 \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 170 = 30 \]
\[2 \) 30 \) \(15 \) \(Q_{10} \)
\[\frac{30}{0} \) \(R_{10} \)

Step 10: 10 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
9 & 10 & 51 & 15 \\
Q_7 Q_8 Q_9 Q_{10} \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 243 = 16\overline{3} \]
\[2 \) 16\overline{3} \) \(81 \) \(Q_{11} \)
\[\frac{16\overline{2}}{1} \) \(R_{11} \)

Step 11: 0 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
10 & 51 & 15 & 81 \\
Q_8 Q_9 Q_{10} Q_{11} \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 228 \]
\[2 \) 228 \) \(11\overline{4} \) \(Q_{12} \)
\[\frac{228}{0} \) \(R_{12} \)

Step 12: 0 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
10 & 51 & 15 & 81 \\
Q_9 Q_{10} Q_{11} Q_{12} \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 42 \]

Step 13: 0 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
15 & 81 & 114 & 21 \\
Q_9 Q_{10} Q_{12} Q_{11} \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 570 \]

Step 14: 0 -
\[
\begin{array}{cccc}
1 & 3 & 4 & 5 \\
81 & 114 & 21 & 285 \\
Q_{10} Q_{12} Q_{13} Q_{14} \\
D_1 D_2 D_3 D_4
\end{array}
\]
\[= 513 \]
Vedic Mathematics

Division

Step 15: 10 -

\[
\begin{pmatrix}
1 & 3 & 4 & 5 \\
114 & 21 & 285 & 256
\end{pmatrix}
\]

= 6 \overline{15}

Q_{12} Q_{13} Q_{14} Q_{15}

= .0037 4 5 24 21 17 67

= .0037 4 5 24 21 17 67

Problem 15: 15628 + 234  
(Vinculum of Example 16 Page No. )

\[
\begin{array}{c|cccccccc}
D_1 & D_2 & 3 & 4 & 1 & 5 & 6 & 2 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
2 & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9
\end{array}
\]

\[
\begin{array}{c|cccccccc}
7 & 2 & 15 & 30 & 45 & 87 & 35 & 126 & 254 \\
\hline
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10}
\end{array}
\]

Step 1: 2) 15 (7 (Q_1)

\[
\begin{pmatrix}
14 \\
1 \\
3 \\
7
\end{pmatrix}
\]

16 - \(\dfrac{1}{7}\) = 16 - 2\overline{1} = \overline{5}

Step 2: 2) \overline{5} (2 (Q_2)

\[
\begin{pmatrix}
4 \\
1 \\
D_1 & D_2
\end{pmatrix}
\]

\[
\begin{pmatrix}
3 & 4 \\
7 & 2
\end{pmatrix}
\]

\(\overline{12} - \overline{6} = \overline{12} + 28\)

\[
\begin{array}{c|c}
Q_1 & Q_2
\end{array}
\]

\(\overline{12} - 22 = \overline{12} + \overline{22} = \overline{30}

Step 3: 2) \overline{30} (\overline{15} (Q_3)

\[
\begin{array}{c|c}
30 \\
0
\end{array}
\]

\(\overline{30} \)

(R_3)
\[
\begin{align*}
\text{Step 4: } & \quad 2 \) \ 61 \ (30) \\
& \quad 60 \\
& \quad 1 \\
& \quad \underline{15} \\
& \quad \underline{30} \\
& \quad Q_3 \quad Q_4 \\
& \quad 10 \ - \ \left( \begin{array}{c} 3 \\ 15 \\ 30 \end{array} \right) \ = \ 10 \ - \ [ \ 90 \ + \ 60 \ ] \ = \ 10 \ - \ 150 \ = \ 20 \\
\text{Step 5: } & \quad 2 \) \ 2 \ 0 \ (10) \\
& \quad 2 \ 0 \\
& \quad 0 \\
& \quad 10 \\
& \quad 0 \\
& \quad Q_3 \quad Q_4 \\
& \quad 0 \ - \ \left( \begin{array}{c} 3 \\ 30 \\ 10 \end{array} \right) \ = \ 0 \ - \ [ \ 30 \ + \ 120 \ ] \ = \ 0 \ - \ 150 \ = \ 90 \\
\text{Step 6: } & \quad 2 \) \ 9 \ 0 \ (45) \\
& \quad 9 \ 0 \\
& \quad 0 \\
& \quad 10 \\
& \quad 45 \\
& \quad Q_3 \quad Q_4 \\
& \quad 0 \ - \ \left( \begin{array}{c} 3 \\ 30 \\ 10 \end{array} \right) \ = \ 0 \ - \ [ \ 40 \ + \ 135 \ ] \ = \ 0 \ - \ 175 \ = \ 175 \\
\text{Step 7: } & \quad 2 \) \ 175 \ (87) \\
& \quad 174 \\
& \quad 1 \\
& \quad 87 \\
& \quad \underline{175} \\
& \quad Q_6 \quad Q_5 \\
& \quad 0 \ - \ \left( \begin{array}{c} 3 \\ 10 \\ 45 \end{array} \right) \ = \ 0 \ - \ [ \ 45 \ + \ 87 \ ] \ = \ 0 \ - \ 132 \ = \ 132 \\
\end{align*}
\]
Vedic Mathematics

Division

\[
\begin{array}{c}
D_1 \quad D_2 \\
3 \quad 4 \\
\hline
45 \quad 87 \\
\hline
Q_6 \quad Q_7 \\
\end{array}
\]

\[
10 - \left( \frac{3 \times 4}{45 \cdot 87} \right) = 10 \cdot \left( 261 + 180 \right) = 10 - 81 = 7 \overline{1}
\]

\[
\text{Step 8 : } 2 \overline{7} \overline{1} (3 \overline{5}) \quad (Q_8)
\]

\[
\begin{array}{c}
70 \\
\hline
1 \\
\hline
\end{array}
\quad \begin{array}{c}
D_1 \quad D_2 \\
\hline
3 \quad 4 \\
\hline
87 \quad 35 \\
\hline
Q_7 \quad Q_8 \\
\end{array}
\]

\[
\begin{array}{c}
10 = \left( \frac{3 \times 4}{87 \cdot 35} \right) = \overline{6} \overline{10} - \left( \frac{105}{348} \right) = \overline{10} - (243) = 253
\end{array}
\]

\[
\text{Step 9 : } 2 \overline{253} (1 \overline{2} \overline{6}) \quad (Q_9)
\]

\[
\begin{array}{c}
\frac{252}{1} \\
\hline
\end{array}
\quad \begin{array}{c}
(R_4)
\end{array}
\]

To get the final answer the decimal in the Quotient is shifted towards Right by 2 digits.

\[
Q = 7 \overline{2} : 15 \overline{30} \overline{10} \overline{45} \overline{87} \overline{35} \overline{126}
\]

\[
= 7 \overline{33} \overline{94} \overline{337} \overline{6}
\]

\[
= 6 \overline{67.863224} \quad \text{As the divisors has one digit after decimal.}
\]

\[
\text{Problem 16: (Vinculum of example 19 Page No )}
\]

\[
0.461397 + 123.4
\]

(a) Vinculum details of example 19 Page 81

\[
\begin{array}{c|cccccccccc}
D_1 \cdot D_2 \\
3 \quad 4 \\
\hline
\begin{array}{cccccccccc}
\end{array}
\begin{array}{cccccccccc}
4 & 6 & 1 & 3 & 9 & 7 & 0 & 0 & 0 & 0 \\
\hline
1 & 4 & 10 & 8 & 2 & 1 & 4 & 8 & 11 & 11 & 0 & 6
\end{array}
\begin{array}{cccccccccc}
R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \quad R_{11}
\end{array}
\begin{array}{cccccccccc}
\hline
\end{array}
\begin{array}{cccccccccc}
0 & 0 & 0 & 3 & 7 & 4 & 1 & 0 & 3 & 5 & 6 & 6 \overline{4}
\end{array}
\begin{array}{cccccccccc}
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} \quad Q_{11}
\end{array}
\end{array}
\]
Vedic Mathematics

Step 1: 12) 4 (0 (Q₁)

\[
\begin{array}{c}
4 \\
\hline
0
\end{array} \quad \text{(R₁)}
\]

\[
D₁ \\
\begin{pmatrix}
3 \\
0
\end{pmatrix}
\]

\[46 - \frac{3}{0} = 46\]

Q₁

Step 2: 12) 46 (3 (Q₂)

\[
\begin{array}{c}
36 \\
10
\end{array} \quad \text{(R₂)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
0
\end{pmatrix} \begin{pmatrix}
4 \\
3
\end{pmatrix} = 92
\]

Q₁Q₂

Step 3: 12) 92 (7 (Q₃)

\[
\begin{array}{c}
84 \\
8
\end{array} \quad \text{(R₃)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
0
\end{pmatrix} \begin{pmatrix}
4 \\
7
\end{pmatrix} = 50
\]

Q₃Q₄

Step 4: 12) 50 (4 (Q₄)

\[
\begin{array}{c}
48 \\
2
\end{array} \quad \text{(R₄)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
7
\end{pmatrix} \begin{pmatrix}
4 \\
4
\end{pmatrix} = \text{iii}
\]

Q₅Q₆

Step 5: 12) i (i (Q₅)

\[
\begin{array}{c}
i \\
\hline
12
\end{array} \quad \text{(R₅)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
4
\end{pmatrix} \begin{pmatrix}
4 \\
1
\end{pmatrix} = 17 - \left[3 + 16\right]
\]

Q₅Q₆

\[= 17 - 13 = 4\]

Step 6: 12) 4 (0 (Q₆)

\[
\begin{array}{c}
0 \\
4
\end{array} \quad \text{(R₆)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
1
\end{pmatrix} \begin{pmatrix}
4 \\
0
\end{pmatrix} = 40 - [4] = 44
\]

Q₆Q₇

Step 7: 12) 44 (3 (Q₇)

\[
\begin{array}{c}
36 \\
8
\end{array} \quad \text{(R₇)}
\]

\[
D₁D₂ \\
\begin{pmatrix}
3 \\
6
\end{pmatrix} \begin{pmatrix}
4 \\
3
\end{pmatrix} = 80 - 9 = 71
\]

Q₇Q₈
Vedic Mathematics

Step 8: \[ 12 \) 71 ( \ 5 (Q_8) \]
\[ \begin{array}{c}
60 \\
11 \\
D_1D_2
\end{array}
\]
\[ \frac{3\ 4}{110 - 27 = 83} \]

\[ 110 - \frac{3\ 4}{5 \ 6} = 72 \]

Step 9: \[ 12 \) 83 ( \ 6 (Q_9) \]
\[ \begin{array}{c}
72 \\
11 \\
D_1D_2
\end{array}
\]

Step 10: \[ 12 \) 72 ( \ 6 (Q_{10}) \]
\[ \begin{array}{c}
72 \\
0 \\
D_1D_2
\end{array}
\]

Step 11: \[ 12\) \ 42 ( \ 4 (Q_{11}) \]
\[ \begin{array}{c}
4\ 8 \\
6 \\
D_1D_2
\end{array}
\]

Quotient = 0.000 3 7 4 1 0 3 5 6 6 4
= 0.000 3 7 3 9 0 3 5 6 5 6

For obtaining the final answer one has to consider the decimal in divisor and shift decimal point by one on division towards right. The final answer thus becomes = 0 003739035656

(b) \[ 0.461397 + 123.4 \] (Reduction method of example: 19 Page: )

\[ \begin{array}{c}
D_1D_2 \\
3 \ 4
\end{array}
\]

\[ \begin{array}{ccccccccc}
.4 & 6 & 1 & 3 & 9 & 7 & 0 & 0 & 0 \\
4 & 10 & 8 & 14 & 4 & 8 & 8 & 9 \\
R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 \\
000 & 3 & 7 & 3 & 9 & 0 & 3 & 6 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8
\end{array}
\]
Vedic Mathematics

Step 1: 12 \) 4 ( 0 \ (Q_1)
\[ \frac{4}{0} \]
\[ \frac{4}{0} \ (R_1) \]
\[ D_1 \]
\[ \begin{array}{c} 3 \\ 0 \end{array} = 46 \]
\[ Q_1 \]

Step 2: 12 \) 46 ( 3 \ (Q_2)
\[ 36 \]
\[ 10 \ (R_2) \]
\[ D_1 D_2 \]
\[ = 101 - \begin{array}{c} 3 \\ 0 \end{array} = 101 - 9 = 92 \]
\[ Q_1 Q_2 \]

Step 3: 12 \) 92 ( 7 \ (Q_3)
\[ 84 \]
\[ 8 \ (R_3) \]
\[ D_1 D_2 \]
\[ = 83 - \begin{array}{c} 3 \\ 1 \end{array} = 83 - 33 = 50 \]
\[ Q_2 Q_3 \]

Step 4: 12 \) 50 ( 4 \ (Q_4)
\[ 48 \]
\[ 2 \ (R_4) \]
\[ D_1 D_2 \]
\[ = 29 - \begin{array}{c} 3 \\ 7 \end{array} = 29 - [12 + 28] = -ve \]
\[ Q_1 Q_4 \]

\[ = 0.0003739036 \]

The final answer is obtained by shifting the decimal towards right by one digit. Thus the final answer is 0.003739036
Problem 17: (Vinculum details for example 14 Pa\textsuperscript{v}c )

\[ 8969 + 243 \]

\[ \begin{array}{cccccccc}
\text{D1 D2} & 4 & 3 & : 8 & 9 & . & 9 & 0 & 0 & 0 & 0 & 0 \\
\hline
: & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
\hline
2 & \text{R}_1 & \text{R}_2 & \text{R}_3 & \text{R}_4 & \text{R}_5 & \text{R}_6 & \text{R}_7 & \text{R}_8 \\
\hline
& 4 & 3 & 2 & 13 & 23 & 26 & 12 & 10 \\
\text{Q}_1 & \text{Q}_2 & \text{Q}_3 & \text{Q}_4 & \text{Q}_5 & \text{Q}_6 & \text{Q}_7 & \text{Q}_8 \\
\end{array} \]

As the Dhvajanka has two digits the decimal in the dividend is shifted by two digits to the left of the dividend. The dividend is now 18969.

Step 1: \[ 2 \) \ 8 \ (4 \ (Q_1) \]

\[ \begin{array}{c}
\begin{array}{c}
\text{D}_1 \\
4 \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{R}_1 \\
0 \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Q}_1 \\
9 - \left( \begin{array}{c}
\begin{array}{c}
\text{D}_1 \\
4 \\
\end{array}
\end{array} \right) = 9 - 16 = -7
\end{array}
\end{array} \]

Step 2: \[ 2 \) \ 7 \ (3 \ (Q_2) \]

\[ \begin{array}{c}
\begin{array}{c}
\text{D}_1 \text{D}_2 \\
4 \ 3 \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{R}_2 \\
6 \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Q}_2 \\
16 - \left( \begin{array}{c}
\begin{array}{c}
\text{D}_1 \text{D}_2 \\
4 \ 3 \\
\end{array}
\end{array} \right) = 16 - (12 + 12) = 16 = 4
\end{array}
\end{array} \]

Step 3: \[ 2 \) \ 4 \ (2 \ (Q_3) \]

\[ \begin{array}{c}
\begin{array}{c}
\text{R}_3 \\
4 \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Q}_3 \\
0 \\
\end{array}
\end{array} \]
Vedic Mathematics

\[ \begin{array}{c}
D_1 D_2 \\
\begin{array}{c}
4 \\
3
\end{array} \\
\begin{array}{c}
\overline{3} \\
2
\end{array}
\end{array} = 9 - \left( \frac{8}{1} + \frac{9}{7} \right) = 9 - (1 \frac{6}{7}) = 26
\]

Step 4:

\[
2 \div 26 \ (13) (Q_4)
\]

\[
\begin{array}{c}
D_1 D_2 \\
\begin{array}{c}
26 \\
0
\end{array} \\
\begin{array}{c}
3 \\
3
\end{array}
\end{array}
\]

\[
= 0 - \left( \frac{4}{3} \right) = 0 - \left( \frac{52}{6} \right) = -(46) = \overline{46}
\]

\[
\begin{array}{c}
2 \\
13
\end{array}
\]

Step 5:

\[
2 \div \overline{46} \ (23) (Q_5)
\]

\[
\begin{array}{c}
D_1 D_2 \\
\begin{array}{c}
46 \\
0
\end{array} \\
\begin{array}{c}
3 \\
23
\end{array}
\end{array}
\]

\[
0 - \left( \frac{4}{13} \right) = - \left( \frac{92}{39} \right) = -(5 \overline{3}) = 53
\]

Step 6:

\[
2 \div 53 \ (26) (Q_6)
\]

\[
\begin{array}{c}
D_1 D_2 \\
\begin{array}{c}
53 \\
26
\end{array}
\end{array}
\]

\[
10 - \left( \frac{4}{23} \right) = 10 - \left( \frac{104}{26} \right) = 10 - 38 = 2 \overline{5}
\]

Step 7:

\[
2 \div \overline{25} \ (12) (Q_7)
\]

\[
\begin{array}{c}
\overline{24}
\end{array}
\]

\[
\begin{array}{c}
1 \ (R_7)
\end{array}
\]
Vedic Mathematics

Division

\[ \frac{D_1 D_2}{10 - \frac{4}{26 \, 12}} \]

\[ = 10 - \frac{4 \times 8 - 78}{30} = 10 - 30 = 10 + 30 = 40 \]

\[ Q_4, Q_7 \]

Step 8:

\[ 2 \frac{40}{40} (2 \, 0) \quad (Q_4) \]

\[ \frac{40}{0} \quad (R_4) \]

\[ 0.4 \, 3 \, 2 \, 13 \, 23 \, 26 \, 12 \, 20 \]

\[ 0.4 \, 3 \, 1 \, 1 \, 5 \, 4 \, 0 \]

\[ 0.36909460 \]

\[ \therefore \text{ Quotient} = 0.36909460 \]

Problem 18:

\[ 134 \, 289 \div 2 \, 76 \quad (\text{Reduction Method of Example 17 Page No.}) \]

\[ 76 \]

\[ 2 \]

\[ 1 \]

\[ 3 \]

\[ 5 \]

\[ 8 \]

\[ 0 \]

\[ 0 \]

\[ 0 \]

\[ 0 \]

\[ Q_1(m) \]

\[ 5 \]

\[ 12 \]

\[ 10 \]

\[ 8 \]

\[ 8 \]

\[ 6 \]

\[ 5 \]

\[ 0 \]

\[ Q_2(m) \]

\[ 4 \]

\[ 11 \]

\[ 9 \]

\[ 7 \]

\[ 7 \]

\[ 5 \]

\[ 1 \]

\[ Q_3(m) \]

\[ 10 \]

\[ 8 \]

\[ 6 \]

\[ 6 \]

\[ 4 \]

\[ Q_4(m) \]

\[ 9 \]

\[ 7 \]

\[ 5 \]

\[ 5 \]

\[ Q_5(m) \]

\[ 8 \]

\[ 6 \]

\[ Q_6(m) \]

\[ 1 \]

\[ Q_7(m) \]

\[ 8 \]
Vedic Mathematics

(1) 2) 1 (0 (Q₁) \[ \begin{array}{c}
0 \\
1 \\
\end{array} \] \[ \overset{\ldots}{Q₁ (0)} \]

(2) 13 \[ \begin{array}{c}
9 \\
0 \\
\end{array} \] = 13

2) 13 (6 (Q₂)

(3) 14 \[ \begin{array}{c}
7 \\
6 \\
0 \\
\end{array} \] = -28 (negative)

Q₂ is reduced by 1 \[ \Rightarrow Q₂ (m) = 5 \]

R₂ is raised by 2 \[ \Rightarrow R₂ (m) = 3 \]

34 \[ \begin{array}{c}
7 \\
6 \\
5 \\
\end{array} \] = -1 (still negative)

Q₂ is further reduced to 4 and R₂ is modified to 5

54 \[ \begin{array}{c}
7 \\
6 \\
4 \\
\end{array} \] = 26

2) 26 (13 (Q₃)

(4) 2 \[ \begin{array}{c}
7 \\
6 \\
13 \\
\end{array} \] = negative

22 \[ \begin{array}{c}
2 \\
2 \\
12 \\
\end{array} \] negative

42 \[ \begin{array}{c}
4 \\
6 \\
11 \\
\end{array} \] negative

62 \[ \begin{array}{c}
4 \\
6 \\
10 \\
\end{array} \] = negative

82 \[ \begin{array}{c}
4 \\
6 \\
9 \\
\end{array} \] = negative \[ \overset{\ldots}{Q₁ (m) = 8} \]

102 \[ \begin{array}{c}
7 \\
6 \\
8 \\
\end{array} \] = 22

2) 22 (11 (Q₄)

22

0 (R₄)
Vedic Mathematics

(5) \(08 - \left\{ \begin{array}{c} 8 \times 11 \end{array} \right\} = \text{negative}\)

\(28 - \left\{ \begin{array}{c} 8 \times 10 \end{array} \right\} = \text{negative}\)

\(- \left\{ \begin{array}{c} 8 \times 9 \end{array} \right\} = \text{negative}\)

\(68 - \left\{ \begin{array}{c} 8 \times 8 \end{array} \right\} = \text{negative}\)

\(88 - \left\{ \begin{array}{c} 8 \times 8 \end{array} \right\} = \text{negative}\)

\(108 - \left\{ \begin{array}{c} 6 \times 6 \end{array} \right\} = 18\)

2) \(18 (Q_s) \quad \frac{18}{\text{Q} (R_s)}\)

(6) \(09 - \left\{ \begin{array}{c} 9 \times 9 \end{array} \right\} = \text{negative}\)

\(29 - \left\{ \begin{array}{c} 8 \times 8 \end{array} \right\} = \text{negative}\)

\(49 - \left\{ \begin{array}{c} 6 \times 7 \end{array} \right\} = \text{negative}\)

\(69 - \left\{ \begin{array}{c} 6 \times 6 \end{array} \right\} = \text{negative}\)

\(89 - \left\{ \begin{array}{c} 6 \times 5 \end{array} \right\} = 18\)

2) \(18 (Q_s) \quad \frac{18}{\text{Q} (R_s)}\)
Vedic Mathematics

(7) \[ 00 - \left[ \begin{array}{c}
\text{negative}
\end{array} \right] \]

20 - \[ \left[ \begin{array}{c}
\text{negative}
\end{array} \right] \]

40 - \[ \left[ \begin{array}{c}
\text{negative}
\end{array} \right] \]

60 - \[ \left[ \begin{array}{c}
\text{negative}
\end{array} \right] \]

80 - \[ \left( \begin{array}{c}
7
\end{array} \right) \]

\[ \begin{array}{c}
5
\end{array} \]

\[ \begin{array}{c}
6
\end{array} \]

\[ 15 \quad 2) \quad 15 (\text{Q}_7) \]

\[ \frac{14}{1} (R_7) \]

(8) \[ 0 - \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

20 - \[ \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

40 - \[ \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

60 - \[ \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

(9) \[ 0 - \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

20 - \[ \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

40 - \[ \left( \begin{array}{c}
\text{negative}
\end{array} \right) \]

\[ Q_8(m) = 3 \]

\[ Q_8(m) = 4 \]

\[ Q_8(m) = 1 \]

\[ Q_8(m) = 3 \]

\[ 22 (11 (Q_8)) \]

\[ 22 \]

\[ \bot (R_8) \]
Vedic Mathematics

Ans = 0.48655418

(As these are two digits after decimal in the divisor, the decimal is shifted to right by two numbers)

Quotient = 48.655418
= 48.655398

Problem 19: 2.1387 ÷ 312 (Reduction Method of Example 18 Page No.)

\[
\begin{array}{c}
12 \\
2. \\
\hline
2 & 1 & 3 & 8 & 7 & 0 & 0 & 0 & 0 & 0 \\
& 2 & 0 & 0 & 1 & 2 & 1 & 0 & 2 & 0 \\
\hline
0 & 0 & 6 & 8 & 5 & 4 & 8 & 1 & 3 & 9 \\
Q_1 & Q_2 & Q_3(m) & Q_4(m) & Q_5(m) & Q_6(m) & Q_7 & Q_8(m) & Q_9(m) & Q_{10}(m) & Q_{11}(m) \\
\end{array}
\]

(1) $Q_1 = 0$ (Due to partition rule Page: )

(2) $2 - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 2$

3) $2 \ (Q_3)$

(3) $21 - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 21$

3) $21 \ (Q_3)$
Vedic Mathematics

(4) \( 03 - \begin{pmatrix} 1 \\ 0 \\ 2 \\ 7 \end{pmatrix} = \text{negative} \quad Q_3(m) = 6 \)

\[ \begin{array}{c}
33 - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 27 \\
3) 27 (Q_4) \\
\quad 27 \\
\quad 0 (R_4)
\end{array} \]

(5) \( 08 - \begin{pmatrix} 2 \\ 9 \end{pmatrix} = \text{negative} \quad Q_4(m) = 8 \)

\[ \begin{array}{c}
38 - 18 \\
3) 18 (Q_5) \\
\quad 18 \\
\quad 0 (R_5)
\end{array} \]

(6) \( 07 - \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \text{negative} \quad Q_5(m) = 5 \)

\[ \begin{array}{c}
37 - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 16 \\
3) 16 (Q_6) \\
\quad 15 \\
\quad 1 (R_6)
\end{array} \]

(7) \( 10 - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \text{negative} \quad Q_6(m) = 4 \)

\[ \begin{array}{c}
40 - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 26 \\
3) 26 (Q_7) \\
\quad 24 \\
\quad 2 (R_7)
\end{array} \]

(8) \( 20 - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 4 \quad Q_7(m) = 8 \)

\[ \begin{array}{c}
8 \end{array} \]

\[ \begin{array}{c}
3) 4 (Q_8) \\
\quad 3 \\
\quad 1 (R_8)
\end{array} \]

(9) \( 10 - \begin{pmatrix} 1 \\ 8 \end{pmatrix} = \text{negative} \quad Q_8(m) = 0 \)

\[ \begin{array}{c}
40 - \begin{pmatrix} 8 \\ 0 \end{pmatrix} = 24 \\
3) 24 (Q_9) \\
\quad 24 \\
\quad 0 (R_9)
\end{array} \]
Vedic Mathematics

\begin{align*}
\text{(10)} & \quad 0 - \left[ \begin{array}{c}
\text{1} \\
\text{7}
\end{array} \right] = \text{negative} \quad \boxed{Q_{10}(m) = 7} \\
\text{)} - \left[ \begin{array}{c}
\text{1} \\
\text{7}
\end{array} \right] = 23 \\
\text{(11)} & \quad 20 - \left[ \begin{array}{c}
\text{2} \\
\text{7}
\end{array} \right] = \text{negative} \quad \boxed{Q_{11}(m) = 6} \\
50 - \left[ \begin{array}{c}
\text{2} \\
\text{6}
\end{array} \right] = 30 \\
\text{(12)} & \quad 0 - \left[ \begin{array}{c}
\text{2} \\
\text{6}
\end{array} \right] = \text{negative} \quad \boxed{Q_{12}(m) = 9} \\
30 - \left[ \begin{array}{c}
\text{2} \\
\text{6}
\end{array} \right] = 30 \\
\text{Ans} = 0.006854807693
\end{align*}

Since divisor has two digits after decimal, the decimal in the answer has to be shifted by two digits

\therefore \quad \text{Quotient} \quad 0.6854807693

\textbf{Problem 20:} \quad 11 + 111 \quad (\text{Reduction})

\begin{align*}
\text{11} & \quad 1 \quad 1 \quad 0 \quad 0 \quad 11 \\
\text{1} & \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\text{1} & \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \\
Q_{1} & \quad \boxed{1} \quad \boxed{9} \quad \boxed{1} \quad \boxed{1} \quad \boxed{9} \quad \boxed{1} \quad \boxed{9} \quad \boxed{1} \quad \boxed{1}
\end{align*}

\begin{align*}
Q_{2}(m) \quad Q_{3}(m) \quad Q_{4}(m) \quad Q_{5}(m) \quad Q_{6}(m) \quad Q_{7}(m) \quad Q_{8}(m)
\end{align*}
(1) \[ \frac{1}{1} (Q_1) \]
\[ \frac{1}{1} (R_1) \]
\[ Q_1 = 1 \]

(2) \[ 01 - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \]
\[ 1) 0 (Q_2) \]
\[ 0 \]
\[ \underline{0} (R_2) \]

(3) \[ 00 - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \text{negative} \]
\[ 10 - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 10 \]
\[ \underline{Q_3(m) = 1} \]
\[ 1) 10 (Q_3) \]
\[ 10 \]
\[ \underline{0} (R_3) \]

(4) \[ 00 - \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \text{negative} \]
\[ 10 - \begin{bmatrix} 1 \\ 9 \end{bmatrix} = 2 \]
\[ 1) 2 (Q_4) \]
\[ 2 \]
\[ \underline{0} (R_4) \]

(5) \[ 00 - \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \text{negative} \]
\[ 10 - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \]
\[ \underline{Q_4(m) = 1} \]
\[ 1) 0 (Q_5) \]
\[ 0 \]
\[ \underline{0} (R_5) \]

(6) \[ 00 - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \text{negative} \]
\[ 10 - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 10 \]
\[ 1) 10 (Q_6) \]
\[ 10 \]
\[ \underline{0} (R_6) \]

(7) \[ 00 - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \text{negative} \]
\[ 10 - \begin{bmatrix} 1 \\ 9 \end{bmatrix} = 2 \]
\[ \underline{Q_6(m) = 0} \]
\[ 1) 2 (Q_7) \]
\[ 2 \]
\[ \underline{0} (R_7) \]
Vedic Mathematics

Division

(8) $00 - \left( \begin{array}{c} 9 \\ 0 \end{array} \right) = \text{negative}$

$10 - \left( \begin{array}{c} 9 \\ 1 \end{array} \right) = 0$

(9) $00 - \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = \text{negative}$

$10 - \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = 10$

Quotient $= 0 \quad 1 \quad 1 \quad 9 \quad 1 \quad 1 \quad 9 \quad 1 \quad 1$

$= 0 \quad 0 \quad 9 \quad 0 \quad 9 \quad 9 \quad 0 \quad 9 \quad 0 \quad 9$

Current Method

$1 \quad 1 \quad 1 \quad 1 \quad \overline{1 \quad 1 \quad 0 \quad 0 \quad (0.09909909)}$

$\overline{9 \quad 9 \quad 9}$

$\overline{1 \quad 0 \quad 1 \quad 0}$

$\overline{9 \quad 9 \quad 9}$

$\overline{1 \quad 1 \quad 0 \quad 0}$

$\overline{9 \quad 9 \quad 9}$

$\overline{1 \quad 0 \quad 1 \quad 0}$

$\overline{1 \quad 1 \quad 0 \quad 0}$

$\overline{9 \quad 9 \quad 9}$
Problem 21 (a) \( 1 + 11 \) (Reduction Method)

\[
\begin{array}{cccccccc}
1 & : & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & : & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
: & : & 1 & 10 & 1 & 10 & 1 & 10 & 1 & 10 & 1 \\
\end{array}
\]

(1) \( 1 ) 1 (Q_1) \\
\[\frac{1}{10} (R_1)\]

(2) \[0 - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \text{ (negative)}\]
Reducing \( Q_1 \) by 1 we get \( Q_1(m) = 0 \)
Adding 1 to \( R_1 \) we get \( R_1(m) = 1 \)

\[10 - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 10 \]

\[1) 10 \begin{pmatrix} 10 \\ 0 \end{pmatrix} (Q_2)\]
\[\begin{pmatrix} 10 \\ 0 \end{pmatrix} (R_2)\]

(3) \[0 - \begin{pmatrix} 1 \\ 10 \end{pmatrix} = -10 \text{ (negative)}\]
Reducing \( Q_2 \) by 1 we get \( Q_2(m) = 9 \)
Adding 1 to \( R_2 \) we get \( R_2(m) = 1 \)

\[10 - \begin{pmatrix} 1 \\ 9 \end{pmatrix} = 1 \]

\[1) 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} (Q_3)\]
\[\begin{pmatrix} 1 \\ 0 \end{pmatrix} (R_3)\]

(4) \[0 - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \text{ (negative)}\]
Reducing \( Q_3 \) we get \( Q_3(m) = 0 \)

\[10 - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 10 \]

\[1) 10 \begin{pmatrix} 10 \\ 0 \end{pmatrix} (Q_4)\]
\[\begin{pmatrix} 10 \\ 0 \end{pmatrix} (R_4)\]
(5) \[ 0 - \left( \begin{array}{c} 1 \\ 10 \end{array} \right) = -10 \text{ (negative)} \]

Reducing \( Q_4 \), we get \( Q_4(m) = 9 \)

\[ 10 - \left( \begin{array}{c} 1 \\ 9 \end{array} \right) = 1 \]

\[ \frac{1}{Q(R_3)} \]

\[ 1 \] \( Q_5 \)

(6) \[ 0 - \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = -1 \text{ (negative)} \]

Reducing \( Q_5 \), we get \( Q_5(m) = 0 \)

\[ 10 - \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = 10 \]

\[ \frac{10}{Q(R_4)} \]

\[ 1 \] \( Q_6 \)

(7) \[ 0 - \left( \begin{array}{c} 1 \\ 10 \end{array} \right) = -10 \text{ (negative)} \]

Reducing \( Q_6 \), we get \( Q_6(m) = 9 \)

\[ 10 - \left( \begin{array}{c} 1 \\ 9 \end{array} \right) = 1 \]

\[ \frac{1}{Q(R_7)} \]

\[ 1 \] \( Q_7 \)

(8) \[ 0 - \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = -1 \text{ (negative)} \]

Reducing \( Q_7 \), we get \( Q_7(m) = 0 \)

Quotient = 0.090909 ...
Problem 21: (b) \( \frac{1}{11} \) (Vinculum Method)

\[
\begin{array}{ccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\text{R}_1 & \text{R}_2 & \text{R}_3 & \text{R}_4 & \text{R}_5 & \text{R}_6 \\
\hline \\
1 & \ddots & 1 & 1 & 1 & 1 & 1 \\
\text{Q}_1 & \text{Q}_2 & \text{Q}_3 & \text{Q}_4 & \text{Q}_5 & \text{Q}_6 \\
\end{array}
\]

(1) \( 1 \) \( 1 \) (1) \( \text{Q}_1 \) \\
\( \frac{1}{0} \) (R_1)

(2) \( 00 - \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} \) \( 1 \) \( 1 \) (1) \( \text{Q}_2 \) \\
\( \frac{1}{0} \) (R_2)

(3) \( 00 - \begin{pmatrix} 1 \\ \hline \hline 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \hline \hline 1 \end{pmatrix} \) \( 1 \) \( 1 \) (1) \( \text{Q}_3 \) \\
\( \frac{1}{0} \) (R_3)

(4) \( 00 - \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} \) \( 1 \) \( 1 \) \( \text{Q}_4 \) \\
\( \frac{1}{0} \) (R_4)

(5) \( 00 - \begin{pmatrix} 1 \\ \hline \hline 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \hline \hline 1 \end{pmatrix} \) \( 1 \) \( 1 \) (1) \( \text{Q}_4 \) \\
\( \frac{1}{0} \) (R_5)

(6) \( 00 - \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \hline 1 \end{pmatrix} \) \( 1 \) \( \text{Q}_6 \) \\
\( \frac{1}{0} \) (R_6)

Quotient = 011111 = 0.090909 ..
Vedic Mathematics

Problem 22: \[ 1 \div 111 \] (Reduction)

\[ 11 : \begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \]

\[ 1 : \begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array} \]

0 \[ \begin{array}{c}
1 \\
0 \\
10 \\
9 \\
\end{array} \]

1) \[ 1 \text{ } (1 \text{ Q}_1 \]
\[ \begin{array}{c}
1 \\
0 \\
(\text{R}_1) \\
\end{array} \]

2) \[ 0 - \begin{array}{c}
1 \\
1 \\
\end{array} = -1 \text{ (negative)} \]

\[ Q_1(m) = 0, \text{ R}_1(m) = 1 \]

\[ 10 - \begin{array}{c}
1 \\
\end{array} = 10 \]

1) \[ 10 \text{ (10 } (\text{Q}_2) \\
\begin{array}{c}
10 \\
0 \\
(\text{R}_2) \\
\end{array} \]

3) \[ 0 - \begin{array}{c}
10 \text{ (negative)} \\
\end{array} \]

\[ Q_3(m) = 9, \text{ R}_3(m) = 1 \]

\[ 10 - \begin{array}{c}
9 \\
1 \\
\end{array} = 1 \]

1) \[ 1 \text{ (1 } (\text{Q}_3) \\
\begin{array}{c}
1 \\
0 \\
(\text{R}_3) \\
\end{array} \]

4) \[ 0 - \begin{array}{c}
10 \text{ (negative)} \\
9 \\
\end{array} \]

\[ Q_4(m) = 0, \text{ R}_4(m) = 1 \]

\[ 10 - \begin{array}{c}
9 \\
1 \\
\end{array} = 1 \]

1) \[ 1 \text{ (1 } (\text{Q}_4) \\
\begin{array}{c}
1 \\
0 \\
(\text{R}_4) \\
\end{array} \]

5) \[ 0 - \begin{array}{c}
10 \text{ (negative)} \\
0 \\
\end{array} \]

\[ Q_5(m) = 0, \text{ R}_5(m) = 1 \]

\[ 10 - \begin{array}{c}
0 \\
1 \\
\end{array} = 10 \]

1) \[ 10 \text{ (10 } (\text{Q}_5) \\
\begin{array}{c}
10 \\
0 \\
(\text{R}_5) \\
\end{array} \]

6) \[ 0 - \begin{array}{c}
10 \text{ (negative)} \\
0 \\
\end{array} \]

\[ Q_6(m) = 9, \text{ R}_6(m) = 1 \]

\[ 10 - \begin{array}{c}
0 \\
9 \\
\end{array} = 1 \]

1) \[ 1 \text{ (1 } (\text{Q}_6) \\
\begin{array}{c}
1 \\
0 \\
(\text{R}_6) \\
\end{array} \]

7) \[ 0 - \begin{array}{c}
10 \text{ (negative)} \\
9 \\
\end{array} \]

\[ Q_6(m) = 0, \text{ R}_6(m) = 1 \]

Quotient = 0 0090090

Division
Combined Division and Multiplication:

a)

Combined Division and Multiplication can be worked out by applying the principles that are enumerated under division.

1) \[ 978534 + (23 \times 519) \]

**Current Method**

\[
\begin{align*}
519 & \\
\times 23 & \\
1557 & \\
1038 & \\
11937 & \\
\hline
11937 & 978534 \\
& 819748 \\
& 23574 \\
& 11937 \\
& 107433 \\
& 89370 \\
& 83559 \\
& 58110 \\
& 47748 \\
& 103620 \\
& 95496 \\
& 81240
\end{align*}
\]

**Vedic Method**

\[
\begin{array}{c|c}
D & 9 \ 7 \ 8 \ 5 \ 3 : 4 \ 0 \ 0 \\
3 & R_1 \ R_3 \ R_3 \ R_4 : R_5 \ R_7 \ R_8 \\
2 & 1 \ 1 \ 2 \ 2 \ 3 \ 3 \ 3 \ 3 \ 1^{st} \ part \\
19 & Q_1 \ Q_2 \ Q_3 \ Q_4 \ Q_5 \ Q_6 \ Q_7 \ Q_8 \\
5 & R_1 \ R_2 \ R_3 \ R_4 \ R_5 \ R_6 \ R_7 \ R_8 \\
& 0 \ 8 \ 1 \ 9 \ 7 \ 4 \ 8 \ 8 \ 2 \ 2^{nd} \ part \\
& Q_1 \ Q_2 \ Q_3 \ Q_4 \ Q_5 \ Q_6 \ Q_7 \ Q_8
\end{array}
\]
Vedic Mathematics

**Step 1:**

\[
\begin{array}{c}
2) 9 \left( \frac{4}{8} (Q_1) \\
\frac{8}{1} (R_1) \\
\frac{3}{1} D \\
17 - \left( \frac{4}{4} \right) = 5 \\
Q_2
\end{array}
\]

**Step 2:**

\[
\begin{array}{c}
2) 5 \left( \frac{2}{4} (Q_2) \\
\frac{4}{1} (R_2) \\
\frac{3}{1} D \\
18 - \left( \frac{2}{2} \right) = 12 \\
Q_2
\end{array}
\]

**Step 3:**

\[
\begin{array}{c}
2) 12 \left( \frac{6}{12} (Q_3) \\
\frac{12}{0} (R_3) \\
\frac{3}{0} D \\
1 - \left( \frac{0}{1} \right) \rightarrow 18 \text{ (negative)} \\
Q_3
\end{array}
\]

\[Q_3 \rightarrow \text{Q}_3 \text{ is reduced by 1}\]

\[
\begin{array}{c}
2) 12 \left( \frac{5}{10} Q_3 (m) \\
\frac{10}{2} R_3 (m) \\
\frac{3}{2} D \\
25 - \left( \frac{5}{5} \right) = 10 \\
Q_3(m)
\end{array}
\]

**Step 4:**

\[
\begin{array}{c}
2) 10 \left( \frac{5}{10} (Q_4) \\
\frac{10}{0} (R_4) \\
\frac{3}{0} D \\
03 - \left( \frac{5}{5} \right) = -12 (-\ell) \\
Q_3
\end{array}
\]

**Reduction Q_4 by 1**

\[
\begin{array}{c}
2) 10 \left( \frac{4}{8} Q_4(m) \\
\frac{8}{2} (R_4(m)) \\
\frac{3}{2} D \\
23 - \left( \frac{4}{4} \right) = 11 \\
Q_4(m)
\end{array}
\]

**Step 5:**

\[
\begin{array}{c}
2) 11 \left( \frac{5}{10} (Q_5) \\
\frac{10}{1} (R_5) \\
\frac{5}{1} D \\
14 - \left( \frac{3}{3} \right) = -1 \text{ (negative)} \\
Q_5
\end{array}
\]

**Reduction Q_5 by 1**

\[
\begin{array}{c}
2) 11 \left( \frac{4}{8} Q_5(m) \\
\frac{8}{3} (R_5) \\
\frac{3}{3} D \\
34 - \left( \frac{4}{4} \right) = 22 \\
Q_5(m)
\end{array}
\]
Step 6:  
2) 22 (11) \((Q_6)\)  
\[
\begin{array}{c}
22 \\
\hline \\
0 \\
\hline \\
-1 \\
\hline
22 \quad \text{(R\(_6\))} \\
D \\
\hline \\
3 \\
\hline
11 \\
\hline
Q_6 \\
\text{Reducing } Q_6 \text{ by 1}
\end{array}
\]

0 - \(\uparrow\)\(= -33 \text{ (negative)}\)

\[
\begin{array}{c}
20 \\
\hline \\
2 \\
\hline
D \\
\hline \\
3 \\
\hline
10 \\
\hline
Q_6(m) \\
\text{Reducing } Q_6(m) \text{ by 1}
\end{array}
\]

2) 22 (10) \((Q_6(m))\)  
\[
\begin{array}{c}
20 \\
\hline \\
2 \\
\hline
D \\
\hline \\
3 \\
\hline
10 \\
\hline
Q_6(m) \\
\text{Reducing } Q_6(m) \text{ by 1}
\end{array}
\]

2) 22 (9) \((Q_6(m))\)  
\[
\begin{array}{c}
18 \\
\hline \\
4 \\
\hline \\
18 \\
\hline \\
4 \\
\hline
D \\
\hline \\
3 \\
\hline
9 \\
\hline
Q_6(m) \\
\text{Reducing } Q_6(m) \text{ by 1}
\end{array}
\]

Step 7:  
2) 13 (6) \((Q_7)\)  
\[
\begin{array}{c}
13 \\
\hline \\
1 \\
\hline
D \\
\hline \\
3 \\
\hline
6 \\
\hline
Q_7 \\
\text{Reducing } Q_7 \text{ by 1}
\end{array}
\]

10 - \(\uparrow\)\(= -8 \text{ (negative)}\)

\[
\begin{array}{c}
10 \\
\hline \\
3 \\
\hline
D \\
\hline \\
3 \\
\hline
5 \\
\hline
Q_7(m) \\
\text{Reducing } Q_7(m) \text{ by 1}
\end{array}
\]

2) 13 (5) \((Q_7(m))\)  
\[
\begin{array}{c}
10 \\
\hline \\
3 \\
\hline
D \\
\hline \\
3 \\
\hline
5 \\
\hline
Q_7(m) \\
\text{Reducing } Q_7(m) \text{ by 1}
\end{array}
\]

30 - \(\uparrow\)\(= 15\)

Step 8:  
2) 15 (7) \((Q_8)\)  
\[
\begin{array}{c}
15 \\
\hline \\
1 \\
\hline
D \\
\hline \\
3 \\
\hline
7 \\
\hline
Q_8 \\
\text{Reducing } Q_8 \text{ by 1}
\end{array}
\]

10 - \(\uparrow\)\(= 11\text{ (negative)}\)

\[
\begin{array}{c}
10 \\
\hline \\
3 \\
\hline
D \\
\hline \\
3 \\
\hline
7 \\
\hline
Q_8(m) \\
\text{Reducing } Q_8(m) \text{ by 1}
\end{array}
\]

2) 15 (6) \((Q_8(m))\)  
\[
\begin{array}{c}
12 \\
\hline \\
D \\
\hline \\
3 \\
\hline
6 \\
\hline
Q_8(m) \\
\text{Reducing } Q_8(m) \text{ by 1}
\end{array}
\]

30 - \(\uparrow\)\(= 12\)
II Part division 1 e, 42 544 956 + 519
Step 1: \[ 5 ) 4 (0 \quad (Q_1) \]
\[ \quad \quad 0 \]
\[ \quad 4 \quad (R_1) \]

42 - \[ \uparrow \]

Step 2: \[ 5 ) 42 \quad (8 \quad (Q_2) \]
\[ \quad 40 \]
\[ \quad \quad 2 \quad (R_2) \]
\[ \quad D_1 \quad D_2 \]

\[ 25 - \]
\[ \quad 0 \quad 8 \quad Q_1Q_2 \]

Step 3: \[ 5 ) 17 \quad (3 \quad (Q_3) \]
\[ \quad 15 \]
\[ \quad \quad 2 \quad (R_3) \]
\[ \quad D_1 \quad D_2 \]

24 - \[ 3 \quad = 24 - (3 + 72) = -ve \]

Reducing \( Q_3 \) by 1

\[ 5 ) 17 \quad (2 \quad (Q_3(m)) \]
\[ \quad 10 \]
\[ \quad \quad 7 \quad (R_3) \]

Reducing \( Q_3(m) \) by 1

\[ 5 ) 17 \quad (1 \quad (Q_3(m)) \]
\[ \quad 5 \]
\[ \quad 12 \quad (R_3) \]
Vedic Mathematics

\[ 74 - \left( \begin{array}{c} 9 \\ -2 \end{array} \right) = ( \]

\[ 04 - \left( \begin{array}{c} 2 \\ 0 \end{array} \right) = \text{negative} \]

\[ \begin{array}{c} 2 \end{array} \]
\[ \begin{array}{c} 0 \end{array} \]
\[ \begin{array}{c} 0 \\ 0 \end{array} \]
\[ \begin{array}{c} Q_4 \end{array} \]
\[ \begin{array}{c} D_1 \end{array} \]
\[ \begin{array}{c} D_2 \end{array} \]

\[ 124 - \quad 124 - 73 = 51 \]
\[ 8 \]
\[ 1 \]

\[ \begin{array}{c} Q_2 \end{array} \]
\[ \begin{array}{c} Q_3 \end{array} \]

Step 4:
\[ \begin{array}{c} 5 \end{array} \]
\[ \begin{array}{c} 51 \end{array} \]
\[ \begin{array}{c} 10 \end{array} \]
\[ \begin{array}{c} (Q_4) \end{array} \]
\[ \begin{array}{c} 50 \end{array} \]
\[ \begin{array}{c} -1 \end{array} \]
\[ \begin{array}{c} (R_4) \end{array} \]
\[ \begin{array}{c} D_1 \end{array} \]
\[ \begin{array}{c} D_2 \end{array} \]

\[ 14 - \quad = -5 \quad (\text{ve}) \]
\[ 1 \]
\[ 10 \]

\[ \begin{array}{c} Q_3 \end{array} \]
\[ \begin{array}{c} Q_4 \end{array} \]

Reducing \( Q_4 \) by 1
\[ \begin{array}{c} 5 \end{array} \]
\[ \begin{array}{c} 51 \end{array} \]
\[ \begin{array}{c} 9 \end{array} \]
\[ \begin{array}{c} (Q_4(m)) \end{array} \]
\[ \begin{array}{c} 45 \end{array} \]
\[ \begin{array}{c} -6 \end{array} \]
\[ \begin{array}{c} (R_4(m)) \end{array} \]
\[ \begin{array}{c} D_1 \end{array} \]
\[ \begin{array}{c} D_2 \end{array} \]

\[ 64 - \quad = 46 \]
\[ 1 \]
\[ 9 \]

\[ \begin{array}{c} Q_3 \end{array} \]
\[ \begin{array}{c} Q_4(m) \end{array} \]

\[ \begin{array}{c} 5 \end{array} \]
\[ \begin{array}{c} 46 \end{array} \]
\[ \begin{array}{c} 9 \end{array} \]
\[ \begin{array}{c} (Q_5) \end{array} \]
\[ \begin{array}{c} 45 \end{array} \]
\[ \begin{array}{c} -1 \end{array} \]
\[ \begin{array}{c} (R_5) \end{array} \]
19 - | 9 | = negative

Q₃ is reduced by 1

Step 5:

\[
\begin{array}{c}
5) 46 (8 (Q₃) \\
\hline
40 \\
6 (R₃)
\end{array}
\]

\[
D₁D₂
\]

\[
69 - \begin{array}{c} 9 \\
\end{array} = 69 - (81 + 8) = (-ve)
\]

\[
9 8
\]

\[Q₄(m)Q₃\]

Reducing Q₃ by 1

\[
5) 46 (7 (Q₃(m))
\]

\[
\begin{array}{c}
35 \\
11 (R₃(m))
\end{array}
\]

\[
D₁D₂
\]

\[
119 - \begin{array}{c} 19 \\
\end{array} = 119 - (88) = 31
\]

\[Q₄(m)Q₅(m)\]

Step 6:

\[
\begin{array}{c}
5) 31 (6 (Q₆) \\
\hline
30 \\
.1 (R₄)
\end{array}
\]

\[
D₁D₂
\]

\[
15 - \begin{array}{c} 19 \\
\end{array} = 5 - (6 + 63) = -ve
\]

\[Q₅ Q₆\]

Reducing Q₆ by 1
Vedic Mathematics

5 ) 31 ( 5 (Q₆(m))

\[ \frac{25}{\text{D}_1 \text{D}_2} \]

65 - \[5\] 65 - (63 + 5) = negative

\[ \frac{7}{7} \]

Q₅Q₆(m)

Reducing Q₆(m) by 1

5 ) 30 ( 4 (Q₆(m))

\[ \frac{20}{\text{D}_1 \text{D}_2} \]

115 - \[4\] 115 - (4 + 63) = 48

\[ \frac{7}{7} \]

Q₅Q₆(m)

Step 7:

5 ) 48 ( 9 (Q₇)

\[ \frac{45}{\text{D}_1 \text{D}_2} \]

36 - \[9\] = negative

\[ \frac{9}{9} \]

Q₆ Q₇

(m)

Reducing Q₇ by 1

5 ) 48 ( 8 (Q₇(m))

\[ \frac{40}{\text{D}_1 \text{D}_2} \]

86 - \[8\] = 42

\[ \frac{9}{9} \]

Q₆ Q₇

(m)(m)
Vedic Mathematics

Division

5 ) 42 ( 8 (Qₘ)

\[
\begin{array}{c}
40 \\
\hline
2 \text{(Rₙ)}
\end{array}
\]

Final Answer = 81.97488

I Part (Vinculum)

\[
\begin{array}{cccccccccc}
9 & 7 & 8 & 5 & 3: & 4 & 0 & 0 & 0 & 0 \\
\hline
1 & 1 & 0 & 1 & : & 0 & 1 & 1 & 1 & 0 & 0 \\
R₁ & R₂ & R₃ & R₄ & R₅ & R₆ & R₇ & R₈ & R₉ & R₁₀
\end{array}
\]

\[
\begin{array}{cccccccccc}
4 & 2 & 6 & \bar{7} & 17 & \bar{2}3 & 29 & \bar{3}8 & 52 & \bar{7}8 \\
Q₁ & Q₂ & Q₃ & Q₄ & Q₅ & Q₆ & Q₇ & Q₈ & Q₉ & Q₁₀
\end{array}
\]

Step 1:

2 ) 9 ( 4 (Q₁)

\[
\begin{array}{c}
8 \\
1 \text{ (R₄)}
\end{array}
\]

17 -

Step 2:

2 ) 5 ( 2 (Q₂)

\[
\begin{array}{c}
4 \\
1 \text{ (R₅)}
\end{array}
\]

18 - \uparrow = 12

Step 3:

2 ) 12 ( 6 (Q₃)

\[
\begin{array}{c}
12 \\
0 \text{ (R₆)}
\end{array}
\]
Vedic Mathematics

\[ \begin{align*}
\text{Step 4:} & \quad 2) \quad \frac{13}{14} (7 \quad (Q_4) \\
& \quad \frac{14}{14} \quad (R_4)
\end{align*} \]

\[ \text{Step 5:} \]

\[ 13 - \left[ \frac{2}{1} \right] = 13 - (2 \overline{1}) \]

\[ Q_4 = 13 + 21 = 34 \]

\[ 2) \quad 34 (17 \quad (Q_5) \]

\[ \frac{24}{24} \quad (R_5) \]

\[ \text{Step 6:} \]

\[ 4 - \left[ \frac{3}{17} \right] = 4 - 51 = \overline{47} \]

\[ Q_5 = \frac{27}{46} (\overline{23} \quad (Q_6) \]

\[ \frac{46}{46} \quad \frac{1}{1} \quad (R_6) \]

\[ \text{Step 7:} \]

\[ 10 - \left[ \frac{2}{23} \right] = 10 - (69) \]

\[ Q_6 = +69 = 59 \]

\[ 2) \quad 59 (29 \quad (Q_7) \]

\[ \frac{58}{58} \quad \frac{1}{1} \quad (R_7) \]

\[ \text{Step 8:} \]

\[ 10 - \left[ \frac{3}{29} \right] = 10 - 87 = \overline{77} \]

\[ Q_7 \]
2 \) \frac{77}{76} \ (\overline{35}) \ (Q_9) \\
\overline{1} \ (R_9)

\text{Step 9:} \\
\overline{10} - \left[ \frac{3}{\overline{35}} \right] = \overline{10} - (\overline{114}) \\
\quad \overline{Q_9} \\
\quad = \overline{10 + 114} = 104

2 \) \frac{104}{52} \ (\overline{52}) \ (Q_9) \\
\overline{0} \ (R_9)

\text{Step 10:} \\
0 - \frac{3}{\overline{52}} = \overline{156} \\
\quad \overline{Q_9}

2 \) \overline{156} \ (\overline{78}) \ (Q_{10}) \\
\overline{156} \ (R_{10})

Q = 4 \overline{267} \ 17. \overline{23} \ 2938 \ 52 \ 78 \\
= 4 \overline{266} \ 5. \overline{164} \ 5 \overline{8} \\
= 4 \overline{2544} \ .9 \overline{564} \overline{2}
### Vedic Mathematics

#### Division

**II Part (using Vinculum)**

<table>
<thead>
<tr>
<th>19</th>
<th>4 2 5 : 4 . 9</th>
<th>6</th>
<th>4 2 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4 2 : 2 1 3 3 2 1 3 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 0 8 3 . | 10 4 14 11 2 2 1 6 37 |
| Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7 Q_8 Q Q Q_{10} |

- **Step 1:**
  5) 4 (0 (Q_1)
  
  0
  
  4 (R_1)

- **Step 2:**
  42 - \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) = 42

- **Step 3:**
  25 - \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) = 17

- **Step 4:**
  24 - \( \begin{pmatrix} 1 \\ 8 \end{pmatrix} \) = 5 1

- **Step 5:**
  14 - \( \begin{pmatrix} 1 \\ 3 \\ 10 \end{pmatrix} \) = 2 3

- **Step 6:**
  3 9 - \( \begin{pmatrix} 1 \\ 10 \\ 4 \end{pmatrix} \) = 73

- **Step 7:**
  35 - \( \begin{pmatrix} 1 \\ 4 \\ 14 \end{pmatrix} \) = 5 7

- **Step 8:**
  26 - \( \begin{pmatrix} 1 \\ 1 \\ 9 \end{pmatrix} \) = \( \overline{111} \)

  = \( \overline{111} \)

  5) \( \overline{111} \) (2 2 (Q_8)

  \( \overline{110} \) (R_8)
Vedic Mathematics

Step 9: \[ \frac{14}{22} \times 9 = 8 \]

Step 10: \[ \frac{32}{16} \times 9 = 186 \]

Final Answer = 0 8 3 1 0 4 1 4 1 1 2 2 1 6 3 7
= 8 2 0 3 5 1 3 3 7
= 8 1 9 7 4 8 6 7 7

2 \[ 634.12 \times (232 \times 67 \times 13) \]

Current Method

232 \times 67 \times 13
232
\times 67
1624
1392
15544
46632
15544
202072
6345.12
634512
202072
20207200
20207200 (0.03140029)
606216000
28296000
20207200
80888000

Vedic Method

\[ \begin{array}{c|ccccccc}
32 & 6 & 3 & : & 4 & 5 & 1 & 2 & 0 & 0 & 0 \\
2 & 2 & : & 3 & 3 & 4 & 5 & 5 & 4 & 3 & 3 \\
7 & 2 & : & 7 & 3 & 4 & 9 & 6 & 5 & 5 & 1 & 0 \\
6 & 2 & : & 3 & 5 & 6 & 1 & 2 & 7 & 6 & 5 \\
3 & : & 0 & 4 & 0 & 8 & 2 & 0 & 3 & 8 & 0 \\
1 & : & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 3 \\
& & 0 & 3 & 1 & 4 & 0 & 0 & 2 & 9 & 3
\end{array} \]

Final Answer = 0.031400293
Vedic Mathematics

Division

3) \[ 210678 + (1.98 \times 0.267) \]

Current Method

\[ 0.267 \]
\[ \times 1.98 \]
\[ 2136 \]
\[ 2403 \]
\[ 267 \]
\[ 0.52866 \]
\[ \frac{210678}{21067800000} \]
\[ \times 52866 \]
\[ 21067800000 (398513.22210) \]
\[ 158598 \]
\[ 520800 \]
\[ 475794 \]
\[ 450060 \]
\[ 422928 \]
\[ 271320 \]
\[ 264330 \]
\[ 69900 \]
\[ 528866 \]
\[ 170340 \]
\[ 158598 \]
\[ 117420 \]
\[ 105732 \]
\[ 116880 \]
\[ 105732 \]
\[ 111480 \]
\[ 105732 \]
\[ 57480 \]
\[ 52866 \]
\[ 461400 \]
\[ 422928 \]
\[ 39472 \]

Vedic Method

\[ 98 \]
\[ \frac{2}{1.06 : 7800000000} \]
\[ 67 \]
\[ 1064 : 030303030 \]
\[ 0398513.22210 \]

Final Answer = 398513.22210
Combined Addition and Division (Left to Right Operation) (V.M.):

Combined operation of two or more individual mathematical operations such as addition, division, multiplication in general can also be carried out using Vedic principles.

Here we are giving a few examples wherein a combined addition and division is demonstrated both in the Current and Vedic Method.

Examples:

Example 1. \((132 + 255 + 273 + 891) \div 5\)

In the Current Method, the numbers are first added up and then the result is divided by 5 showing the quotient and the remainder.

In the Vedic Method, this has a difference in operation in the sense that addition is carried out from left to right and division is simultaneously carried out as detailed in the examples.

Step 1:

The addition is carried out from left to right and from top to bottom keeping the numbers vertically. While doing so, if one gets a value greater than or equal to the divisor, the result is divided at that stage, and quotient is shown on the left, remainder is carried out to the next value in that column.

The division is carried out in such a way that the quotient is adjusted to have the modulus of the remainder least. For example, if 8 is to be divided by 5, the quotient 1 and remainder 3 is not preferred to the quotient 2 and remainder \(\frac{2}{5}\). This is to be followed throughout.

After the first column is over, all the quotients so obtained are added which shows the corresponding digit in the final result under that column. At the end of the first column, the remainder is carried out to the beginning of the next column. Same procedure of addition and simultaneous division is carried to the rest of the columns representing the addition. The addition of the remainder in each column, which is brought into it from the previous column, is necessarily multiplied by 10 and then proceeded.

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\overline{\begin{array}{ccc} 1 &amp; 3 &amp; 2 \ 2 &amp; 5 &amp; 5 \ 2 &amp; 7 &amp; 3 \ 8 &amp; 9 &amp; 1 \end{array}})</td>
<td>(\overline{\begin{array}{ccc} 2 &amp; 1 \ 1 &amp; 3 &amp; 2 \ 2 &amp; 5 &amp; 5 \ 2 &amp; 7 &amp; 3 \ 8 &amp; 9 &amp; 1 \ 3 &amp; 1 &amp; 0 \end{array}})</td>
</tr>
<tr>
<td>5) 1551 (310</td>
<td>(\text{Quotient} = 310 )</td>
</tr>
<tr>
<td>15</td>
<td>(\text{Remainder} = 1 )</td>
</tr>
<tr>
<td>05</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>01</td>
<td></td>
</tr>
</tbody>
</table>

Quotient = 310
Remainder = 1
Vedic Mathematics

First Column:

Step 1:

\[ 1 + 2 + 2 = 5 \]
\[ 5 + 5 = 1 \]
\[ R = 0 \]

1 is kept as quotient to the left of 2

Step 2:

\[ 0 + 8 = 8 \]
\[ 8 + 5 = 2 \frac{2}{5} \] instead of \[ 1 \frac{3}{5} \]. Quotient is 2 and Remainder is \( \frac{2}{5} \). Quotient is kept to the left of

\[ 8 \] and \( \frac{2}{5} \) is carried to second column as \( \overline{2} \times 10 = \overline{20} \)

\( \frac{2}{5} \) is carried to second column as \( \overline{2} \times 10 = \overline{20} \)

Two quotients are added to give the result under the first column, i.e., \[ 1 + 2 = 3 \]

Second Column:

Step 1:

\[ \overline{20} + 3 = \overline{17} \]
\[ \overline{17} + 5 = \overline{12} \]
\[ \overline{12} + 7 = \overline{5} \]
\[ \overline{5} + 9 = 4 \]

\[ 4 + 5 = 1 \frac{1}{5} \] Quotient is 1 and remainder is \( \frac{1}{5} \).

\( \frac{1}{5} \) is carried to third column as \( \overline{10} \).

Quotient is 1, which is the result in the second column, and is kept to the left of 9

Third Column:

Step 1:

\[ \overline{10} + 2 = \overline{8} \]
\[ \overline{8} + 5 = \overline{3} \]
\[ \overline{3} + 3 = 0 \]
\[ 0 + 1 = 1 \]

When 1 is divided by 5, quotient is 0 and is kept to the left of 1. The remainder is 1. This remainder represents the final remainder.

\( \therefore \) Quotient = 310

Remainder = 1
Combined Addition followed by Division by two digits in the Divisor (V.M.):

The divisor is partitioned into two with the Dhwajanka process, where the lower one is used for division and upper one is used for multiplication, as is described in straight division.

The provision for the digits in the remainder is shown accordingly as determined by
1. The number of digits that are given in the Dhwajanka,
2. Accordingly the partition in the addition is shown as the number of columns in the addition.

While all the other procedure is common as described for the divisor having one digit, one has to take into consideration of the Dhwajanka multiplication, which is applied to the value resulting in each column by the addition of corresponding quotients. From this it is followed by the usual division method. (Refer Straight division). The difference between single digit divisor (5) in ex.1 and multiple digit divisor (54) in ex.2 is that from 2nd column onwards R subtraction similar to that carried out in straight division is also applied here i.e. formation of ID’s and ND’s. If –ve value results as the ND, that is carried out as Vinculum.

All other steps are just the same as in the previous case.

Example 2:

\[(3121 + 9562 + 5321 + 4907) + 54\]

Current Method

\[
\begin{array}{ccc}
3 & 1 & 2 & 1 \\
9 & 5 & 6 & 2 \\
5 & 3 & 2 & 1 \\
4 & 9 & 0 & 7 \\
2 & 2 & 9 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{c}
1 & 2 \\
3 & 1 & 2 & 1 \\
9 & 5 & 6 & 2 \\
5 & 3 & 2 & 1 \\
4 & 9 & 0 & 7 \\
2 & 2 & 9 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{cc}
3 & 5 \\
9 & 6 & 2 \\
5 & 3 & 2 & 1 \\
4 & 9 & 0 & 7 \\
1 & 2 \\
\end{array}
\]

Division

\[
\begin{array}{c}
Q_1 Q_2 Q_3 \\
3 & 1 & 2 & 1 \\
5 & 3 & 2 & 1 \\
4 & 2 & 4 & 15 \\
\end{array}
\]

54) 22911 (424

216

131

108

231

216

15

Vedic Method

Partition divisor 54 into two parts as 5 and 4, where 4 is (Dhwajanka) which is used for multiplication and 5 is part divisor used in Division as in the case of straight divisor.

First Column:

Step 1:

\[3 + 9 = 12\]

\[12 + 5 = 2 \frac{2}{5}\]

2 is kept as quotient to the left of 9.
Step 2:
(Remainder) $2 + 5 = 7$
$7 + 5 = 1 \frac{2}{5}$
1 is kept at the left of 5 in the column.

Step 3:
$2 + 4 = 6$
$6 + 5 = 1 \frac{1}{5}$
1 is kept at the left of 4. Remainder 1 is carried to the next column as 10.
Quotient in the first column = $2 + 1 + 1 = 4$

Second Column:
Step 1:
$10 - \begin{bmatrix} 4 \\ 4 \end{bmatrix} \rightarrow$ (Dhwajanka)
$\begin{bmatrix} 4 \\ \end{bmatrix} \rightarrow$ (Quotient digit in the first column)

$10 - 16 = \frac{6}{1}$

Step 2:
$\frac{6}{1} + 1 = \frac{5}{1}$
$\frac{5}{1} + 5 = 0$
$0 + 3 = 3$
$3 + 9 = 12$
$12 + 5 = 2 \frac{2}{5}$
2 is quotient and remainder 2 is carried out to the next column as 20. Quotient in the second column = 2.

Third Column:
Step 1:
$20 - \begin{bmatrix} 4 \\ 2 \end{bmatrix} \rightarrow$ (Dhwajanka)
$\begin{bmatrix} 4 \\ \end{bmatrix} \rightarrow$ (Quotient digit in the second column)

$12 + 2 = 14$
$14 + 5 = 3 \frac{1}{5}$
$\frac{1}{5} + 6 = 5$
$5 + 5 = 1$
2 is carried as 20 to the next column, i.e., remainder column.
Quotient in the third column = $3 + 1 = 4$
Vedic Mathematics

Division

Remainder Column:

In the remainder column add all the digits of the column along with the modified remainder obtained in third column to get the total remainder

Step 1:
(Dhwajanka)

\[ \begin{align*}
20 - & \cdot 4 \text{ (modified remainder)} \\
\end{align*} \]

(Quotient digit in the third column)

Step 2:

\[ \begin{align*}
4 + 1 + 2 + 1 + 7 &= 15 \\
\therefore \text{Remainder} &= 15 \\
\text{Quotient} &= 424
\end{align*} \]

Example 3 \((54563 + 92821 + 76543 + 24095) + 321\) (In Dhwajanka there are two digits)

**Current Method**

<table>
<thead>
<tr>
<th>5 4 5 6 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 2 8 2 1</td>
</tr>
<tr>
<td>7 6 5 4 3</td>
</tr>
<tr>
<td>2 4 0 9 5</td>
</tr>
<tr>
<td>2 4 8 0 2 2</td>
</tr>
</tbody>
</table>

**Vedic Method**

| \( D \) | \( 2 \) | \( 1 \) | \( 1 \) | \( 0 \) |
|--------|--------|--------|--------|
| 21     | 15     | 34     | 5      | 6      | 3      |
| 2        | 19     | 12     | 8      | 2      | 1      |
| 7        | 37     | 26     | 25     | 4      | 3      |
| 3        | 2      | 14     | 20     | 9      | 5      |

\( PD \) | \( 7 \) | \( 7 \) | \( 2 \) | \( 20 \) | \( 10 = 210 \)

\( Q_1 \) | \( Q_3 \) | \( Q_3 \)

\( \therefore \text{Quotient} = 772 \)

\( \text{Remainder} = 210 \)

Quotient = 772

Remainder = 210

First Column:

Step 1:

\[ \begin{align*}
5 + 3 &= 1 \frac{2}{3} \\
\end{align*} \]

Step 2:

\[ \begin{align*}
2 + 9 &= 11 \\
2 + 7 &= 9
\end{align*} \]

Step 3:

\[ \begin{align*}
11 + 3 &= 3 \frac{2}{3} \\
9 + 3 &= 3
\end{align*} \]
Vedic Mathematics

Division

Quotient in the first column is 7  Remainder is 2, which is carried to the next column as 20

Second Column:

Step 1:

\[
\begin{array}{c}
\text{Dhvajanka} \\
20 - \left( \begin{array}{c} 2 \\ 1 \\ 7 \\ Q_1 \end{array} \right) = 6 \\
\text{Quotient digit in the first column}
\end{array}
\]

Step 2:

\[
\begin{array}{c}
6 + 4 = 10 \\
10 + 3 = 3 \frac{1}{3}
\end{array}
\]

Step 3:

\[
\begin{array}{c}
1 + 2 = 3 \\
3 + 3 = 1
\end{array}
\]

Step 4:

\[
\begin{array}{c}
6 + 3 = 2
\end{array}
\]

Step 5:

\[
\begin{array}{c}
4 + 3 = 3 \frac{1}{3}
\end{array}
\]

Quotient in the second column is 7  Remainder is 1, which is carried to the next column as 10

Third Column:

Step 1:

\[
\begin{array}{c}
10 \left( \begin{array}{c} D_1 \\ D_2 \\ 2 \\ 7 \\ Q_1 \end{array} \right) - 11 \\
\text{Quotient digit in the first column } Q_1
\end{array}
\]

Step 2:

\[
\begin{array}{c}
-11 + 5 = -6 \\
6 + 8 - 2 \\
2 + 5 \ 7 \\
7 
\end{array}
\]

Step 3:

\[
\begin{array}{c}
1 + 0 = 1 \\
1 + 3 = 0 \ (Q) \\
0 \frac{1}{3} \ \text{Remainder 1}
\end{array}
\]

Quotient in the third column is 2  Remainder is 1, which is carried to the remainder column as 10
Fourth Column:

Step 1:

\[
\begin{array}{c}
10 - \\
\begin{array}{c}
D_1 \\
2 \\
7 \\
Q_2 \\
2 \\
Q_3
\end{array}
\end{array}
\]

\[= -1\]

Step 2:

\[-1 + 6 + 2 + 4 + 9 = 20\]

We put 2 in the remainder

Quotient digit in the third column

Fifth Column:

Step 1:

\[0 - \]

\[= -2\]

\[Q_3\] Quotient digit in the third column

Step 2:

\[-2 + 3 + 1 + 3 + 5 = 10\]

\[3 + 1 + 3 + 5 = 12 - 2 = 10\]

\[\therefore\text{Quotient} = 772, \text{Remainder} = 210\]
Vedic Mathematics

Chapter IV

a) Division by Paravartya Method  (Division of Polynomials) (V.M.):

Paravartya Yojayet sutram is applied for division. The modus operandi is as follows

The application of Paravartya is by considering the opposite signs for all coefficients of x excepting for the highest power in the divisor x. The division is carried out with such a re-combination of the coefficients, which is shown below the dividend, after the first term in the quotient is worked out with the first term of the divisor.

Case 1: The divisor having coefficient of highest power of x as 1:

Consider one example $8x^2 - 4x - 24 + x - 2$

Examples:

1. Divide $8x^2 - 4x - 24$ by $x - 2$

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - 2) 8x^2 - 4x - 24 (8x + 12$</td>
<td>$x - 2$ $8x^2 - 4x - 24$</td>
</tr>
<tr>
<td>$8x^2 - 16x$</td>
<td>$+ 2$ $+ 16x + 24$</td>
</tr>
<tr>
<td>$+ 12x - 24$</td>
<td>Quotient after dividing by $x$</td>
</tr>
<tr>
<td>$+ 12x - 24$</td>
<td>$8x + 12$</td>
</tr>
<tr>
<td>$0$</td>
<td></td>
</tr>
</tbody>
</table>

First the dividend and the divisor are written in the decreasing orders of powers of x (zeroes supplemented if any terms of x are missing in the Dividend and the Divisor).

The dividend is partitioned from right end into two parts. The second part is the remainder region, which may contain more than one term, but depends on the number of terms in the Paravartya.

The Paravartya form of $x - 2$ is +2. Division is carried out with 2 as follows.

Step 1:

The first term of the dividend is divided by the first term of divisor to get the first term in the quotient.

$8x^2 / x = 8x (Q_1)$

The Paravartya Division is effective from this step onwards.

Step 2:

The quotient so obtained in the first step is multiplied with the Paravartya form. The result is placed under the next term of the dividend and the corresponding coefficients are suitably added to get the second term of the quotient.
Vedic Mathematics

Division

\[ 8x \times 2 = 16x \; ; \; 16x - 4x = 12x \]

This result is now divided by the highest power of \( x \) in the divisor

Hence \( 12x + x = 12 \)

This is the second term of the quotient \( Q_2 \)

Step 3:

Second term of the quotient \( Q_2 \) is multiplied with the Paravartya followed by addition to get the remainder.

\[ 12 \times 2 = 24; \; 24 - 24 = 0 \]

Some more examples are given below when higher powers are considered for Dividend and Divisor:

2. Divide \( 9x^3 - 7x^2 + 5x + 3 \) by \( x + 3 \)

Current Method

\[
\begin{align*}
x + 3) & 9x^3 - 7x^2 + 5x + 3 \quad (9x^2 - 34x + 107) \\
9x^3 + 27x^2 & \\
- 34x^2 + 5x & \\
\hline
107x + 3 & \\
107x + 321 & \\
\hline
- 318 &
\end{align*}
\]

Quotient = \( 9x^2 - 34x + 107 \)
Remainder = \(-318\)

Vedic Method

\[
\begin{align*}
x + 3 & \quad 9x^3 - 7x^2 + 5x \quad + 3 \\
- 3 & \quad \hline
- 27x^2 + 102x - 321 & \\
9x^2 - 34x + 107 & \\
\hline
- 318 &
\end{align*}
\]

Step 1: \( 9x^2 / x = 9x^2 \) (Q1)
Step 2 \( 9x^2 (-3) = -27x^2 \)
\(-7x^2 - 27x^2 = -34x^2 + x = -34x \) (Q2)
Step 3. \(( -34x) (-3) = -102x \)
\( 102x + 5x = 107x \)
\( 107x + x = 107 \)
Step 4: \((107)(-3) = -321 \)
\(+ 3 - 321 = -318 \) (R)
Ans: \( 9x^2 - 34x + 107 \)
R = \(-318\)

3. Divide \( x^4 - 2x^3 + 5x^2 + x + 4 \) by \( x + 4 \)

Current Method

\[
\begin{align*}
\begin{align*}
(x + 4)x^4 - 2x^3 + 5x^2 + x + 4 & (x^3 - 6x^2 + 29x - 115) \\
- 6x^3 & \\
- x^3 & \\
+ 29x^2 + x & \\
+ 29x^2 + 116x & \\
- 115x + 4 & \\
- 115x - 460 & \\
+ 464 &
\end{align*}
\end{align*}
\]

Vedic Method

\[
\begin{align*}
x + 4 & \quad x^4 - 2x^3 + 5x^2 + x \quad + 4 \\
- 4 & \quad \hline
- 4x^3 + 24x^2 - 116x + 460 & \\
x^3 - 6x^2 + 29x - 115 & \\
\hline
+ 464 &
\end{align*}
\]

Quotient = \( x^3 - 6x^2 + 29x - 115 \)
Remainder = 464
4. Divide \( x^3 - 2x^2 + 5x + 1 \) by \( x - 1 \)

**Current Method**

\[
\begin{align*}
  x - 1) & x^3 + 0x^2 - 2x^2 + 5x + 1 \div x^3 - x^4 + x^3 - x^2 - x + 4 \\
  & + x^3 - 2x^3 \\
  & + x^3 - x^3 \\
  & - x^3 + 0x^2 \\
  & - x^3 + x^2 \\
  & - x^3 + 5x \\
  & - x^2 + x \\
  & 4x + 1 \\
  & 4x - 4 \\
  & + 5
\end{align*}
\]

**Vedic Method**

\[
\begin{align*}
  x - 1 & x^3 + 0x^2 - 2x^2 + 0x^2 + 5x | + 1 \\
  + 1 & 1x^4 + 1x^2 - 1x^2 - 1x | + 4 \\
  & x^4 + x^3 - x^2 - x + 4 | + 5
\end{align*}
\]

Quotient = \( x^4 + x^3 - x^2 - x + 4 \)

Remainder = 5

5. Divide \( x^5 + 4x^4 + 5x^3 + 2x + 1 \) by \( x^2 + 3x + 2 \)

**Current Method**

\[
\begin{align*}
  x^2 + 3x + 2) & x^5 + 4x^4 + 5x^3 + 0x^2 + 2x + 1 \div x^3 + x^2 - 2 \\
  & x^3 + 3x^2 + 2x \\
  & + 3x^2 + 2x \\
  & - 2x^4 + 2x + 1 \\
  & - 2x^4 - 6x - 4 \\
  & 8x + 5
\end{align*}
\]

**Vedic Method**

\[
\begin{align*}
  x^2 + 3x + 2 & x^5 + 4x^4 + 5x^3 + 0x^2 | + 2x + 1 \\
  - 3x - 2 & - 3x^4 - 2x^3 \\
  & + 3x^3 - 2x^2 + 0x^2 \\
  & + 0x + 6x + 4
\end{align*}
\]

Quotient = \( x^3 + x^2 - 2 \)

Remainder = 8x + 5

6. Divide \( x^6 + x^4 + 3x^3 + 4x^2 + 5 \) by \( x^1 + x + 1 \)

**Current Method**

\[
\begin{align*}
  x^1 + x^1 + 4x^2 + 5(\chi^1 + 2) \\
  x^1 + x^1 \\
  2x^1 + 4x^2 + 5 \\
  2x^1 + 2x^2 + 2 \\
  4x^2 - 2x + 3
\end{align*}
\]

**Vedic Method**

\[
\begin{align*}
  x^1 + 0x^2 + x + 1 & x^6 + 0x^5 + x^4 + 3x^2 | + 4x^2 + 0x + 5 \\
  0x^2 - 1x - 1 & 0x^2 - 1x^4 - 1x^3 \\
  & 0x^4 + 0x^3 + 0x^2 \\
  & 0x + 0x + 0x \\
  & 0x^2 - 2x - 2 \\
  & 4x^2 - 2x + 3
\end{align*}
\]

Quotient = \( x^3 + 0x^2 + 0x + 2 \)

Remainder = 4x^2 - 2x + 3
Vedic Mathematics

Case 2: If the coefficient is not 1 for the highest power of \( x \) in the divisor.

The procedure is as follows.

**Method I**

1) To divide the first term of the dividend by the first term of the divisor as it is.
2) To divide each quotient term by the first term in the divisor and the result is used for the multiplication with the Paravartya form.
3) The remainder is left as it is.

**Method II**

One may obtain the unit coefficient for the highest power in the divisor by dividing it through out by that coefficient and taking the corresponding Paravarthy form. Only the quotients at the end are divided by the coefficient of the highest power of \( x \) in the divisor. Both the methods are shown.

**Examples:**

1. Divide \( 6x^3 - 12x^2 + 3x - 10 \) by \( 2x - 5 \)

**Current Method**

\[
\begin{align*}
(2x-5)6x^3 - 12x^2 + 3x - 10(3x^2 + \frac{3}{2}x + \frac{21}{4})
\end{align*}
\]

**Vedic Method I**

\[
\begin{align*}
\frac{2x-5}{6x^3 - 12x^2 + 3x} &- 10 \\
+ 5 &+ 15x^2 + \frac{15}{2}x + \frac{105}{4} \\
\frac{6}{2}x^2 + \frac{3}{2}x + \frac{21}{4} &+ 65 \\
\frac{4}{4} \\
\end{align*}
\]

Quotient = \( 3x^2 + \frac{3}{2}x + \frac{21}{4} \)

Remainder = \( \frac{65}{4} \)

**Vedic Method II**

\[
\begin{align*}
\frac{x - \frac{5}{2}}{6x^3 - 12x^2 + 3x} &- 10 \\
\frac{5}{2} &+ 15x^2 + \frac{15}{2}x + \frac{105}{4} \\
6x^2 + 3x + \frac{21}{2} &+ 65 \\
\frac{4}{4} \\
\end{align*}
\]

Dividing each quotient by 2, the final quotient is

\( 3x^2 + \frac{3}{2}x + \frac{21}{4} \)

Remainder = \( \frac{65}{4} \)

**Working Details of Method I**

Step 1: \( \frac{6x^3}{2x} = 3x^2 \); \((Q_1)\)

Step 2: \((3x^2)(5) - 12x^2 = 3x^2\),

\( \frac{3x^2}{2x} = \frac{3x}{2} \ Q_2 \)

\( \cdots \cdots \cdot (3 \cdots )x) \cdots \cdots 21 \cdots \cdot \)

\( \frac{21}{4} \) \( \left( \frac{1}{2x} \right) = \frac{21}{4} \ Q_3 \)

Step 4: \( \left( \frac{21}{4} \right)(5) = \frac{105}{4} - 10 = \frac{65}{4} \) (R)
(2) Divide $6x^5 + 2x^4 + 5x^3 + 1$ by $3x^2 - 2x + 1$

**Current Method**

\[
\begin{align*}
3x^2-2x+1 & | 6x^5 + 2x^4 + 5x^3 + 1(2x^3+2x^2+\frac{7}{3}x+\frac{8}{9}) \\
6x^4+3x^3 & \quad + 1 \\
6x^4-4x^3+2x^2 & \quad + 1 \\
7x^2-2x^2 & \quad + 1 \\
7x^3-\frac{14}{3}x^2+\frac{7}{3}x & \quad + \frac{8}{3}x^2-\frac{7}{3}x+1 \\
& \quad + \frac{8}{3}x^2-\frac{16}{9}x+\frac{8}{9} \\
& \quad + \frac{5}{9}x+\frac{1}{9}
\end{align*}
\]

**Vedic Method I**

\[
\begin{align*}
3x^2-2x+1 & | 6x^5 + 2x^4 + 5x^3 + 0.x^2 \\
2x-1 & | 4x^4 - 2x^3 \\
& | \frac{14}{3}x^3 - \frac{7}{3}x \\
& | \frac{16}{9}x - \frac{8}{9} \\
& | \frac{5}{9}x + \frac{1}{9}
\end{align*}
\]

Quotient = $2x^3 + 2x^2 + \frac{7}{3}x + \frac{8}{9}$

Remainder = $-\frac{5}{9}x + \frac{1}{9}$

**Vedic Method II**

\[
\begin{align*}
x^2-\frac{2}{3}x+\frac{1}{3} & | 6x^5 + 2x^4 + 5x^3 + 0.x^2 \\
& | 4x^4 - 2x^3 \\
& | \frac{14}{3}x^3 - \frac{7}{3}x \\
& | \frac{16}{9}x - \frac{8}{9} \\
& | \frac{5}{9}x + \frac{1}{9}
\end{align*}
\]

Final quotient = $2x^3 + 2x^2 + \frac{7}{3}x + \frac{8}{9}$

Remainder = $-\frac{5}{9}x + \frac{1}{9}$

Division
3 Divide \( x^3 - 6x^2 + 11x - 6 \) by \( 2x - 1 \)

**Current Method**

\[
\begin{array}{c}
-6x^2 + 11x - 6 (\frac{x^2}{2} - \frac{11}{4}x + \frac{33}{8}) \\
- \frac{x^3}{2} \\
\frac{-11}{2}x^2 + 11x \\
- \frac{-11}{2}x^2 + \frac{11}{4}x \\
\frac{33}{4}x - 6 \\
\frac{33}{4}x - \frac{33}{8} \\
\frac{-15}{8}
\end{array}
\]

**Vedic Method I**

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
2x - 1 & x^3 & -6x^2 & +11x & -6 \\
+1 & \frac{x^3}{2} & -\frac{11}{4}x & \frac{33}{8} \\
& \frac{x^3}{2} & \frac{11}{4}x & \frac{33}{8} & -15 \\
& \frac{2x}{4} & \frac{33}{8} & 8 \\
\hline
\text{Final quotient} & \frac{2x^2}{2} & \frac{11}{4}x & \frac{33}{8} \\
\text{Remainder} & \frac{-15}{8}
\end{array}
\]

**Vedic Method II**

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
x - \frac{1}{2} & x^3 & -6x^2 & +11x & -6 \\
\frac{1}{2} & \frac{x^3}{2} & -\frac{11}{4}x & \frac{33}{8} \\
\frac{1}{2} & \frac{11}{2}x & \frac{33}{4} & -15 \\
\hline
\text{Quotient} & \frac{x^3}{2} & \frac{11}{4}x & \frac{33}{8} \\
\text{Remainder} & \frac{-15}{8}
\end{array}
\]
\[
x^2 - 2x + 1 + x^3 - 3x^2 + 2x + 1
\]

**Current Method**

\[
x^3 - 3x^2 + 2x + 1) x^2 - 2x + 1 \left( \frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^3} + \frac{3}{x^4} \right)
\]

\[
x^2 - 3x + 2 + \frac{1}{x}
\]

\[
x - 1 - \frac{1}{x}
\]

\[
x - 3 + \frac{2}{x} + \frac{1}{x^2}
\]

\[
2 - \frac{3}{x} - \frac{1}{x^2}
\]

\[
2 - \frac{6}{x} + \frac{4}{x^2} + \frac{2}{x^3}
\]

\[
\frac{3}{x} - \frac{5}{x^2} - \frac{2}{x^3}
\]

\[
\frac{3}{x} + \frac{9}{x^2} + \frac{6}{x^3} + \frac{3}{x^4}
\]

\[
\frac{4}{x^4} + \frac{8}{x^5} + \frac{3}{x^6}
\]

**Vedic Method**

\[
x^3 - 3x^2 + 2x + 1 \left( \frac{1}{x^3} - 2x - \frac{1}{x^2} + 3x + 2 - \frac{1}{x} + \frac{2}{x^2} + \frac{4}{x^3} - \frac{2}{x^4} \right)
\]

\[
0 \times x^3
\]

\[
x^3 - 2x + 1
\]

\[
0 + 0 + 0
\]

\[
+ 3x - 2 - \frac{1}{x}
\]

\[
+ 3 - \frac{2}{x} - \frac{1}{x^2}
\]

\[
+ \frac{6}{x} - \frac{4}{x^2} - \frac{2}{x^3}
\]

\[
0 - \frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^3} + \frac{3}{x^4} - \frac{5}{x^5} - \frac{2}{x^6}
\]

**Quotient**

\[
\frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^3} + \frac{3}{x^4}
\]

\[
\text{......}
\]

\[
\text{for } x \neq 0
\]

The Problem on this type will be discussed more clearly in Power Series later in another Lecture Notes.
5. \[ 2x^3 + 4x^2 + 6x - 8 + 2x^4 - x^3 + 4x + 6 \]

**Vedic Method I**

\[
\begin{array}{c}
2x^4 + 4x^2 + 6x - 8 \\
+ x^3 + 0x^2 + 4x + 6 \\
\hline
0x^4 \\
\end{array}
\]

\[
\begin{array}{c}
x^2 + 0x - 4 - \\
+ \frac{6}{x} \\
\hline
\frac{5}{2}x + 0 - \frac{10}{x} - \frac{15}{x^2} \\
\hline
17 - 0 - 17 - 51 \\
+ \frac{4}{x} - \frac{1}{x^2} - 2x \\
\hline-
\frac{31}{8x} + \frac{31}{x^2} + \frac{93}{4x^4} \\
\end{array}
\]

Quotient = \[ \frac{1}{x} + \frac{5}{2x^3} + \frac{17}{4x^4} - \frac{31}{8x^5} \]

**Vedic Method II**

\[
\begin{array}{c}
2x^4 - x^3 + 0x^2 + 4x + 6 \\
\hline
x^3 + \frac{0x^2}{2} + 2x + 3 \\
\hline
x^3 + 0x^2 - 2x - 3; \quad x^2 + 0x - 4 - \frac{5}{x} \\
\hline
+ \frac{5}{2}x + 0 - \frac{10}{x} - \frac{15}{x^2} \\
\hline
17 - 0 - 17 - 51 \\
\end{array}
\]

\[
\frac{2 + \frac{5}{2}}{x} + \frac{17}{2x^3} - \frac{31}{4x^4}
\]

Each term of the quotient is to be divided by 2 to get the final quotient

Quotient = \[ \frac{1}{x} + \frac{5}{2x^3} + \frac{17}{4x^4} - \frac{31}{8x^5} \]
(b) **Applying Paravartya Sutra to Numbers (V.M.)**

The Paravartya form is obtained by taking all the digits with their opposite sign excepting the first digit in the divisor. The procedure is same as explained for polynomials, having the coefficient of the highest power as 1.

**Examples:**

**Example 1:** \[13653 + 111\]

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>[111) 13653 (123]</td>
<td>[1 \mid 1 \mid 1 136]</td>
</tr>
<tr>
<td>[]</td>
<td>[\mid 5 3]</td>
</tr>
<tr>
<td>[111]</td>
<td>[-1 -1]</td>
</tr>
<tr>
<td>[255]</td>
<td>[\mid -1 -1]</td>
</tr>
<tr>
<td>[222]</td>
<td>[]</td>
</tr>
<tr>
<td>[333]</td>
<td>[-2 -2]</td>
</tr>
<tr>
<td>[333]</td>
<td>[]</td>
</tr>
<tr>
<td>[0]</td>
<td>[-3 -3]</td>
</tr>
<tr>
<td></td>
<td>[1 2 3 0 0]</td>
</tr>
</tbody>
</table>

Quotient = 123
Remainder = 0

**Example 2:** \[49897 + 121\]

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>[121) 49897 (412]</td>
<td>[1 2 1 4 9 -1]</td>
</tr>
<tr>
<td>[484]</td>
<td>[-2 -1]</td>
</tr>
<tr>
<td>[149]</td>
<td>[-8 -4]</td>
</tr>
<tr>
<td>[121]</td>
<td>[-2]</td>
</tr>
<tr>
<td>[287]</td>
<td>[]</td>
</tr>
<tr>
<td>[242]</td>
<td>[]</td>
</tr>
<tr>
<td>[45]</td>
<td>[]</td>
</tr>
<tr>
<td></td>
<td>[4 1 2 4 5]</td>
</tr>
</tbody>
</table>

Quotient = 412
Remainder = 45
Example 3:

\[ 159568 + 14312 \]

**Current Method**

\[
\begin{array}{c}
14312) 159568 (11 \\
14312 \\
16448 \\
14312 \\
2136 \\
\end{array}
\]

**Vedic Method**

\[
\begin{array}{cccccc}
1 & 4 & 3 & 1 & 2 & \\
\hline
-4 & -3 & -1 & -2 & \\
-3 & -1 & -2 & \\
-4 & -3 & -1 & -2 & \\
1 & 1 & 2 & 1 & 3 & 6 \\
\end{array}
\]

Quotient = 11
Remainder = 2136

Example 4:

\[ 29721 + 142 \]

**Current Method**

\[
\begin{array}{c}
142) 29721 (209 \\
2840 \\
1321 \\
1278 \\
43 \\
\end{array}
\]

**Vedic Method**

\[
\begin{array}{cccccc}
1 & 4 & 2 & \\
\hline
-4 & -2 & \\
-8 & -4 & \\
-4 & -2 & \\
4 & 2 & \\
2 & 1 & \bar{1} & 4 & 3 & \\
\end{array}
\]

Quotient = 21 \( \bar{1} = 209 \)
Remainder = 43

If the quotient or the remainder consists of Vinculum then it has to be devinculised to get it into the ordinary form. Even after this, if the remainder has a Vinculum form then add 'n' times the original divisor to the remainder to get into normal form (n which is an integer should be the required minimum value). This is followed by subtraction of 'n' from the previous quotient to obtain the final quotient. (refer Example Page No. ) One can also further divide the Vinculum remainder.
Example 5:  

98765 + 1321  
a) Further division of the Remainder

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1321) 98765 (74</td>
<td>1 3 2 1</td>
</tr>
<tr>
<td>9247</td>
<td>9 8 7 6 5</td>
</tr>
<tr>
<td>6295</td>
<td>-27 -18 -9</td>
</tr>
<tr>
<td>5284</td>
<td>+57 +38 +19</td>
</tr>
<tr>
<td>1011</td>
<td>9 1 9</td>
</tr>
<tr>
<td></td>
<td>46 35 24</td>
</tr>
<tr>
<td>Q₁ =</td>
<td>71 4974(R)</td>
</tr>
</tbody>
</table>

Quotient = 9 1 9 = 8 9 = 71  
Remainder = 4974 (R)

since remainder > Divisor R is to be further the divided by the Divisor and the quotient thus obtained is added to the previous quotient

(b) 4974 + 1321

\[
\begin{array}{c|cccc}
1321 & 4 & 9 & 7 & 4 \\
-3-2-1 & -12 & -8 & -4 \\
\hline
Q₂ = & 4 & 3 & 1 & 0 = 1690
\end{array}
\]

(c) Further division of the remainder

\[
\begin{array}{c|cccc}
1321 & 1 & 6 & 9 & 0 \\
-3-2-1 & 3 & 2 & 1 \\
\hline
Q₃ = & 1 & 9 & 11 & 1 = 1011
\end{array}
\]

Final Quotient = Q₁ + Q₂ + Q₃ = 71 + 4 + 1 = 74

Final Remainder = 1011
Example 6: 

\[
\begin{align*}
\text{Current Method} & \quad 29429 + 1463 & \quad \text{a) Further Division} \\
1463) 29429 \ (20) & \quad \text{Vedic Method} & \quad 2 \ 9 \ 4 \ 2 \ 9 \\
29250 & \quad -4 \ -6 \ -3 & \quad -8 \ -12 \ -6 \\
169 & \quad -4 \ -6 \ -3 & \quad 12 \ 10 \ 6 = 1306 \\
& \quad 169 & \quad 169
\end{align*}
\]

Quotient = \(21 + 1 = 20\)

Final Remainder = \(1306 + 1463 = 169\) (n = 1)

If more than one digit results as a single unit in the quotient / remainder the normal Vedic addition holds good.

Example 7: 

\[
\begin{align*}
\text{Current Method} & \quad 7967 + 1627 \\
1627) 17967 \ (11) & \quad \text{Vedic Method} & \quad 1 \ 6 \ 2 \ 7 \ 1 \ 7 \ 6 \ 7 \\
1627 & \quad -6 \ -2 \ -7 & \quad -2 \ -7 \\
1697 & \quad -6 \ -2 \ -7 & \quad 1 \ 1 \ 1 \ 3 \ 0 \\
1627 & \quad 1 \ 1 \ 1 \ 3 \ 0 & \quad 1 \ 1 \ 1 \ 7 \ 0
\end{align*}
\]

Quotient = 11, Remainder = 70

In case the first digit of the divisor is not 1, then Vinculum is tried to see if it can be achieved. This is to see that an easy division with 1 is obtained. This facilitates the secondary multiplication easy. If it is not converted, then each digit of the quotient is to be divided by that number, which probably may result in fractions. These fractions are needed to be carried over properly.
**Example 8:** \(32517 + 987\)

Conversion to Vinculum followed by Paravartya

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>987) 32517 (32</td>
<td>9 8 7 3 2 5 1 7</td>
</tr>
<tr>
<td>2961</td>
<td>10 1 3 0 3 9</td>
</tr>
<tr>
<td>2907</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td></td>
</tr>
<tr>
<td>933</td>
<td></td>
</tr>
</tbody>
</table>

Quotient = 32
Remainder = 933

Divisor is converted into Vinculum form and then Paravartya is applied

b) If the remainder is greater than the original divisor, subtract \(n\) times the divisor from the remainder until the resulting remainder is less than the divisor (\(n\) should be minimum). In this case one has to add "\(n\)" to the previous quotient (see ex...
**Vedic Mathematics**

**Example 9:**

\[25935 \div 829\]

**Current Method**

\[
\begin{array}{c}
829) 25935 (31 \\
2487 \\
1065 \\
\text{ } 829 \\
\text{ } 236
\end{array}
\]

**Vedic Method**

\[
\begin{array}{c|ccc}
829 & 2 & 5 & 935 \\
\hline
1231 & 231 & 4 & 62 \\
\hline
829 & 236 & 18 & 279 \\
\hline
29 & 21 & 22 & 14 = 2114 \\
\hline
Q_1 & 29 & 1894 & R > the divisor
\end{array}
\]

Final Remainder = \(1894 - 2 \times 829 = 236(n = 2)\)

Final Quotient = \(Q_1 + 2 = 29 + 2 = 31\)

(or)

Dividing the remainder further or dividing 2114 by 2 - 31

\[
\begin{array}{c|cc}
829 & 1 & 834 \\
\hline
1231 & 231 & 462 \\
\hline
829 & 236 & 1065 > \text{divisor} \\
\hline
Q_2 & 10 & 6376 \\
\hline
Q_3 & 2236
\end{array}
\]

Final Quotient = \(Q_1 + Q_2 + Q_3 = 29 + 1 + 1 = 31\)

Final Remainder = 236
**Example 10:**

12345 ÷ 7869

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>7869) 12345 (1</td>
<td><strong>7869</strong></td>
</tr>
<tr>
<td><strong>7869</strong></td>
<td>2 3 4 5</td>
</tr>
<tr>
<td>4476</td>
<td>12131</td>
</tr>
<tr>
<td></td>
<td>2131</td>
</tr>
<tr>
<td></td>
<td>4 4 7 6</td>
</tr>
</tbody>
</table>

Quotient = 1  
Remainder = 4476

In order to get ‘1’ as the first digit in the divisor, one may also divide (eg. 11, 12) or multiply (eg. 13) the divisor suitably followed or vice versa by Vinculum, if necessary, and then finally by Paravartya.

When the divisor is multiplied or divided suitably before the actual division is carried out, the final quotient is also multiplied or divided accordingly to obtain the final result.

In doing so, if one gets fractions (eg. 11) then that fraction is carried out to the remainder part of the divisor while retaining the integer part in the quotient.

**Example 11:**

4298 ÷ 273

<table>
<thead>
<tr>
<th>Current Method</th>
<th>Vedic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>273) 4298 (15</td>
<td><strong>273</strong></td>
</tr>
<tr>
<td>273</td>
<td>2 7 3</td>
</tr>
<tr>
<td>1568</td>
<td>4 2 9 8</td>
</tr>
<tr>
<td>1365</td>
<td>3 3 1</td>
</tr>
<tr>
<td>203</td>
<td>1 -1</td>
</tr>
<tr>
<td></td>
<td>6 - 6</td>
</tr>
<tr>
<td></td>
<td>3 4 6</td>
</tr>
<tr>
<td></td>
<td>11 2</td>
</tr>
<tr>
<td><strong>3 4 6</strong></td>
<td><strong>R &lt; 273</strong></td>
</tr>
<tr>
<td><strong>15 1\frac{1}{3}</strong></td>
<td><strong>11 2</strong></td>
</tr>
<tr>
<td>15</td>
<td><strong>9\frac{1}{3}</strong></td>
</tr>
<tr>
<td>15</td>
<td>203</td>
</tr>
</tbody>
</table>

∴ Quotient = 15  
Remainder = 203

I Step for the Divisor – Vinculum  
II Step for the Divisor – sub multiple of the vinculum  
III Step for the Divisor – Paravartya
### Example 12:

**Current Method**

<table>
<thead>
<tr>
<th>486) 101100 (208</th>
<th>972</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3900</td>
</tr>
<tr>
<td></td>
<td>3888</td>
</tr>
<tr>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>

**Vedic Method**

\[
\begin{array}{cccc}
6 | 1 | 0 & 1 & 1 & 0 & 0 \\
81 & 2 & -1 & & & \\
1.21 & & & & 4 & -2 \\
2.1 & & & & & \\
\end{array}
\]

Sub multiple

<table>
<thead>
<tr>
<th>Vinculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 - 4</td>
</tr>
</tbody>
</table>

Paravartya

\[
\begin{array}{cccc}
1 & 2 & 4 & 7 \\
93 & & & \\
207.5 & & & \\
14 & -7 & & \\
\end{array}
\]

\[
\begin{array}{c}
1247 \\
93 \\
\hline
207 \\
408 + 93 \\
\hline
208 \\
486 \\
\hline
\end{array}
\]

Quotient = 208, Remainder = 12

498 > 486 and 498 - 1x 486 = 12

\[1 \text{ is to be added to } Q\]

\[Q = 207 + 1 = 208 \text{ and Remainder } = 12\]

### Example 13:

**Current Method**

<table>
<thead>
<tr>
<th>249) 16770 (67</th>
<th>1494</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1830</td>
</tr>
<tr>
<td></td>
<td>1743</td>
</tr>
<tr>
<td></td>
<td>87</td>
</tr>
</tbody>
</table>

**Vedic Method**

\[
\begin{array}{cccc}
4 \times 249 & 1 & 6 & 7 & 7 & 0 \\
996 & & & & & \\
100.4 & 0 & 0 & 4 & & \\
0.04 & & & & & \\
\hline
\text{Multiple} & & & & 0 & 0 & 24 \\
\text{Vinculum} & & & & & & \\
\text{Paravartya} & 1 & 6 & 7 & 11 & 24 & \\
\hline
4 \times 16 & & & & & 834 \\
\hline
64 & & & & & 834 \text{ R> Divisor 24} \\
\end{array}
\]

Final Remainder = 834 - 3 x 249 = 87

Final Quotient = 64 + 3 = 67

\[\text{If the remainder has Vinculum then add the original divisor once or twice or } n \text{ times etc. as the case may be and remove that } n \text{ where } n = 1, 2, \ldots.\]

\[\text{From the previous quotient.}\]
Vedic Mathematics

When the remainder > Original divisor then divide it further by the same method
For example  \( R = .834 \)

In order to have partition 3 digits, convert this to Vinculum form i.e

\[
\begin{align*}
834 &= 1 \overline{2} 1 \overline{7} 1 \overline{6} = 1 \overline{66} \\
1 \overline{66} + 249
\end{align*}
\]

\[
\begin{array}{c|cc}
4 \times 249 & 1 & \overline{6} \overline{6} \\
996 & 1004 & 004 \\
\hline
& 1 & \overline{6} \overline{2} \\
4 \times 1 & 2 & 4 \overline{9} \\
\hline
& 1 \overline{2} 7 = 87
\end{array}
\]

As the remainder is in the Vinculum add one time 249 which results in 87. This is less than the original divisor. Subtract 1 from previous value of the quotient.

\[
\begin{align*}
\therefore & \text{ From the Quotient subtract 1} \\
\therefore & \text{ the final Quotient is } 64 + 4 - 1 = 67
\end{align*}
\]

Comparison of different methods:

\[
897356 + 721
\]

Current Method

\[
721 \quad 897356 \quad (1244 \cdot 59916)
\]

\[
\begin{array}{ccccccc}
721 & 1763 & 1442 & 3215 & 2884 & 3316 & 2884 \\
& & & & & & 4320 \\
& & & & & & 3605 \\
& & & & & & 7150 \\
& & & & & 6489 & 6610 \\
& & & & & 6489 & 1210 \\
& & & & 721 & 4890 & 4326 \\
& & & & & & 564 \\
\end{array}
\]

\[
\begin{align*}
\text{Straight Division Method} \\
21 : & 8 \overline{9} 7 3 : 5 \overline{6} 0 0 0 \\
7 & 1 3 4 : 5 \overline{8} 9 4 \overline{6} 7 \\
& 1 \overline{2} 4 \overline{4} . 5 \overline{9} 9 1 \overline{6}
\end{align*}
\]

Quotient = 1244

\[
\text{Remainder} = 556 - \frac{2}{4} \times \frac{1}{4} 10 - = 556 - 120 - 4 = 432
\]

Quotient in decimals = 1244.59916
Paravartya Division Method

\[
\begin{array}{c|ccc|ccc}
721 & 8 & 9 & 7 & 3 & 5 & 6 \\
\hline
Vinculum & 1321 & & & 8 & & \\
Paravartya & 321 & 24 & 16 & & & 99 \\
& & & 66 & & & 33 \\
& & & 270 & & & 180 90 \\
\hline
& 8 & 33 & 90 & 201 & 212 & 96 \\
Q_1 & 1220 & & & & & \\
\end{array}
\]

\[= 17736 > \text{divisor}\]

\[17736 > 24 \times 721\]

\[\therefore 17736 - 24 \times 721\]

\[17736 - 17304 - 432 - \text{Remainder}\]

\[24 \text{ is to be added to } Q\]

\[\therefore Q = 1220 + 24 = 1244\]

and \(R = 432\)

or

Dividing the remainder \(\overline{2276}\) similarly by treating the remainder as the dividend

\[
\begin{array}{c|cc|cc|cc|cc}
721 & 2 & \overline{2} & 7 & 6 & (a) & 17736 & + & 721 \\
\hline
1321 & & 4 & \overline{2} & & 3 & 2 & \overline{1} & 7 & 7 & 3 & 6 \\
321 & & & 12 & \overline{8} & 4 & 30 & 20 & 10 \\
\hline
 & 2 & 4 & 14 & 17 & 2 & \text{} & \text{} & 3316 & + & 721 \\
Q_2 & 2 & 4 & & & 1572 & \text{or} & 3316 & + & 721 \\
\hline
\end{array}
\]

\[Q_2 = 20\]

\[
\begin{array}{c|cc|cc|cc|cc}
721 & 2 & \overline{2} & 7 & 6 & (b) & 316 & + & 721 \\
\hline
 & 2 & 4 & 14 & 17 & 2 & & & \text{or} & 316 & + & 721 \\
\hline
Q_2 & 2 & 4 & & & 1572 & \text{} & \text{} & 1253 & + & 721 \\
\hline
\end{array}
\]

\[Q_3 = 316 + 721\]

\[Q_3 = 316 + 721\]

\[Q_4 = 316 + 721\]

\[Q_4 = 316 + 721\]

\[
\begin{array}{c|cc|cc|cc|cc}
1 & 2 & \overline{5} & 3 & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} \\
3 & 2 & \overline{1} & & & & & & & & & \\
\hline
Q_4 = 1 & 5 & 7 & 2 & = & 432 & \text{or} & 1 & 5 & 7 & 2 & = & 432 \\
\end{array}
\]

Final Quotient \(Q\) \(= Q_1 + Q_2 + Q_3 + Q_4\)

\[= 1220 + 20 + 3 + 1 = 1244\]

\[\therefore \text{Quotient } = 1244\]

Final Remainder = 432
Vedic Mathematics

For decimal points in the quotient, add 0 to the remainder and apply the division.

\[
\begin{array}{c|cccc}
721 & 4 & 3 & 2 & 0 \\
1321 & 12 & 8 & 4 \\
321 & 4 & 15 & 6 & 4 \\
\hline
& 4 & 14 & 3 & 6 > 721 \\
\hline
Q_1 & 5 & 7 & 1 & 5 & R_1 \\
\end{array}
\]

\[Q = Q_1 + Q_2 = 4 + 1 = 5\]

\[\therefore\text{ further divide}\]

\[
\begin{array}{cccc}
3 & 2 & 1 & 436 \\
\overline{2} & \overline{1} & & \\
Q_2 & 7 & 1 & 5 & R_1 \\
\end{array}
\]

or

\[
\begin{array}{c}
1436 \\
-721 \quad (1 \times 721) \\
715 \quad n = 1 \\
\hline
\end{array}
\]

\[Q = 4 + 1 = 5\]

\[R = 715\]

\[
\begin{array}{c|cccc}
721 & 7 & 1 & 5 & 0 \\
1321 & 21 & 14 & \bar{7} \\
321 & 7 & 22 & 11 & 7 \\
\hline
& 7 & 21 & 1 & 7 \\
\hline
Q_1 & 7 & 21 & 0 & 3 > 721 \therefore \text{ further division} \\
9 & 6 & 6 & 1 \\
\end{array}
\]

\[Q_1 + Q_2 = 7 + 2 = 9\]

\[
\begin{array}{cccc}
3 & 2 & 1 & 103 \\
\overline{2} & \overline{1} & & \\
3 & \bar{4} & \bar{2} & \\
Q_2 & 7 & 4 & 1 \\
\hline
& 6 & 6 & 1 \\
\hline
\end{array}
\]

or

\[
\begin{array}{c}
2103 \\
1442 \quad (2 \times 721) \\
\overline{661} \quad n = 2 \\
\hline
661 \\
Q = 7 + 2 = 9 \\
R = 661 \\
\end{array}
\]
## Vedic Mathematics

### Division

<table>
<thead>
<tr>
<th>721</th>
<th>6</th>
<th>6</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>321</td>
<td>18</td>
<td>12</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>24</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>23</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

3rd decimal digit

<table>
<thead>
<tr>
<th>Q1</th>
<th>6</th>
<th>22</th>
<th>8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

\[ Q_1 \div 2 \div 8 \div 4 \div 2 > 721 \]

\[ 22384 \times 3 \times 721 = 121 \]

\[ Q_1 = 6 + 3 = 9 \]

II) one can write Remainder part 842 into Vinculum form so that it can have 4 digits and then divide

\[ 842 = 1 \bar{2} \quad 1 \bar{6} \quad 1 \bar{8} = 1 \bar{1} \bar{5} \bar{8} \]

<table>
<thead>
<tr>
<th>Q3</th>
<th>1279</th>
</tr>
</thead>
</table>

4th decimal digit

<table>
<thead>
<tr>
<th>721</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>321</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

5th decimal digit

<table>
<thead>
<tr>
<th>721</th>
<th>4</th>
<th>8</th>
<th>9</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1321</td>
<td>12</td>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>321</td>
<td>4</td>
<td>20</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Q1</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ Q = Q_1 + Q_2 = 4 + 2 \]

\[ Q = 4 + 2 \]

Quotient in decimals = 1244.59916

\[ 2006 - 1442 \overset{\text{(-2x721)}}{\quad} = 564 \quad n = 1 \]
Division by Nikhilam Method Applying the Nikhilam sutram to the Divisor

\[
\begin{array}{cccccc}
721 & 8 & 9 & 7 & 3 & 5 & 6 \\
279 & 16 & 56 & 72 & \\
50 & 175 & 225 & \\
226 & 791 & 1017 & \\
\end{array}
\]

\[Q_1 = \frac{8 \times 25 + 113}{1021} = 1023\]

\[58833 \ (R_1) > 721\]

Hence further division

\[Q_1 = 1163 \quad 79 \]

\[\begin{array}{cccc}
5 & 8 & 8 & 3 & 3 \\
10 & 35 & 45 & \\
36 & 126 & 162 & \\
\end{array}
\]

\[Q_2 = 68 \quad 5 \quad 18 \quad 79 \quad 174 \quad 165 \]

\[9805 \ R_2 > 721\]

\[9805 \quad -9373 \quad (13 \times 721) \quad 432 \quad n = 13 \]

\[Q = 1163 + 68 + 13 \]

\[Q = 1163 + 68 + 19 + 4\]

\[Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 \]

\[\therefore \ Quotient = 1163 + 68 + 9 + 3 + 1 = 1244\]

Remainder = 432
**Vedic Mathematics**

For decimal points in the quotient

```
\[
\begin{array}{c|ccc}
721 & 4 & 3 & 2 & 0 \\
\hline
279 & 8 & 28 & 36 \\
\hline
Q_1 & 11 & 30 & 36 & R = 1436 > 721 hence further division \\
\hline
721 & 1 & 4 & 3 & 6 \\
\hline
279 & 2 & 7 & 9 & or \\
\hline
Q_2 & 1 & 6 & 10 & 15 & R = 715
\end{array}
\]
```

or

\[
\begin{array}{c}
1436 \\
-721 \text{ (1 x 721) } n = 1 \\
715 \\
Q = 4 + 1 = 5
\end{array}
\]

Quotient = 4 + 1 = 5, Remainder = 715

```
\[
\begin{array}{c|ccc}
721 & 7 & 1 & 5 & 0 \\
\hline
279 & 14 & 49 & 63 \\
\hline
Q_1 & 15 & 54 & 63 & R = 2103 > 721 hence further division \\
\hline
721 & 2 & 1 & 0 & 3 \\
\hline
279 & 14 & 14 & 18 \\
\hline
Q_2 & 2 & 5 & 14 & 21 & R = 661
\end{array}
\]
```

or

\[
\begin{array}{c}
2103 \\
-1442 \text{ (2 x 721) } \\
661 \\
Q = 7 + 2 = 9
\end{array}
\]

Quotient = 7 + 2 = 9, Remainder = 661.

```
\[
\begin{array}{c|ccc}
721 & 6 & 6 & 1 & 0 \\
\hline
279 & 12 & 42 & 54 \\
\hline
6 & 18 & 43 & 54 & R = 2284 > 721 \\
\hline
\end{array}
\]
```

hence further division

\[
\begin{array}{c}
2284 \\
-2163 \text{ (3 x 721) } \\
121 \\
Q = 8 + 1
\end{array}
\]

or we can simply subtract 721 from 842 once and then add 1 to the Quotient

```
\[
\begin{array}{c|ccc}
721 & 2 & 8 & 4 \\
\hline
279 & 4 & 14 & 18 \\
\hline
Q_2 & 6 & 22 & 22 \\
\hline
8 & 4 & 2 > 721 \\
\hline
1 & 1 & 2 & 1
\end{array}
\]
```

842 can be written as 

```
\[
\begin{array}{c}
842 \\
1 \frac{2}{4} \text{ to facilitate division} \\
121 \\
Q = 8 + 1
\end{array}
\]
```

or

```
\[
\begin{array}{c|ccc}
279 & 2 & 4 & 2 \\
\hline
1 & 2 & 7 & 9 \\
\hline
1 & 0 & 11 & 11 \\
\hline
121
\end{array}
\]
```

Division
Vedic Mathematics

\[ \text{Quotient} = 6 + 2 + 1 = 9, \text{ Remainder} = 121 \]

\[
\begin{array}{c|ccc}
 & 721 & 1 \, 2 \, 1 \, 0 \\
279 & 2 \, 7 \, 9 \\
- & 1 \, 4 \, 8 \, 9 \\
\hline
& 1
\end{array}
\]

Quotient = 1
Remainder = 489

4th decimal point

\[
\begin{array}{c|ccc}
 & 721 & 4 \, 8 \, 9 \, 0 \\
279 & 8 \, 28 \, 36 \\
- & 4 \, 16 \, 37 \, 36 \\
\hline
Q_1 & 2 \, 0 \, 0 \, 6 \\
& 4 \, 14 \, 18 \\
\hline
Q_2 & 2 \, 4 \, 14 \, 24 \\
& 5 \, 6 \, 4 \\
\hline
\end{array}
\]

5th decimal point

or

\[
\begin{array}{c|c|c}
& 2006 \\
\text{or} & 1442 \\
& 564 \\
& n = 2 \\
\hline
Q = 4 + 2 = 6 \\
\end{array}
\]

\[2006 > 721 \text{ hence further division}\]

\[\text{Quotient} = 4 + 2 = 6\]
Remainder = 564

\[\therefore \text{Quotient in decimal points} = 1244.59916\]
Chapter V

a) Argumental Division: (Significance of Left Hand to Right Hand Multiplication)(V.M.)

By extending a simple method of the application of Urdhva Tiryagbhyaam, one can obtain the quotient and remainder by an argumentation, (in a converse manner, converting a division into multiplication).

Example: consider \(2x^2 + 5x - 5 + x + 3\)

The procedure is as follows.

Write down the quotient as \(Ax + B\) form and multiply it by the divisor \(x + 3\), the value is compared with the given dividend to obtain the quotient and the remainder by the argumentation process. A and B are to be determined.

\[
\begin{array}{c}
\text{Quotient} \\
2x^2 + 5x - 5 \\
\leftarrow \text{(Given dividend)} \\
\end{array}
\]

Step 1: \[
\begin{array}{c}
Ax \\
\uparrow \\
x \\
\end{array} = Ax^2 \text{ (Urdhva)}
\]

It is obvious that the division is now converted into multiplication.

Starting from the left hand, the vertical multiplication is \(Ax^2\), which gives value 2 for A when compared with \(2x^2\).

Now the Quotient is \(2x + B\),

Step 2: \[
\begin{array}{c}
Ax + B \\
\times \\
\times \\
\end{array} = 3Ax + Bx = 6x + Bx \quad (\because A = 2)
\]

Applying the sutram Tiryak and comparing the \(x\) terms we get,

\(6x + Bx = 5x\)

\(\therefore B = -1\)

Step 3: Applying Urdhva to the last column

\(3B = 3(-1) = -3\)

On comparison, constant term \(-3\) is different from the value \(-5\) of the dividend and hence remainder resulting from this is \(-5 - (-3) = -2\)

\(\therefore \text{Quotient} = 2x - 1\)

\(\text{Remainder} = -2\)

Example 2: \(x^2 + 6x + 12 + x + 2\)

\[
\begin{array}{c}
Ax + B \\
\times + \\
\times + \\
\end{array}
\]

\[
\begin{array}{c}
x + 2 \\
\end{array}
\]

\[
\begin{array}{c}
x^2 + 6x + 12 \\
\end{array}
\]
Vedic Mathematics

Step 1: \[ Ax^2 = x^2 \]
\[ A = 1 \]
\[ \therefore \text{Quotient} = x + 4 \]
\[ \text{Remainder} = 4 \]

Step 2: \[ 2Ax + Bx = 6x \]
\[ 2x + Bx = 6x \]
\[ \therefore B = 4 \]

Step 3: \[ 2B = 12. \quad \text{But} \ B = 4 \Rightarrow 2B = 8 \]
\[ \therefore R = 12 - 8 = 4 \text{ (on companions with constant of dividend)} \]

Example 3:

\[ 3x^3 + 6x^2 + 5x + 13 + x + 5 \]

\[ Ax^2 + Bx + C \]
\[ \underline{x + 5} \]
\[ 3x^3 + 6x^2 + 5x + 13 \]
Step 1:
\[ Ax^3 = 3x^3 \]
\[ \therefore A = 3 \]

Step 2:
\[ 5Ax^2 + Bx^2 = 6x^2 \]
\[ 15x + Bx = 6x \]
\[ \therefore B = -9 \]

Step 3:
\[ 5Bx + Cx = 5x \]
\[ -45x + Cx = 5x \]
\[ \therefore C = 50 \]

Step 4:
\[ 5C = 250 \]
\[ R = 13 - 250 = -237 \text{ (on comparison with constant in the dividend)} \]

Quotient = 3x^2 - 9x + 50

Remainder = -237

Example 4:

\[ x^4 + 10x^3 + 35x^2 + 50x + 24 + x + 4 \]

\[ Ax^3 + Bx^2 + Cx + D \]
\[ \underline{x + 4} \]
\[ x^4 + 10x^3 + 35x^2 + 50x + 24 \]
\[ \therefore \text{Quotient} = x^3 + 6x^2 + 11x + 6 \]
\[ \text{Remainder} = 0 \]

Step 1:
\[ Ax^4 = x^4 \]
\[ \therefore A = 1 \]

Step 2:
\[ 4Ax^3 + Bx^3 = 10x^3 \]
\[ 4x^3 + Bx^3 = 10x^3 \]
\[ \therefore B = 6 \]

Step 3:
\[ 4Bx^2 + Cx^2 = 35x^2 \]
\[ 24x^2 + Cx^2 = 35x^2 \]
\[ Bx^3 = 6x^3 \]
\[ \therefore C = 11 \]

Step 4:
\[ 4Cx + Dx = 50x \]
\[ 44x + Dx = 50x \]
\[ \therefore D = 6 \]

Step 5:
\[ 4D = 24 \]
\[ \therefore R = 24 - 24 = 0 \text{ (on comparison with constant in the dividend)} \]
Example 5: \(6x^2 + 5x + 10 + 2x + 1\)

\[
\begin{align*}
Ax + B \\
2x + 1 \\
\hline
6x^2 + 5x + 10
\end{align*}
\]

Step 1:

\[
2Ax^2 = 6x^2
\]
\[
\therefore A = 3
\]

Step 2:

\[
Ax + 2Bx = 5x
\]
\[
3x + 2Bx = 5x
\]
\[
\therefore B = 1
\]

Step 3:

\[
B = 1
\]
\[
\therefore R = 10 - 1 = 9 \text{ (on comparison with the constant term in the dividend)}
\]

\[
\therefore \text{Quotient} = 3x + 1
\]

Remainder = 9

Example 6: \(24x^4 + 50x^3 + 35x^2 + 10x + 13 + 4x + 1\)

\[
\begin{align*}
Ax^3 + Bx^2 + Cx + D \\
4x + 1 \\
\hline
24x^4 + 50x^3 + 35x^2 + 10x + 13
\end{align*}
\]

Step 1:

\[
4Ax^4 = 24x^4
\]
\[
\therefore A = 6
\]

Step 2:

\[
Ax^3 + 4Bx^3 = 50x^3
\]
\[
6x^3 + 4Bx^3 = 50x^3
\]
\[
\therefore C = 6
\]

Step 3:

\[
Bx^2 + 4Cx^2 = 35x^2
\]
\[
11x^2 + 4Cx^2 = 35x^2
\]
\[
\therefore D = 1
\]

Step 4:

\[
Cx + 4Dx = 10x
\]
\[
6x + 4Dx = 10x
\]
\[
\therefore D = 1
\]

\[
\therefore R = 13 - 1 = 12 \text{ (on comparison with the constant in the dividend)}
\]

\[
\therefore \text{Quotient} = 6x^3 + 11x^2 + 6x + 1
\]

Remainder = 12
Example 7: \[ 10x^4 + 17x^3 + 20x^2 + 6x + 3 + 2x^2 + 3x + 3 \]

\[ \frac{Ax^2 + Bx + C}{2x^2 + 3x + 3} \]

\[ 10x^4 + 17x^3 + 20x^2 + 6x + 3 \]

Step 1: \[ 2Ax^4 = 10x^4 \]

\[ \therefore A = 5 \]

Step 2: \[ 3Ax^3 + 2Bx^2 = 17x^3 \]

\[ 15x^3 + 2Bx^2 = 17x^3 \]

\[ \therefore B = 1 \]

Step 3: \[ 3Ax^2 + 2Cx^2 + 3Bx^2 = 20x^2 \]

\[ 15x^2 + 2Cx^2 + 3x^2 = 20x^2 \]

\[ \therefore C = 1 \]

Step 4:

\[ 3Bx + 3Cx = 6x \]

\[ \therefore B = 1, C = 1, R_1 = 0 \]

Also \( C = 1, R_2 = 0 \)

\[ \therefore \text{Remainder} = 0 \]

( on comparison with the x coeff and constant the remainders \( R_1 \) and \( R_2 \) are zero)

Quotient = \( 5x^2 + x + 1 \)

Example 8:

\[ (2x^{10} + 4x^9 + 9x^8 + 14x^7 + 17x^6 + 20x^5 + 15x^4 + 16x^3 + 16x^2 + 8x + 10) + (2x^5 + 2x^4 + 3x^3 + x^2 + 2x + 3) \]

\[ \frac{Ax^2 + Bx^4 + Cx^2 + Dx^2 + Ex + F}{2x^5 + 2x^4 + 3x^3 + x^2 + 2x + 3} \]

\[ 2x^{10} + 4x^9 + 9x^8 + 14x^7 + 17x^6 + 20x^5 + 15x^4 + 16x^3 + 16x^2 + 8x + 10 \]

Step 1: \[ 2Ax^{10} = 2x^{10} \]

\[ \therefore A = 1 \]

Step 2: \[ 2Ax^9 + 2Bx^9 = 4x^9 \]

\[ 2x^9 + 2Bx^9 = 4x^9 \]

\[ \therefore B = 1 \]

Step 3: \[ 3Ax^8 + 2Cx^8 + 2x^8 = 9x^8 \]

\[ 3x^8 + 2Cx^8 + 2x^8 = 9x^8 \]

\[ 5x^8 + 2Cx^8 = 9x^8 \]

\[ \therefore C = 2 \]

Step 4:

\[ Ax^7 + 2Dx^7 + 3Bx^7 + 2Cx^7 = 14x^7 \]

\[ x^7 + 2Dx^7 + 3x^7 + 4x^7 + 14x^7 \]

\[ (2D + 8)x^7 = 14x^7 \]

\[ \Rightarrow 2D + 8 = 14 \]

\[ \therefore D = 3 \]

Step 5:

\[ 2Ax^6 + 2Ex^6 + Bx^6 + 2Dx^6 + 3Cx^6 = 17x^6 \]

\[ 2x^6 + 2Ex^6 + x^6 + 6x^6 + 6x^6 = 17x^6 \]

\[ \therefore E = 1 \]
Vedic Mathematics

Step 6:

\[ 3Ax^5 + 2Fx^3 + 2Bx^2 + 2Ex + Cx + 3Dx^3 \]
\[ = 20x^3 \]
\[ 3x^5 + 2Fx^3 + 2x^2 + 2x + 9x^3 = 20x^3 \]
\[ \therefore F = 1 \]

Step 8:

\[ 3Cx^3 + 3Fx^3 + 2Dx^2 + Ex^3 \]
\[ = 6x^3 + 3x^3 + 6x^2 + x^3 \]
\[ = 16x^3 \text{ Same as the dividend value} \]
\[ R_2 = 0 \]

Step 10:

\[ 3Ex + 2Fx = 8x \]
\[ 3Ex + 2Fx = 3x + 2x = 5x \]
\[ R_4 = 8x - 5x = 3x \quad R_3 \]
\[ \therefore \text{Quotient} = x^5 + x^4 + 2x^3 + 3x^2 + x + 1 \]
\[ \text{Remainder} = 4x^2 + 3x + 7 \]

b) Argumental division as applied to numbers (V.M.):

Consider one example

**Example 1:** \[ 438 + 23 \]

438 is the dividend and 23 is the divisor

Obviously the divisor \( \times \) quotient + remainder = dividend.

The problem is to find out the quotient considering the dividend as a result of the multiplication of the quotient with the divisor. In this process the remainder, if any, can also be obtained. The procedure is to apply Urdhva Tiryak sutram between the quotient and the divisor followed by a comparison with the given dividend.

Considering the above examples, one can write down quotient AB as multiplicand (to be determined), the divisor as multiplier and the dividend as the result of multiplication, which includes the remainder if any.

Now the process is with the left-hand side multiplication using Urdhva Tiryak.
Vedic Mathematics

Step 1:

\[ \begin{array}{ccc} \uparrow & B & \text{Quotient} \\ A & \text{Divisor} \\ 4 & 3 & 8 \text{ Dividend (given)} \\ \end{array} \]

The vertical multiplication (left to right multiplication)

\[ 2A = 4 \]
\[ A = 2 \]

Since \[ 2A = 4 \] on comparison with the given Dividend. The remainder is zero

\[ R_1 = 0 \]

Step 2: Substituting the value of \( A = 2 \) and the Tiryak multiplication

\[ \begin{array}{c} A = 2 \\ B \end{array} \]

\[ \begin{array}{c} 2 \times 3 \quad \text{3} \\ 4 \quad 3 \quad 8 \quad 0 \end{array} \]

Step 2: One can also continue the procedure by considering negative value in Vinculum.

\[ 2B = \frac{-3}{2} \]

\[ B = 1 \]

\[ \begin{array}{c} A \quad B \\ 2 \quad 3 \quad 3 \quad 8 \quad 0 \quad 1 \quad R_1 \quad R_2 \end{array} \]

To avoid \(-ve\) value we will reduce \( A \) by 1, i.e., \( A = 1 \). Now \( 2A = 2 \). But on comparison \( 2A = 4 \). \( \therefore \) Remainder is 2. The remainder \( R_1 \) is changed to 2 (modified) from 0.

\[ \begin{array}{c} 2 \quad 3 \\ 4 \quad 3 \quad 8 \\ 2 \quad \text{R}_1 \text{ (modified)} \end{array} \]

Now \( 3A + 2B = 23 \)
\[ 3 + 2B = 23 \]
\[ 2B = 20 \]
\[ B = 10 \] with \( R_2 = 0 \)

Step 3:

\[ A = 1 \quad B = 10 \]

\[ \begin{array}{c} 4 \quad 3 \quad 8 \\ 2 \quad 0 \quad R_1 \quad R_2 \end{array} \]
Vedic Mathematics

On comparison with 8 on Urddha multiplication
Excess will be 8 - 30 = -22
3B = 30, which is greater than 8.
By reducing the value of B by 1 to B = 9
we get a remainder R₂ (m) as 2

(Modified)
A = 1  B = 9
  
2
4  3  8
2  2
R₁(m)  R₂(m)

Now 3B = 3 × 9 = 27
On comparison with the value 28
the remainder is 1.
∴ Quotient comes out as A = 1, B = 9,
i.e., AB Quotient is 19
Remainder = 1

This procedure is calledArgumental Division and can be applied to division involving
any number of digits.

Example 2: 28556 ÷ 32

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

3  2
28  5  5  6
1  0  1

Step 1:

3A = 28
A is 9 with remainder R₁ as 1

A = 9  B  C
  
3  2
28  5  5  6
1  R₁

Division

Step 3:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

4  3  8
0  1

3B = 18
But 3B = 3 (on substitution of B = 1)
∴ R₃ = 18 + 3
    = 18 + 3
    = 1
∴ Final Quotient = AB = 21 = 19
     Final Remainder = 1

Step 2:

2A + 3B = 15
18 + 3B = 15
3B = -3
B = -1 = 1 one can continue the
procedure by considering -ve value in Vinculum
Remainder R₂ = 0

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| 3 | 2 |

28  5  5  6
1  0
R₁  R₂
Vedic Mathematics

Step 3:

\[ 2B + 3C = 5 \]
\[ \overline{2} + 3C = 5 \]
\[ 3C = 7 \]

C is 2 with remainder \( R_3 \) as 1

\[ 2C = 16 \]

Since \( C = 2 \)

\[ \therefore \text{Remainder is } 16 - 4 = 12 \]

Quotient = \( \overline{912} = 892 \)

Verification

\[ 32 \times 892 + 12 = 28556 \]

Step 4:

\[ \begin{array}{ccc}
2 & 3 & 6 \\
\hline
8 & 1 & 4 \\
& 0 & 14 \\
\end{array} \]

28
1
0
1

Example 3:

\[ 81420 + 236 \]

\[ \begin{array}{ccc}
A & B & C \\
2 & 3 & 6 \\
\hline
8 & 1 & 4 \\
& 2 & 0 \\
& 0 & 14 \\
\end{array} \]

Step 1:

\[ 2A = 8 \]

A = 4 with remainder \( R_1 \) as 0

\[ \begin{array}{ccc}
A & B & C \\
4 & 3 & 6 \\
\hline
8 & 1 & 0 \\
& 0 & & \text{R}_1 \\
\end{array} \]
Vedic Mathematics

Step 2:

\[
\begin{align*}
3A + 2B &= 1 \\
12 + 2B &= 1 \\
2B &= -11
\end{align*}
\]

To avoid negative value we reduce A value by 1
A = 3 with remainder R_1 as 2

\[
\begin{array}{ccc}
A = 3 & B & C \\
2 & 3 & 6
\end{array}
\]

\[
\begin{array}{cccc}
8 & 1 & 4 & 2 & 0 \\
2 & R_1(m)
\end{array}
\]

\[
\begin{align*}
3A + 2B &= 21 \\
9 + 2B &= 21 \\
2B &= 12 \\
B &= 6 with R_3 = 0
\end{align*}
\]

\[
\begin{array}{ccc}
A = 3 & B = 6 & C \\
2 & 3 & 6
\end{array}
\]

\[
\begin{array}{cccc}
8 & 1 & 4 & 2 & 0 \\
2 & 0 & R_1(m) & R_3
\end{array}
\]

Step 3:

\[
\begin{align*}
6A + 2C + 3B &= 4 \\
18 + 2C + 18 &= 4 \\
2C &= 32 \\
C &= 16 R_3 = 0
\end{align*}
\]

One can keep the value in Vinculum

\[
\begin{array}{ccc}
A = 3 & B = 6 & C = \overline{16} \\
2 & 3 & 6
\end{array}
\]

\[
\begin{array}{cccc}
8 & 1 & 4 & 2 & 0 \\
2 & 0 & .0 & R_1(m) & R_3
\end{array}
\]

Using Vinculum

Step 2: 2B = \overline{11}

\[
\begin{align*}
B &= 5 , R_2 = \overline{1}
\end{align*}
\]

Step 3:

\[
\begin{array}{ccc}
A = 4 & B = \overline{5} & C \\
2 & 3 & 6
\end{array}
\]

\[
\begin{array}{cccc}
8 & 1 & 4 & 2 & 0 \\
0 & 1 & 1 & R_1 & R_2 & R_3
\end{array}
\]

\[
\begin{align*}
6A + 2C + 3B &= \overline{14} \\
24 + 2C + \overline{15} &= \overline{14} \\
2C + 9 &= \overline{14} \\
2C = \overline{15} \Rightarrow C = \overline{7}, R_3 = \overline{1}
\end{align*}
\]

Step 4

\[
\begin{array}{ccc}
A = 4 & B = \overline{5} & C = \overline{7} \\
2 & 3 & 6
\end{array}
\]

\[
\begin{array}{cccc}
8 & 1 & 4 & 2 & 0 \\
0 & 1 & 1 & R_1 & R_2 & R_3
\end{array}
\]

\[
\begin{align*}
6B + 3C &= \overline{12} \\
But\ 6B + 3C &= 6(\overline{5}) + 3(\overline{7}) = \overline{51} \\
\therefore R_4 &= \overline{12} - \overline{51} = 43
\end{align*}
\]
Vedic Mathematics

Step 5:

\[ 6C = -96 \]
\[ \therefore R_3 = 140 - (-96) = 236 \quad \text{Or} \]

Remainder is equal to divisor
\[ \therefore \text{We can further divide remainder with divisor and get 1 as Quotient and 0 as final remainder} \]
We add this 1 to the previous Quotient.

\[ A = 3, \ B = 6, \ C = \overline{16} \]
\[ ABC = 36 \overline{16} = 356 \]
Final Quotient = \[ 1 + 35\overline{6} = 35\overline{5} = 345 \]
Final Remainder = 0

Example 4: \[ 89765 + 321 \]

\[
\begin{array}{ccc}
A & B & C \\
\hline
8 & 9 & 7 \\
\hline
2 & 4 & 4 \\
\hline
2 & 4 & 21 \\
\hline
R_1 & R_2 & R_3 & R_4 \\
\hline
\end{array}
\]

Step 1:
\[ 3A = 8 \]
\[ A = 2 \text{ with remainder } R_1 \text{ as 2} \]

\[
\begin{array}{ccc}
A & B & C \\
\hline
3 & 2 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccc}
8 & 9 & 7 & 6 & 5 \\
\hline
2 & \text{ } & \text{ } & \text{ } & \text{ } \\
R_1 & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\end{array}
\]

Step 2:
\[ 2A + 3B = 29 \]
\[ 4 + 3B = 29 \]
\[ 3B = 25 \]
B is 8 with remainder \( R_2 \) as 1

\[
\begin{array}{ccc}
A & B & C \\
\hline
3 & ? & 1 \\
\hline
8 & 9 & 7 & 6 & 5 \\
\hline
2 & 1 & \text{ } & \text{ } & \text{ } \\
R_1 & R_2 & \text{ } & \text{ } & \text{ } \\
\hline
\end{array}
\]

Division

Step 5

\[ A = 4, \ B = 5, \ C = \overline{7} \]

\[
\begin{array}{ccc}
8 & 1 & 4 \\
\hline
2 & 3 & 6 \\
\hline
0 & 1 & 1 \\
\hline
R_1 & R_2 & R_3 & R_4 \\
\hline
43 & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\end{array}
\]

\[ 6C = 430 \]
But \[ 6C = 6(\overline{7}) = \overline{42} \]
\[ \therefore R_3 = 430 - \overline{42} = 472 \]
Remainder = 472 > 236 divisor
\[ \therefore 472 - 2 \times 236 = 0(n=2) \]
Final Quotient = \[ ABC = 45\overline{7} + 2 = 45\overline{5} = 345 \]
Final Remainder = 0
Vedic Mathematics

Step 3:

\[ A + 3C + 2B = 17 \]
\[ 2 + 3C + 16 = 17 \]
\[ C = -1/3 \]

To avoid negative value, we reduce B value
\[ \therefore B = 7 \text{ with remainder } R_2 = 4 \]

\[ A = 2 \quad B = 7 \quad C \]

\[ \begin{array}{ccc}
8 & 9 & 7 \\
2 & 4 & 1 \\
R_1 & R_2 & R_3 \\
\end{array} \]

\[ A + 3C + 2B = 47 \]
\[ 2 + 3C + 14 = 47 \]

C is 10 with remainder \( R_3 \) as 1

Step 4:

\[ A = 2 \quad B = 7 \quad C = 10 \]

\[ \begin{array}{ccc}
8 & 9 & 7 \\
2 & 4 & 1 \\
R_1 & R_2 & R_3 \\
\end{array} \]

B + 2C = 16
B + 2C = 7 + 20 = 27 which is greater than 16. Hence reduction in the value of c
\[ \therefore C = 9 \text{ with remainder } R_3 \text{ as 4} \]

\[ A = 2 \quad B = 7 \quad C = 9 \]

\[ \begin{array}{ccc}
8 & 9 & 7 \\
2 & 4 & 4 \\
R_1 & R_2 & R_3 \\
\end{array} \]

B + 2C = 46
But B + 2C = 7 + 18 = 25
\[ \therefore \text{Remainder, } R_4 = 46 - 25 = 21 \]

Using Vinculum

Step 3

\[ A + 3C + 2B = 17 \]
\[ 2 + 3C + 16 = 17 \]
\[ 3C = \bar{1} \]
\[ C = 1 \text{ With } R_3 = 2 \]

\[ A = 2 \quad B = 8 \quad C = \bar{1} \]

\[ \begin{array}{ccc}
8 & 9 & 7 \\
2 & 1 & 2 \\
R_1 & R_2 & R_3 \\
\end{array} \]

B + 2C = 26
But B + 2C = 8 + \bar{2} = 6
\[ \therefore \text{Remainder } R_4 = 26 - 6 = 20 \]
Step 5:

\[ A = 2 \quad B = 7 \quad C = 9 \]

\[
\begin{array}{c}
8 \\
9 \\
2 \\
\hline
3 \\
2 \\
R_1 \\
\end{array}
\]

\[
\begin{array}{c}
7 \\
4 \\
\hline
5 \\
6 \\
R_2(m) \\
\end{array}
\]

\[
\begin{array}{c}
7 \\
4 \\
\hline
5 \\
21 \\
R_3(m) \\
\end{array}
\]

\[
\begin{array}{c}
1 \\
\hline
R_4 \\
\end{array}
\]

\[
\begin{array}{c}
\therefore C = 215 \\
\text{But } C = 9 \\
\therefore \text{Remainder } = 215 - 9 = 206 \\
\therefore \text{Final Quotient } = A \cdot B \cdot C \\
\quad = 279, \\
\quad \text{Remainder } = 206
\end{array}
\]

Step 5:

\[ A = 2 \quad B = 8 \quad C = \overline{1} \]

\[
\begin{array}{c}
8 \\
9 \\
2 \\
\hline
3 \\
2 \\
R_1 \\
\end{array}
\]

\[
\begin{array}{c}
7 \\
4 \\
\hline
5 \\
6 \\
R_2 \\
\end{array}
\]

\[
\begin{array}{c}
7 \\
4 \\
\hline
5 \\
20 \\
R_3 \\
\end{array}
\]

\[
\begin{array}{c}
1 \\
\hline
R_4 \\
\end{array}
\]

\[
\begin{array}{c}
C = 205 \\
\text{But } C = \overline{1} \\
\therefore \text{Remainder } R_3 = 205 - \overline{1} = 206 \\
\therefore \text{Final Quotient } = A \cdot B \cdot C \\
\quad = 28\overline{1} = 279 \\
\text{Quotient } = 28\overline{1} = 279 \\
\text{Final Remainder } = 206
\end{array}
\]

The ease within which the Vinculum method is worked out can be understood also from Ex. 5

Example 5:

109876548 + 6783

\[
\begin{array}{cccccc}
A & B & C & D & E \\
6 & 7 & 8 & 3 \\
\hline
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8
\end{array}
\]

Step 1:

\[ 6A = 10 \]

\[ \therefore A \text{ is 1 with remainder } R_1 \text{ as 4} \]

\[
\begin{array}{cccccc}
A = 1 & B & C & D & E \\
6 & 7 & 8 & 3 & 1 \\
\hline
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8
\end{array}
\]

\[
\begin{array}{c}
4 (R_1)
\end{array}
\]
Vedic Mathematics

Step 2:

7A + 6B = 49
7 + 6B = 49
6B = 42

B is 7 with remainder R_2 as 0

\[
\begin{array}{cccccc}
A & B & C & D & E \\
1 & 7 & 8 & 3 & 2 \\
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
4 & 0 & R_1 & R_2 \\
\end{array}
\]

Step 3:

Using Vinculum

8A + 7B + 6C = 8
8 + 49 + 6C = 8
6C = -49 = 4 \bar{9}

OR

We reduce B value by 1

\[
\begin{array}{cccccc}
A & B & C' & D & E \\
1 & 6 & 7 & 8 & 2 \\
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
R_1 & R_2(m) \\
\end{array}
\]

8A + 7B + 6C = 68
8 + 42 + 6C = 68
6C = 18
C is 3 with remainder R_3 as 0

Step 4:

\[
\begin{array}{cccccc}
A & B & C & D & E \\
1 & 6 & 3 & 7 & 8 \\
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
4 & 6 & 0 & R_1 & R_2(m) & R_3 \\
\end{array}
\]
3A + 8B + 6D + 7C = 7
3 + 48 + 6D + 21 = 7
6D = -65

\( \therefore \text{We reduce C value by 1} \)

C is 2 with remainder \( R_3 \) as 6

\( A = 1 \quad B = 6 \quad C = 2 \quad D \quad E \)

6 7 8 3

10 9 8 7 6 5 4 8

4 6 6

\( R_1 \quad R_2(m) \quad R_3(m) \)

3A + 8B + 6D + 7C = 67
3 + 48 + 6D + 14 = 67
6D = 2

\( \text{We reduce C value further by 1} \)

C is 1 with remainder \( R_3 \) as 12

\( A = 1 \quad B = 7 \quad C = 8 \quad D \quad E \)

6 7 8 3

10 9 8 7 6 5 4 8

4 0 1 0

\( R_1 \quad R_2 \quad R_3 \quad R_4 \)

3B + 6E + 8C + 7D = 66
21 + 6E + 6 4 + 7 = 66
6E+ = 56

\( F = 9, \ R_1 = 2 \)
### Vedic Mathematics

#### Step 5:

\[ 3B + 6E + 8C + 7D = 36 \]
\[ 18 + 6E + 8 + 77 = 36 \quad \text{E = -ve} \]

\[ \therefore \text{We reduce D value by 1} \]

D is 10 with remainder \( R_4 \) as 9

\[ A = 1 \quad B = 6 \quad C = 1 \quad D = 10 \quad E \]

#### Step 6:

\[ A = 1 \quad B = 7 \quad C = 8 \quad D = 1 \quad E = 9 \]

#### Division

\[ 3A + 8B + 6D + 7C = 127 \]
\[ 3 + 48 + 6D + 7 = 127 \]
\[ 6D = 69 \]

D is 11 with Remainder \( R_4 \) as 3

\[ \begin{array}{cccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
\hline
4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array} \]

\[ 3C + 7E + 8D = 24 + 63 + 8 \]
\[ = 95 \]

\[ \therefore \quad R_4 = 25 \quad 31 = 6 \]

#### Step 7:

\[ A = 1 \quad B = 7 \quad C = 8 \quad D = 1 \quad E = 9 \]

\[ \begin{array}{cccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
\hline
4 & 0 & 1 & 0 & 2 & 6 & 6 & 6 \\
\hline
\end{array} \]

\[ 3D + 8E = 6 + 6 \]

But \[ 3D + 8E = 3 + 72 = 69 \]

\[ \therefore \quad R_7 = 64 - 69 \]
\[ = 12 \]

#### Step 8:

\[ A = 1 \quad B = 7 \quad C = 8 \quad D = 1 \quad E = 9 \]

\[ \begin{array}{cccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
\hline
4 & 0 & 1 & 0 & 2 & 6 & 12 & 5 \\
\hline
\end{array} \]

\[ \begin{array}{cccccc}
R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 \\
\hline
\end{array} \]
Vedic Mathematics

\[ \begin{align*}
A &= 1 & B &= 6 & C &= 1 & D &= 9 & E \\
6 & \quad 7 & 8 & 3
\end{align*} \]

\[ \begin{array}{cccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 \\
4 & 6 & 12 & 15 & & & & \\
R_1 & R_2(m) & R_3(m) & R_4(m) & R_5 & \\
\end{array} \]

\[3B + 6E + 8C + 7D = 156\]
\[18 + 6E + 8 + 63 = 156\]
\[6E = 67\]

E is 11 with Remainder \(R_3\) as 1

\[ \begin{align*}
A &= 1 & B &= 6 & C &= 1 & D &= 9 & E &= 11 \\
6 & \quad 7 & 8 & 3
\end{align*} \]

\[ \begin{array}{cccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 \\
4 & 6 & 12 & 15 & 1 & & & \\
R_1 & R_2(m) & R_3(m) & R_4(m) & R_5 & \\
\end{array} \]

Step 6:

\[3C + 7E + 8D = 3 + 77 + 72 = 152, \text{ which is greater than } 15\]

\[\therefore \text{ We reduce } E \text{ value by 1}\]

E is 10 with Remainder \(R_3\) as 7

\[ \begin{align*}
A &= 1 & B &= 6 & C &= 1 & D &= 9 & E &= 10 \\
6 & \quad 7 & 8 & 3
\end{align*} \]

\[ \begin{array}{cccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 \\
4 & 6 & 12 & 15 & 7 & & & \\
R_1 & R_2(m) & R_3(m) & R_4(m) & R_5 & \\
\end{array} \]

\[3C + 7E + 8D = 145, \text{ which is greater than } 75\]

\[\therefore \text{ We reduce } E \text{ value further by 1}\]

E is 9 with Remainder \(R_3\) as 13

\[ \begin{align*}
\text{Division} \\
3E &= \underline{1 \overline{2 \overline{5 \overline{8}}}} \\
\text{But } 3E &= 27 \\
\text{Remainder} &= \underline{1 \overline{2 \overline{5 \overline{8}}}} - 27 \\
&= \underline{1 \overline{2 \overline{7 \overline{1}}}} \\
&= \underline{1 \overline{2 \overline{6 \overline{9}}}}
\end{align*} \]

Since the remainder is negative add quotient \(n\) times the divisor until it becomes positive.

\[= \underline{1 \overline{2 \overline{6 \overline{9}}}} + 6783 = \underline{5 \overline{5 \overline{2 \overline{6}}}}
= 5514 \]

Final Quotient \(= 17 \overline{8} \overline{19} - 1\)
\[= 17 \overline{8} \overline{18} \]
\[= 16198 \]

Final Remainder \(= 5514\)

\[\begin{align*}
\text{Note:} \\
m - n + 1 &= \text{Number of constants} \\
where m &= \text{Number of digits in the dividend} \\
n &= \text{Number of digits in the divisor}
\end{align*} \]
A = 1  B = 6  C = 1  D = 9  E = 9

6   7   8   3

10  9  8  7  6  5  4  8

R₁  R₂(m)  R₃(m)  R₄(m)  R₅(m)

3C + 7E + 8D = 3 + 63 + 72 = 138 which is greater than 135
∴ We reduce E value further by 1
∴ E is 8 with Remainder R₅ as 19

A = 1  B = 6  C = 1  D = 9  E = 8

6   7   8

10  9  8  7  6  5  4  8

R₁  R₂(m)  R₃(m)  R₄(m)  R₅(m)

3C + 7E + 8D = 131
R₆ = 195 - 131 = 64

A = 1  B = 6  C = 1  D = 9  E = 8

6   7   8   3

10  9  8  7  6  5  4  8

R₁  R₂(m)  R₃(m)  R₄(m)  R₅(m)  R₆
Step 7:

\[3D + 8E = 27 + 64 = 91\]
\[R_7 = 644 - 91 = 553\]

\[A = 1 \quad B = 6 \quad C = 1 \quad D = 9 \quad E = 8\]

6 7 8

\[
\begin{array}{cccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 8 \\
4 & 6 & 12 & 15 & 19 & 64 & 553 & \\
R_1 & R_2(m) & R_3(m) & R_4(m) & R_5(m) & R_6 & R_7 & \\
\end{array}
\]

Step 8:

\[3E = 24\]
\[R_8 = 5538 - 24 = 5514\]

\[\therefore \text{Quotient} = 16198, \text{Remainder} = 5514\]
Vedic Mathematics

Extension of Argumental Division For Finding Decimals

Example 6: 7652 + 23

\[ m = \text{Number of digits in dividend} = 4 \]
\[ n = \text{Number of digits in divisor} = 2 \]
\[ \therefore \text{Number of constants to be assumed} = m - n + 1 = 4 - 2 + 1 = 3 \]

\[ m = 4 \quad n = 2 \quad \therefore 3 \text{ constants ABC are to be determined} \]

\[ \begin{array}{ccc}
2 & 3 & \text{A} \text{ B} \text{ C} \\
7 & 6 & 5 \text{ (R1)} \text{ (R2)} \text{ (R3)} \\
1 & 1 & 0 \\
\end{array} \]

Step 1: \[ 2A = 7 \]
\[ A = 3, \quad \text{R}_1 = 1 \]

Step 2: \[ 3A + 2B = 16 \]
\[ 9 + 2B = 16 \]
\[ 2B = 7 \]
\[ B = 3, \quad \text{R}_2 = 1 \]

Step 3: \[ 3B + 2C = 15 \]
\[ 9 + 2C = 15 \]
\[ 2C = 6 \]
\[ C = 3, \quad \text{R}_3 = 0 \]

Step 4: \[ 3C = 2 \]
\[ \text{But} \quad 3C = 9 \]
\[ \therefore \quad \text{R}_4 = 2 - 9 = -7 \]

Since \( \text{R}_4 \) is negative, add divisor 23, once to get the final remainder and subtract ‘1’ from quotient.

\[ 3C + 2D = 2 \]
\[ 9 + 2D = 2 \]
\[ 2D = 7 \]
\[ D = 3, \quad \text{R}_4 = 1 \]

\[ \therefore \quad \text{Final Reminder} = -7 + 23 = 16 \]

\[ \text{Quotient} = \text{ABC} - 1 = 333 - 1 = 332 \]
\[ R = 16 \]
Vedic Mathematics

Step 6:  
\[ 3D + 2E = 10 \]
\[ 9 + 2E = 10 \]
\[ 2E = 19 = 1 \]
\[ E = 0, \ R_3 = 1 \]

Step 7:  
\[ 3E + 2F = 10 \]
\[ 0 + 2F = 10 \]
\[ 2F = 10 \]
\[ F = 5, \ R_4 = 0 \]

Step 8:  
\[ 3F + 2G = 0 \]
\[ 15 + 2G = 0 \]
\[ 2G = 15 \]
\[ G = 7, \ R_7 = 1 \]

Quotient = AB C D E F G H

\[ = 333.30575 \]
\[ = 332.69565 \]

Step 9:  
\[ 3G + 2H = 10 \]
\[ 21 + 2H = 10 \]
\[ 2H = 11 \]
\[ H = 5, \ R_8 = 1 \]

Example 7:  
91267 + 231

Method I  
Argumental Division

M = 5, N = 3

No. of decimal required = 5

\[ \begin{array}{cccccccc}
A & B & C & D & E & F & G & H \\
2 & 3 & 1 & & & & & \\
9 & 1 & 2 & 6 & 7 & 0 & 0 & 0 \end{array} \]

\( (R_1)(R_2)(R_3)(R_4)(R_5)(R_6)(R_7)(R_8) \)

Step 1:  
\[ 2A = 9 \]
\[ \boxed{A = 4} \]
\[ R_1 = 1 \]

Step 2:  
\[ 3A + 2B = 11 \]
\[ 12 + 2B = 11 \]
\[ 2B = 1 \]
\[ \boxed{B = 0} \]
\[ R_2 = 1 \]

Step 3:  
\[ A + 3B + 2C = 12 \]
\[ 4 + 0 + 2C = 12 \]
\[ 2C = 12 - 4 = 12 \]
\[ \boxed{C = 6} \]
\[ R_3 = 0 \]

Step 4:  
\[ B + 3C + 2D = 06 \]
\[ 0 + 18 + 2D = 6 \]
\[ 2D = 24 \]
\[ \boxed{D = 12} \]
\[ R_4 \]

Step 5:  
\[ 2E + 3D + C = 7 \]
\[ 2E + 36 + 6 = 7 \]
\[ 2E = 23 \]
\[ \boxed{E = 11} \]
\[ R_5 = 1 \]
### Vedic Mathematics

**Step 6:**

2F + 3E + D = 10
2F + 33 + 12 = 10
2F = 11

\[ F = 5 \quad R_4 = 1 \]

**Step 7:**

2G + 3F + E = 10
2G + 15 + 11 = 10
2G = 6

\[ G = 3 \quad R_7 = 0 \]

**Step 8:**

2H + 3G + F = 0
2H + 9 + 5 = 0
2H = 14

\[ H = 7 \quad R_8 = 0 \]

Quotient = ABCD E FGH
= 406.11537
= 405.11523
= 395.09523

**Method II Straight Division using Vinculum**

<table>
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<tr>
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<th>9</th>
<th>1</th>
<th>2</th>
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<td>0</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>R_1</td>
<td>R_2</td>
<td>R_3</td>
<td>R_4</td>
<td>R_5</td>
<td>R_6</td>
<td>R_7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>6</td>
<td>12</td>
<td>11</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) \[ 2 ) 9 ( 4 (Q_1) \]
\[ \frac{8}{1} \quad (R_1) \]

(2) \[ 11 - \left( \begin{array}{c} 3 \\ 4 \end{array} \right) = 1 \]

(2) \[ \frac{12}{1} \quad (R_2) \]

(3) \[ \frac{\frac{12}{1}}{4} \quad (R_3) \]

(4) \[ 06 - \left( \begin{array}{c} 3 \\ 6 \end{array} \right) = 24 \]

(2) \[ \frac{24}{0} \quad (R_4) \]

(5) \[ 07 - \left( \begin{array}{c} 3 \\ 12 \end{array} \right) = 23 \]

(2) \[ \frac{23}{1} \quad (R_5) \]
Vedic Mathematics

(6) \[ 10 - \left( \frac{3}{12} \right) = 1 \]

(7) \[ 10 - \left( \frac{1}{11} \right) = 6 \]

(8) \[ 00 - \left( \frac{3}{3 \times 3} \right) = 14 \]

Quotient = 406 1211537 = 395.09523

Current Method

\[ 231 \) \[ 91267(39509523 \]

Example 8: \[ 23 + 122 \]

\[ m = 2, \ n = 3 \]

\[ \therefore \text{Number of constants} = 2 - 3 + 1 = 0 \] (This denotes that the quotient part starts with decimal point.)

Example 8: \[ 23 + 122 \]

\[ m = 2, \ n = 3 \]

\[ \therefore \text{Number of constants} = 2 - 3 + 1 = 0 \] (This denotes that the quotient part starts with decimal point.)
### Vedic Mathematics

**Step 1:** \[ A = 2 \], \( R_1 = 0 \)

**Step 2:**
- \( 2A + B = 3 \)
- \( 4 + B = 3 \)

\[ R = \square \]

\[ R_2 = 0 \]

**Step 3:**
- \( 2A + 2B + C = 0 \)
- \( 4 + 2 + C = 0 \)

\[ C = \square \]

\( R_3 = 0 \)

**Quotient:** \( 0.2 \hat{1} 2 \bar{6} \bar{8} \)

0.18852

---

**Example 9:** \( 1 + 111 \)

\( m = 1, \ n = 3 \)

Number of constants = \( 1 - 3 + 1 = -1 \)

This denotes that in the quotient the decimal point is followed by one zero.

\[ \therefore \text{Quotient} \quad 0 \quad 0 \quad A \quad B \quad C \quad D \quad E \]

\[
\begin{array}{c}
A & B & C & D & E \\
1 & 1 & 1 & 1 & \\
0 & 0 & 0 & 0 & 0 \\
(R_1) & (R_2) & (R_3) & (R_4) & (R_5)
\end{array}
\]

**Step 1**
\[ A = 1 \], \( R_1 = 0 \)

**Step 2:**
- \( A + B = 0 \)
- \( 1 + B = 0 \)

\[ B = \hat{1} \]

\[ R_2 = 0 \]

**Step 3:**
- \( A + B + C = 0 \)
- \( 1 + \hat{1} + C = 0 \)

\[ C = 0 \]

\( R_3 = 0 \)

**Quotient:** \( 0.0 \quad A \quad B \quad C \quad D \quad E \)

\( = 0.01 \hat{1} 01 \hat{1} \)

\( = 0009009 \)

---

### Division

**Step 4:**
- \( D + 2B + 2C = 0 \)
- \( D + 2 + 4 = 0 \)

\[ D = \square \]

\[ R_4 = 0 \]

**Step 5:**
- \( E + 2D + 2C = 0 \)
- \( E + 12 + 4 = 0 \)

\[ E = \bar{8} \]

\( R_5 = 0 \)
Example 10: \(134.289 + 2.76\)

\[ m = 3, \ n = 1 \]
\[ \text{Number of constants} = 3 - 1 + 1 = 3 \]

If one wants 5 decimal digits then the quotient is

\[
\begin{array}{cccccccccc}
A & B & C & D & E & F & G & H & I \\
\hline
2 & 7 & 6 & 1 & 3 & 4 & 2 & 8 & 9 & 0 & 0 & 0 & 0
\end{array}
\]

\[
(R_1) (R_2) (R_3) (R_4) (R_5) (R_6) (R_7) (R_8) (R_9)
\]

Step 1: \(2A = 1\)
\[ A = 0, \ R_1 = 1 \]

Step 2: \(7A + 2B = 13\)
\[ 0 + 2B = 13 \]
\[ B = 6, \ R_2 = 1 \]

Step 3: \(6A + 7B + 2C = 14\)
\[ 0 + 42 + 2C = 14 \]
\[ 2C = 2.8 \]
\[ C = 1.4, \ R_3 = 0 \]

Step 4: \(2D + 6B + 7C = 2\)
\[ 2D + 36 + 9.8 = 2 \]
\[ 2D = 64 \]
\[ D = 32, \ R_4 = 0 \]

Step 5: \(2E + 7D + 6C = 8\)
\[ 2E + 224 + 8.4 = 8 \]
\[ 2E = 6.6 \]
\[ E = 6.6, \ R_5 = 0 \]

Step 6: \(2F + 7E + 6D = 9\)
\[ 2F + 4632 + 192 = 9 \]
\[ 2F = 279 \]

\[ F = 139, \ R_6 = 1 \]

Step 7: \(2G + 7F + 6E = 10\)
\[ 2G + 973 + 396 = 10 \]
\[ 2G = 567 \]

\[ G = 2.83, \ R_7 = 1 \]

Step 8: \(2H + 7G + 6F = 10\)
\[ 2H + 9731 + 834 = 10 \]
\[ 2H = 1137 \]

\[ H = 568, \ R_8 = 1 \]

Step 9: \(2I + 7H + 6G = 10\)
\[ 2I + 3976 + 1698 = 10 \]
\[ 2I = 2268 \]

\[ I = 1.134, \ R_9 = 1 \]

Quotient = \(A \ B \ C \ E \ F \ G \ H \ I\)

\[
= 0.6 \ 14.32 \ 6 \ 6 \ 139 \ 283 \ 568 \ 1134
\]

\[
= 5 \ 1 \ 3 \ 5 \ 5 \ 2 \ 5 \ 4
\]

\[
= 4 \ 8 \ 6 \ 5 \ 5 \ 2 \ 4 \ 6
\]
(c) Problems from Swamiji's Text and Hall and Knight Algebra
Argumental Division(For Polynomials) - simplified method

The procedure in brief can be explained as follows:

1. One should write down the dividend and divisor in descending order of power of $x$.

2. To divide the highest power of $x$ in the dividend with the highest power of $x$ in the divisor, which gives the first quotient($Q_1$).

3. Leaving the first term in the divisor, the rest of the terms are used for successive multiplications in Urdhva Tiryak manner. i.e., quotients are to be multiplied with the divisor terms.

4. While doing so, a comparison is made between the result of multiplication with the corresponding terms of the dividend, to establish the difference.*

5. The difference is now divided by the highest power of $x$ in the divisor to get the quotients and finally the absolute term.

   The method explained by swamiji in his book is exemplified through a number of problems in the book.
   A different method of division is also explained at the end of the notes.

6. For a few problems the current method is demonstrated and for the rest of the problems the reader is expected to complete.

Polynomial Division using Urdhva- Tiryak Sutram.
(ARGUMENTAL DIVISION)
Given the dividend and the divisor, the quotient can be worked out (or) given the dividend and quotient the divisor can be found out, both by the division method. Both these methods make use of the Urdhva Tiryagbyham Sutram used for multiplication. This method is very simple and division can be worked out with ease.

The following are a few examples and the various steps are explained.

Example 1: $x^3 - x^2 - 9x - 12 + x^2 + 3x + 3$

   The dividend is $x^3 - x^2 - 9x - 12$

   The divisor is $x^2 + 3x + 3$

Step 1. Divide $x^3$ by $x^2$

   $(x^3)$

   i.e., $x^3/x^2 = x Q_1($quotient$) Q_1=x$

Carry out Urdhva Tiryak multiplication of the part divisor $3x + 3$ with the successive quotients.

*"Original co - efficient" refers to that in the Dividend. "Difference" means subtracting of the multiplication result from that of the corresponding dividend term. i.e., original.
Vedic Mathematics

Division

Step 2: Now concentrating on the $x^2$-term of the dividend which is $-x^2$. This can be compared with the $x^2$ term obtained by the multiplication of quotients so obtained with the suitable terms in the divisor.

$$\text{Divisor} = x^2 + 3x + 3 \quad (\text{Urdhva})$$

$$\begin{array}{c}
\uparrow \\
3x (Q_1)
\end{array} = 3x^2 \quad (\text{Urdhva})$$

But the co-efficient of $x^2$ in the dividend is $-1$. In order to get $-x^2$ of the dividend, we have to now subtract $3x^2$ from $-x^2 = -x^2 - 3x^2 = -4x^2$

This is to be divided by $x^2$ to get $Q_2$ (the highest term in the divisor).

Hence $-4x^2/x^2 = -4(Q_2)$

\[Q_2 = -4\]

Step 3: Now the Tiryak multiplication is to be carried out between Quotient $Q_1$ $Q_2$ and the part divisor.

$$\begin{array}{c}
\downarrow \\
12x + 3x = -9x \quad (\text{Tiryak})
\end{array}$$

On Tiryak multiplication we get the value as $-9x$. The original co-efficient of $x$ is also $-9$. So, the difference between these two is zero.

Step 4: By Urdhva multiplication of last term of divisor and quotient.

$$Q_2 \quad \text{we get} \quad \begin{array}{c}
3 \\
4
\end{array} = 12 \quad (\text{Urdhva})$$

But the original absolute term is $12$.

.: The difference zero is the remainder.

i.e., Quotient $= Q_1 + Q_2 = x - 4$

Remainder $= 0$

Example 2: $28y^4 - 71y^3 - 35y^2 + 30y + 9 + 4y^2 - 13y + 6$

Step 1: $28y^4 / 4y^2 = 7y^2$ \[Q_1 = 7y^2\]

Carry out Urdhva Tiryak Multiplication of the part divisor $- 13y + 6$ with the successive quotients as follows.
Vedic Mathematics

Step 2: \(4y^2 - 13y + 6 = -91y^3\) (Urdhvva)

\[
\begin{array}{c}
4y^2 - 13y + 6 \\
\uparrow \\
7y^2 \\
Q_1
\end{array}
\]

But the original co-efficient of \(y^3 = -71\)

\[
\therefore \text{the difference is} = -71y^3 + 91y^3 = 20y^3
\]

\[
Q_2 = 20y^3 / 4y^2 = 5y
\]

\[
\boxed{Q_2 = 5y}
\]

Step 3:

\[
\begin{array}{c}
4y^2 + 13y + 6 \\
\uparrow \\
7y^2 + 5y \\
Q_1 \\
Q_2
\end{array}
\]

\[
42y^2 - 65y^2 = -23y^2 \text{(Tiryak)}
\]

But the original co-efficient of \(y^2 = -35\)

\[
\therefore \text{Difference} = -35y^2 + 23y^2 = -12y^2
\]

\[
-12y^2 / 4y^2 = -3
\]

\[
\boxed{Q_3 = 3}
\]

Original coefficient refers to that in the Dividend. Difference means subtracting of the multiplication result from that of the corresponding dividend term.

Step 4: \(4y^2 - 13y + 6\)

\[
\begin{array}{c}
4y^2 - 13y + 6 \\
\uparrow \\
7y^2 + 5y \\
Q_1 \\
Q_2 \\
Q_3
\end{array}
\]

\[39y + 30y = 69y \text{ (Tiryak)}\]

But the original co-efficient of \(y = 30\)

\[
\therefore \text{The difference} \ 30y - 69y = -39y \quad \text{This is Remainder} \ R_1
\]

\[
\boxed{R_1 = -39y}
\]

Step 5:

The last term by Urdhva multiplication \(-18\) of the last term with \(Q_3\)

\[
4y^2 - 13y + 6 \\
\uparrow \\
7y^2 + 5y - 3
\]

But the original absolute term is \(9\)

\[
\therefore \text{The difference is} \ 9 + 18 = 27 \quad \text{This gives the remainder} \ R_2
\]

\[
\boxed{R_2 = 27}
\]

\[
\therefore \text{Quotient} = Q_1 + Q_2 + Q_3 = 7y^2 + 5y - 3
\]

\[
\text{Remainder} = R_1 + R_2 = -39y + 27
\]
Example 3:  \[ 3\left(a^3/27 - a^2/12 + a/16 - 1/64\right) + \left(a^3 - 1/4\right) \]

Step 1: \( (a^3/27) + (a/3) = a^2/9 \) (Q₁)

Carry out Urdhva Tiryak Multiplication of the part divisor \(-\frac{1}{4}\)

\[ Q₁ = \frac{a^2}{9} \]

Step 2: \( a/3 - \frac{1}{4} \)

\( a^2/9 \)

\( a^2 - a^2/36 \) (Urdhva)

\( a^2/9 \)

But the original co-efficient of \( a^2 = -1/12 \)

\[ (-a^2/18) / (a/3) = -a/6 \) (Q₂)

\[ Q₂ = -\frac{a}{6} \]

Step 3: \( a/3 - \frac{1}{4} \)

\( a^{2/9} - a/6 \)

\( a/24 \) (Urdhva)

\( a^{2/9} - a/6 \)

\( Q₁, Q₂ \)

But the original co-efficient of \( a = 1/16 \)

\[ \therefore \text{The difference} = a/16 - a/24 = a/48 \]

\( (a/48) / (a/3) = 1/16 \) (Q₃)

\[ Q₃ = \frac{1}{16} \]

Step 4: \( \frac{a}{3} - \frac{1}{4} \)

(Absolute term)

\[ \frac{a}{9} - \frac{a}{6} + \frac{1}{16} \]

\( a^3/9 \)

\( a/6 \)

\( 1/64 \)

\( Q₁, Q₂, Q₃ \)

The original constant = \(-1/64\)

The difference is \( -\frac{1}{64} + \frac{1}{64} = 0 \)

Remainder = 0

Quotient \( Q₁ + Q₂ + Q₃ = a^2/9 - a/6 + 1/16 \)
Vedic Mathematics

Current Method:

\[
\begin{array}{c}
\frac{a^2}{3} - \frac{a}{4} + \frac{1}{16} - \frac{9}{64} + \frac{6}{16} + \frac{1}{36} \\
\end{array}
\]

Example 4: \[\frac{x^3 - 4x^4 + 3x^3 + 3x^2 - 3x + 2}{x^3 - x - 2}\]

Step 1

\[
\begin{array}{c}
\frac{x^3}{x^3} = x^3 (Q_1) \\
\end{array}
\]

\[Q_1 = x^3\]

Carry out Urdhva Tiryak Multiplication of part of the divisor (the Dhwajanka) \(- x - 2\) with successive quotients as follows:

Step 2

\[
\begin{array}{c}
\frac{x^2 - x - 2}{x^3} \\
\frac{x}{Q_1} \\
\end{array}
\]

\[Q_2 = -3x\] (Urdhva)

But the original co-efficient is \(-4x^4\)

\[\therefore \text{The difference is } -4x^4 + x^4 = -3x^4\]

\[-3x^4 \div x^2 = -3x^2\]

Step 3:

\[
\begin{array}{c}
\frac{x^2 - x - 2}{x^3} \\
\frac{3x^2 - 2x^3}{x^3} \\
\end{array}
\]

But the original coefficient is \(3x^3\): The difference is \(3x^3 - x^3 = 2x^3, \frac{2x^3}{x^2} = 2x (Q_3)\)

\[Q_3 = \]

Step 4:

\[
\begin{array}{c}
\frac{x^2 - x - 2}{x^3} \\
\frac{x^3 - 3x^2 + 2x}{Q_1, Q_2, Q_3} \\
\end{array}
\]

\[-2x^2 + 6x^2 = 4x^2 (Tiryak)\]
But the original co-efficient of $x^2$ is 3

\[ \therefore \text{The difference is } 3x^2 - 4x^2 - x^2, \frac{x^2}{x^2} = -1 \ (Q_4) \quad \therefore Q_4 = -1 \]

Step 5:

\[ x^2 - x - 2 \]

\[ x^3 - 3x^2 + 2x - 1 \]

\[ Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \]

(by Tiryak)

But the original co-efficient of $x^2$ is 3

\[ \therefore \text{The difference is 0 which is } R_1 \]

Step 6: (To get the absolute term)

\[ x^2 - x - 2 \]

\[ x^3 - 3x^2 + 2x - 1 \]

\[ Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \]

(Urdhva)

But the original absolute term is 3

\[ \therefore \text{The difference is } R_2 = 0 \]

Quotient is

\[ x^3 - 3x^2 + 2x - 1 \]

\[ Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \]

\[ R = R_1 + R_2 = 0 \]

Example 6:

\[ \frac{2y^3 - 3y^2 - 6y - 1}{2y^2 - 5y - 1} \]

Step 1:

\[ \frac{2y^3}{2y^2} = y \ (Q_1) \]

\[ Q_1 = y \]

carrying out Urddhva Tiryak Multiplication of the part divisor (the part of the divisor means, the divisor excluding the highest term which is used for division.) - 5y - 1 with successive quotients as follows

Step 2:

\[ 2y^2 - 5y - 1 \]

\[ y \]

\[ Q_1 \]

(Urdhva)

But the original coefficient of $y^1$ is 2

\[ \therefore \text{The difference is } -3y^2 - (-5y^2) = 2y^2 \]

\[ \frac{2y^2}{2y^2} = 1 \ (Q_2) \]

\[ Q_2 = 1 \]
Vedic Mathematics

Division

Step 3: \[ 2y^2 - 5y - 1 \] 
\( (y) \) 
\[ y + 1 \] 
\( Q_1 \quad Q_2 \) 
\[ = -5y - y = -6y \quad \text{(Tiryak)} \]

The original co-efficient of \( y \) is \( -6 \)
\[ \therefore \text{The difference is} \]
\[ -6y - (-6y) = 0 \]
\( R_1 = 0 \)

Step 4:
\[ 2y^2 - 5y \] 
\( (\text{to get the absolute term}) \) 
\[ y + 1 \] 
\( Q_1 \quad Q_2 \) 
\[ = -1 \quad \text{(Urdhva)} \]

The original absolute term is \(-1\)
\[ \therefore \text{The difference is} \]
\[ -1 - (-1) = 0 \]
\( R_2 = 0 \)

Hence Quotient = \( y + 1 \) and remainder = 0

Example 6:

\[ \frac{6m^3 - m^2 - 14m + 3}{3m^2 + 4m - 1} \]

Step 1:
\[ \frac{6m^3}{3m^3} = 2m \quad (Q_1) \]

Carrying out Urdhva Tiryak Multiplication of the part of the divisor \( 4m - 1 \) with successive quotients as follows.

Step 2:
\[ 3m^2 + 4m - 1 \] 
\( (m^2) \) 
\[ = 8m^2 \quad (Urdhva) \]
\[ 2m \]
\( Q_1 \)

The original co-efficient of \( m^2 \) is \(-1\)
\[ \therefore \text{Difference} = -m^2 - 8m^2 = -9m^2 \]
\[ -9m^2 = -3 \quad (Q_2) \]
\( Q_2 = -3 \)

Step 3:
\[ 3m^2 + 4m - 1 \] 
\( (m) \) 
\[ = -12m - 2m = -14m \quad (Tiryak) \]
\[ 2m - 3 \]
\( Q_1 \quad Q_2 \)

But the original co-efficient of \( m \) is \(-14\)
\[ \therefore \text{The difference is} \]
\[ -14m - (-14m) = 0 \quad (R_1) \]
\( R_1 = 0 \)
Vedic Mathematics

Division

Step 4: \[3m^2 + 4m + 1\]
(The absolute term) \[
\frac{2m - 3}{2m - 3} = 3\quad \text{(Urdhva)}
\]
\[
Q_1, Q_2
\]
The original absolute term is also 3
\[\therefore \text{The difference is } 3 - 3 = 0 \quad (R_2)\]

Quotient = \[Q_1 + Q_2 = 2m - 3\]
Remainder = \[R_1 + R_2 = 0\]

Example 7: \[
\frac{6a^5 - 13a^4 + 4a^3 + 3a^2}{3a^2 - 2a^2 - a}
\]

Step 1: \[
6a^5 = 2a^2 \quad (Q_1) \quad R_1 = 2a^2
\]

Carrying out Urdhva Tiryak Multiplication of the part divisor \[-2a^2 - a\] with successive quotients as follows

Step 2: \[
3a^3 - 2a^2 - a
\]
(a') \[
\frac{2a^2}{2a^2} = -4a^4 \quad (Urdhva)
\]
\[
Q_1
\]
But the co-efficient of original \[a^4\] is \[-13\]
\[\therefore \text{The difference is } -13a^4 - (-4a^4) = -9a^4\]
\[\frac{-9a^4}{3a^3} = -3a \quad (Q_2) \quad Q_2 = -3a
\]

Step 3: \[
3a^3 - 2a^2 - a
\]
(a') \[
\frac{2a^2}{2a^2} = 6a^3 - 2a^4 - 4a^5 \quad (Tiryak)
\]
\[
Q_1, Q_2
\]
But the co-efficient of original \[a^3\] is 4
\[\therefore \text{The difference is } 4a^3 - 4a^3 = 0\]

Step 4:
(a') \[
\frac{2a^2 - 3a}{2a^2 - 3a} = 3a^2 \quad (Urdhva)
\]
\[
Q_1, Q_2 \quad R_1
\]
But the coefficient of original \[a^2\] is 3
\[\therefore \text{The difference is } 3a^2 - 3a^2 = 0 \quad R_2 = 0\]
\[Q = Q_1 + Q_2 = 2a^2 - 3a \quad R = R_1 + R_2 = 0\]
Example 8

\[ x^4 + x^3 + 7x^2 - 6x + 8 \]
\[ x^4 + 2x + 8 \]

Step 1:

\[ \frac{x^4}{x^2} = x^2 \quad (Q_1) \]

\[ Q_1 = x^2 \]

Carry out the Urdhva and Tiryak multiplication of 2x + 8 of the dividend with successive quotient as follows:

Step 2:

\[ x^2 + 2x + 8 \]
\[ \frac{x^2}{x^3} = 2x^3 \quad (Urdhva) \]

\[ Q_1 \]

But the coefficient of original x is 1

\[ \therefore \text{ the difference is} \]

\[ x^3 - 2x^3 = -x^3 \quad & \quad \frac{-x^3}{x^2} = -x \quad (Q_2) \]

\[ Q_2 = -x \]

Step 3:

\[ x^2 + 2x + 8 \]
\[ \frac{x^2}{x^3} = 8x^2 - 2x^2 = 6x^2 \quad (Tiryak) \]

But the coefficient of original x is 7

\[ \therefore \text{ the difference is} \]

\[ 7x^2 - 6x^2 = x^2 \quad \frac{x^2}{x^2} = 1 \quad (Q_1) \]

\[ Q_1 = 1 \]

Step 4:

\[ x^2 + 2x + 8 \]
\[ \frac{x^2}{x} = 2x - 8x = -6x \quad (Tiryak) \]

But the coefficient of original x term is -6

\[ \therefore \text{ the difference is} \]

\[ -6x + 6x = 0 \quad \frac{R_1}{R_1} = 0 \]

Step 5:

(To get the Absolute

\[ x^2 + 2x + 8 \]
\[ \frac{x^2}{x} = x^2 - x + 1 \quad (Urdhva) \]

But the original absolute term is 8

\[ \therefore \text{ the difference is} \]

\[ 8 - 8 = 0 \quad \frac{R_2}{R_2} = 0 \]

Quotient, \[ Q = Q_1 + Q_2 + Q_3 = x^2 - x + 1 \]

Remainder, \[ R = R_1 + R_2 = 0 \]
Vedic Mathematics

Example 2: \[ \frac{a^4 - a^3 - 8a^2 + 12a - 9}{a^2 + 2a - 3} \]

Step 1:
\[ a^4 = a^2 \quad (Q_1) \]
\[ a^4 = a^2 \] \[ Q_1 = a^2 \]

Carrying out Urdhva Tiryak multiplication of the remaining part of the divisor \(2a - 3\) by the successive quotients

Step 2:
\[ a^2 + 2a - 3 \]
\[ (a^3) \]
\[ a^2 \]
\[ = 2a \]
\[ Q_1 \]

But the coefficient of original \(a^1\) is -1
\[ \therefore \text{ the difference is} \]
\[ -a^3 - 2a^3 = -3a^3; \]
\[ \frac{-3a^3}{a^2} = -3a \quad (Q_2) \]
\[ Q_2 = -3a \]

Step 3:
\[ a^2 + 2a - 3 \]
\[ (a^3) \]
\[ a^2 - 3a \]
\[ = -3a^2 - 6a^2 = -9a^2 \]
\[ Q_1 \]
\[ Q_2 \]

But the coefficient of original \(a^2\) is 8
\[ \therefore \text{ the difference is} \]
\[ -8a^2 - (-9a^2) = a^2 \]
\[ a^2 = 1 \quad (Q_3) \]
\[ Q_3 = 1 \]

Step 4:
\[ a^2 + 2a - 3 \]
\[ (a) \]
\[ a^2 - 3a + 1 \]
\[ = 9a + 2a = 11a \quad (Tiryak) \]
\[ Q_1 \]
\[ Q_2 \]
\[ Q_3 \]

But the coefficient of original is 12
\[ \therefore \text{ the difference is} \]
\[ 12a - 11a = a \quad (R_1) \]
\[ R_1 = a \]

Step 5:
\[ a^2 + 2a - 3 \]
\[ (\text{To get the absolute term}) \]
\[ a^2 - 3a + 1 \]
\[ Q_1 \]
\[ Q_2 \]
\[ Q_3 \]

But the original absolute term is 9
\[ \therefore \text{ the difference is} \]
\[ -9 - (-3) = -6 \quad (R_2) \]
\[ R_2 = -6 \]

\[ Q = Q_1 + Q_2 + Q_3 = a^2 - 3a + 1 \quad \& \quad R = R_1 + R_2 = a - 6 \]
Example 10

\[ \frac{a^4 + 6a^3 + 13a^2 + 12a + 4}{a^3 + 3a^2 + 2} = \frac{Q}{R} \]

**Step 1:**

\[ \frac{a^4}{a^3} = a \quad (Q_1) \]

\[ Q_1 = a^2 \]

Carrying out Urdhva Tryak multiplicaton of the remaining part \(3a + 2\) of the divisor with the successive quotients

**Step 2:**

\[ \frac{a^2 + 3a + 2}{a^3} = 3a^3 \text{(Urdhva)} \]

But the coefficient of the given \(a^3\) is 3

\[ 6a^3 - 3a^3 = 3a^3 \; ; \quad \frac{3a^3}{a^2} = 3a \quad (Q_2) \]

\[ Q_2 = 3a \]

**Step 3:**

\[ \frac{a^2 + 3a + 2}{a^2} = 2a^2 + 9a^2 = 11a^2 \text{ (Tryak)} \]

But the coefficient of given \(a^2\) is 13

\[ 13a^2 - 11a^2 = 2a^2, \quad \frac{2a^2}{a^2} = 2 \quad (Q_3) \]

\[ Q_3 = 2 \]

**Step 4:**

\[ \frac{a^2 + 3a + 2}{a} = 6a + 6a = 12a \text{ (Tryak)} \]

But the coefficient of given \(a\) is 12

\[ 12a - 12a = 0 \quad (R_1) \]

\[ R_1 = 0 \]

**Step 5:**

\[ \frac{a^2 + 3a + 2}{a} = 4 \text{ (Urdhva)} \]

But the given absolute term is 4

\[ Q = Q_1 + Q_2 + Q_3 = a^2 + 3a + 2 \quad & \quad R = R_1 + R_2 = 0 + 0 = 0 \]

Example 11

\[ \frac{2x^4 - x^3 + 4x^2 + 7x + 1}{x^2 - x + 3} = \frac{Q}{R} \]

**Step 1:**

\[ \frac{2x^4}{x^2} = 2x^2 \quad (Q_1) \]

\[ Q_1 = 2x^2 \]
Carrying out the multiplication of the remaining part $x + 3$ of the divisor with the successive quotients as follow

**Step 2:**

\[
\begin{align*}
(x^2) & \\
\frac{x^2}{2x^2} - \frac{x}{Q_1} + \frac{3}{Q_1} &= -2x \quad \text{(Urdhva)}
\end{align*}
\]

But the coefficient of original $x^3$ is $-1$

\[
\therefore \text{ the difference is } -x^3 - (-2x^3) = x^3 & \quad \frac{x^3}{x^2} = x (Q_2) \quad Q_2 = x
\]

**Step 3:**

\[
\begin{align*}
(x^2) & \\
\frac{x^2}{2x^2} - \frac{6x^2}{Q_1} + \frac{3}{Q_1} &= 5x^2 \quad (Tiryak)
\end{align*}
\]

the coefficient of the original $x^2$ is $4$

\[
\therefore \text{ the difference is } 4x^2 - 5x^2 = -x^2 & \quad \frac{-x^2}{x^2} = -1 \quad (Q_3) \quad Q_3 = -1
\]

**Step 4:**

\[
\begin{align*}
(x) & \\
\frac{x^2}{2x^2} - \frac{3}{Q_1} + \frac{x}{Q_2} &= 3x + x = 4x \quad (Tiryak)
\end{align*}
\]

the coefficient of the original $x$ is $7$

\[
\therefore \text{ the difference is } 7x - 4x = 3x \quad (R_1) \quad R_1 = 3x
\]

**Step 5:**

(To get the Absolute term) \[
\begin{align*}
\frac{x^2}{2x^2} - \frac{1}{Q_1} + \frac{3}{Q_1} &= -3 \quad \text{(Urdhva)}
\end{align*}
\]

The absolute term of the original is $1$

\[
\therefore \text{ the difference is } 1 - (-3) = 4 \quad (R_2) \quad R_2 = 4
\]

\[
Q = Q_1 + Q_2 + Q_3 = 2x^2 + x - 1 & \quad R = R_1 + R_2 = 3x + 4
\]

**Example 12:**

\[
\frac{x^4 - 5x^4 + 9x^3 - 6x^2 - x + 2}{x^2 - 3x + 2} \quad Q = x^3 - 2x^2 + x + 1 \quad R = 0
\]

**Step 1:**

\[
\begin{align*}
(x^3) & \\
\frac{6}{x^3} &= x^1 \quad (Q_1) \quad Q_1 = x^1
\end{align*}
\]

Carrying out Urdhva Tiryak multiplication of the part divisor $-3x + 2$ with successive quotients as follows
Vedic Mathematics

Step 2:
\[
x^2 - 3x + 2 \quad (x^4)
\]
\[
(x^4)
\]
\[
Q_1
\]
The coefficient of the original \(x^4\) is 5
\[
\therefore \text{the difference is}
\]
\[
-5x^4 - (-3x^4) = -2x^4 \quad \& \quad -2x^4 = -2x^2
\]
\[
Q_2 = -2x^2
\]

Step 3:
\[
x^2 - 3x + 2 \quad (x^3)
\]
\[
(x^3)
\]
\[
Q_1 \quad Q_2
\]
The coefficient of the original \(x^3\) is 9
\[
\therefore \text{the difference is}
\]
\[
9x^3 - 8x^3 = x^3 \quad \& \quad x^3 = x
\]
\[
Q_3 = x
\]

Step 4:
\[
x^2 - 3x + 2 \quad (x^2)
\]
\[
(x^2)
\]
\[
Q_1 \quad Q_2 \quad Q_3
\]
The coefficient of the original \(x^2\) is 6
\[
\therefore \text{the difference is}
\]
\[
-6x^2 - (-7x^2) = x^2 \quad \& \quad x^2 = 1
\]
\[
Q_4 = 1
\]

Step 5:
\[
x^2 - 3x + 2 \quad (x)
\]
\[
(x)
\]
\[
Q_1 \quad Q_2 \quad Q_3 \quad Q_4
\]
The coefficient of the original \(x\) is also 1
\[
\therefore \text{the difference is}
\]
\[
-x - (-x) = 0 \quad (R_1)
\]

Step 6:
\[
x^2 - 3x + 2 \quad \text{(To get the Absolute Term)}
\]
\[
(x^2)
\]
\[
Q_1 \quad Q_2 \quad Q_3 \quad Q_4
\]
The original absolute term is 2
\[
\therefore \text{the difference is}
\]
\[
Q = Q_1 + Q_2 + Q_3 + Q_4 = x^3 - 2x^2 + x + 1 \quad \& \quad R = R_1 + R_2 = 0
\]

Example 13
\[
x^5 - 4x^4 + 3x^3 - 3x^2 + 2 \quad \frac{x^3}{x^2 - x - 2}
\]
\[
Q = x^3 - 3x^2 + 2x - 1
\]
\[
R = 0
\]

Step 1:
\[
x^5 \quad (x^7)
\]
\[
(x^7)
\]
\[
Q_1 = x^3
\]

Carrying out Urdhva-Tiryak multiplication of the remaining part of the divisor \(-x - 2\) by the successive quotients
Step 2:
\[ x^2 - x - 2 \]
\[ (x^4) \]
\[ \begin{array}{c}
\text{Q}_1 \\
\hline
\text{Q}_2 \\
\end{array} \]
\[ -x (x^2) = -x^4 \quad \text{(Urdhva)} \]
But the coefficient of the original is \(-4x^4\)
\[ \therefore \text{the difference is} \]
\[ -4x^4 - (-x^4) = -3x^4 \]
\[ -3x^4 = 3x^2 \quad \text{(Q}_2\text{)} \]
\[ Q_2 = -3x^4 \]

Step 3:
\[ x^2 - x - 2 \]
\[ (x^3) \]
\[ \begin{array}{c}
\text{Q}_1 \\
\hline
\text{Q}_2 \\
\end{array} \]
\[ \begin{array}{c}
\text{Q}_3 \\
\hline
\end{array} \]
\[ 2x^3 + 3x^3 = x^3 \quad \text{(Tiryak)} \]
But the coefficient of original is \(x^3\) is 3
\[ \therefore \text{the difference is} \]
\[ 3x^3 - x^3 = 2x^3 \]
\[ \frac{2x^3}{x^2} = 2x \quad \text{(Q}_3\text{)} \]
\[ Q_3 = 2x \]

Step 4:
\[ x^2 - x - 2 \]
\[ (x^3) \]
\[ \begin{array}{c}
\text{Q}_1 \\
\hline
\text{Q}_2 \\
\end{array} \]
\[ \begin{array}{c}
\text{Q}_3 \\
\hline
\text{Q}_4 \end{array} \]
\[ (-2) (-3x^2) + (2x) (-x) = 6x^2 - 2x^2 = 4x^2 \quad \text{(Tiryak)} \]
But the coefficient of original is \(3x^2\)
\[ \therefore \text{the difference is} \]
\[ 3x^2 - 4x^2 = -x^2 \quad \text{&} \quad -x^2 = -1 \quad \text{(Q}_4\text{)} \]
\[ Q_4 = -1 \]

Step 5:
\[ x^2 - x - 2 \]
\[ (x) \]
\[ \begin{array}{c}
\text{Q}_1 \\
\hline
\text{Q}_2 \\
\end{array} \]
\[ \begin{array}{c}
\text{Q}_3 \\
\hline
\text{Q}_4 \end{array} \]
\[ 4x + x = -3x \quad \text{(Tiryak)} \]
But the coefficient of original is also \(-3x\)
\[ \therefore \text{the difference is} \]
\[ -3x - (-3x) = 0 \quad \text{(R}_1\text{)} \]
\[ R_1 = 0 \]

Step 6:
\[ x^2 - x - 2 \]
\[ \text{Absolute} \]
\[ \begin{array}{c}
\text{Q}_1 \\
\hline
\text{Q}_2 \\
\end{array} \]
\[ \begin{array}{c}
\text{Q}_3 \\
\hline
\text{Q}_4 \end{array} \]
\[ 2 \]
But the original absolute is also 2
\[ \therefore 2 - 2 = 0 \quad \text{(R}_3\text{)} \]
\[ R_3 = 0 \]

\[ Q = Q_1 + Q_2 + Q_3 + Q_4 = x^3 - 3x^2 + 2x - 1 \quad \text{&} \quad R = R_1 + R_2 = 0 + 0 = 0 \]
Example 14

\[
\frac{30x^4 + 11x^3 - 82x^3}{3x^3 + 2x - 4} \div \frac{5x + 3}{x^4} = 10x \quad (Q_1) \quad Q_1 = 10x
\]

Carrying out the Urthva Tryak multiplication of the remaining part of the divisor 0.x^2 + 2x - 4 by the successive quotients.

Step 2.

\[
3x^3 + 0x^2 + 2x - 4
\]

\[\begin{array}{c}
+ 10x \\
Q_1 \\
\end{array}
\]

= 0 x^3 \quad \text{(Urthva)}

But the coefficient of the original x^4 is -71

the difference is

\[-71x^3 - 0 \times x^3 = -71x^3 \quad \& \quad \frac{-71x^3}{3x^3} = -\frac{71}{3} \]

\[Q_2 = -\frac{71}{3}\]

Step 3:

\[
3x^3 + 0x^2 + 2x - 4
\]

\[\begin{array}{c}
10x \times -\frac{71}{3} \\
Q_1 \\
Q_2 \\
\end{array}
\]

\[= 20x^2 \quad \text{(Tryak)}
\]

But the coefficient of the original x^3 is 0

∴ the difference is

\[0 \times x^2 - 20x^2 = -20x^2 \]

\[R_1 = -20x^2\]

Step 4:

\[
3x^3 + 0x^2 + 2x - 4
\]

\[\begin{array}{c}
10x \times -\frac{71}{3} \\
Q_1 \\
Q_2 \\
\end{array}
\]

\[= -\frac{142}{3}x - 40x = -\frac{262}{3}x \quad \text{(Tryak)}
\]

But the coefficient of original x is -5

∴ the difference

\[-5x + \frac{262}{3}x = \frac{247}{3}x \]

\[R_2 = \frac{247}{3}x\]

Step 5:

\[
3x^3 + 0x^2 + 2x - 4
\]

(Absolute)

\[\begin{array}{c}
10x \times -\frac{71}{3} \\
Q_1 \\
Q_2 \\
\end{array}
\]

\[= \frac{284}{3} \quad \text{(Urthva)}
\]

But the original absolute value is 3

∴ the difference is

\[-\frac{284}{3} = \frac{-275}{3} \]

\[R_3 = -\frac{275}{3}\]

Quotient = 10x - \frac{71}{3}

Remainder = -20x^2 + \frac{247}{3}x - \frac{275}{3}
Vedic Mathematics

Example 15:

\[
\frac{6k^5 - 15k^4 + 4k^3 + 7k^2}{3k^3 - k + 1} \cdot Q = 2k^2 - 5k + 2
\]

Step 1:

\[\frac{6k^5}{3k^3} = 2k^2 \quad (Q_1) \quad Q_1 = 2k^2\]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor
\(0k^2 - k + 1\) by the successive quotients

Step 2:

\[
\frac{3k^3}{2k^2} \cdot 0 = 0 \quad k^4 \quad (Urdhva)
\]

But the coefficient of original \(k^4\) is \(-15k^4\)
	he difference is

\[-15k^4 - 0 = -15k^4 \quad & \quad -15k^4 \cdot -5k \quad Q_2 = -5k\]

Step 3:

\[
\frac{3k^3 + 0k^2 - k + 1}{2k^2 - 5k} = -2k^3 \quad (Tiryak)
\]

The coefficient of original \(k^3\) is 4
	he difference is

\[4k^3 - (-2k^3) = 6k^3 \quad & \quad 6k^3 = 2 \quad Q_3 = 2\]

Step 4:

\[
\frac{3k^3 + 0k^2 - k + 1}{2k^2 - 5k + 2} = -2k^3 + 5k^2 = 7k^2 \quad (Tiryak)
\]

The coefficient of original \(k^3\) is also 7

\therefore \quad The \ difference \ is \ 0

Step 5:

\[
\frac{3k^3 + 0k^2}{2k^2} \quad \frac{k + 1}{2k^2} = 2k \quad 5k = -7k \quad (Tiryak)
\]

The coefficient of original k is also -7

\therefore \quad the \ difference \ is \ 0

Step 6:

\[
\frac{3k^3 + 0k^2 - k + 1}{2k^2 + 5k + 2} \quad (Urdhva)
\]

The original absolute value is also 2

\therefore \quad: \quad Quotient = 2k^2 - 5k + 2, \quad Remainder \ 0
Vedic Mathematics

Example 16

\[
\frac{15 + m^5 + 2m^4 + 4m^3 + 9m^2 - 31m}{3 - 2m - m^2}
\]

\[Q = 5 - 7m - m^3\]

given problem can be written as

\[
\frac{m^5 + 2m^4 + 4m^3 + 9m^2 - 31m + 15}{-m^2 - 2m + 3}
\]

Step 1:

\[
\frac{-m^3}{m^3} = -m^3 (Q_1)
\]

\[Q_1 = -m^3\]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor \(-2m + 3\) with the successive quotients

Step 2:

\[
\frac{-m^2 + 2m + 3}{m^4} = 2m^4
\]

But the original coefficient of \(m^4\) is also 2

\[\therefore \text{the difference} = 2m^4 - 2m^4 = 0 (Q_2)\]

\[Q_2 = 0m^3\]

Step 3:

\[
\frac{-m^2 - 2m + 3}{m^3} \quad \frac{-m^3}{m^3} = -3m^3 \quad \text{(Tiryak)}
\]

But the original coefficient of \(m^3\) is 4

\[\therefore \text{difference is} = 4m^3 - (-3m^3) = 7m^3\]

\[\frac{7m^3}{-m^3} = -7m (Q_3)\]

\[Q_3 = -7m\]

Step 4:

\[
\frac{-m^2 - 2m + 3}{m^2} \quad \frac{-m^3}{m^3} = 14m^2 \quad \text{(Tiryak)}
\]

But the original coefficient of \(m^2\) is 14

\[\therefore \text{difference is} = 9m^2 - 14m^2 = -5m^2\]

\[-5m^2 = 5 (Q_4)\]

\[Q_4 = 5\]

Step 5:

\[
\frac{-m^2 - 2m + 3}{m} \quad \frac{-m^3}{m^3} = -10m - 21m = -31m \quad \text{(Tiryak)}
\]

But the original coefficient of \(m\) is \(-31\)

\[\therefore \text{the difference is} = -31m - (-31m) = 0 (R_1)\]

\[R_1 = 0\]
Vedic Mathematics

Division

Step 6: \(-m^2 - 2m + 3\)

Absolute \[ \frac{m^3 + 0m^2 - 7m + 5}{Q_1 \quad Q_2 \quad Q_3 \quad Q_4} = 15 \]

The original absolute value is also 15.

\[ \therefore \text{The difference} = 15 - 15 = 0 \quad (R_2) \]

\[ R_2 = 0 \]

\[ Q = Q_1 + Q_2 + Q_3 \quad \& \quad R = R_1 + R_2 \]

\[ Q = -m^3 - 7m + 5 \quad \& \quad R = 0 \]

Example 17:

\[ \frac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{x^2 - 2x - 9} \]

Step 1:

\[ \frac{6x^4}{x^2} \cdot 6x^2 \quad (Q_1) \]

\[ Q_1 = 6x^2 \]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor \(-2x - 9\) with the successive quotients

Step 2:

\[ \frac{x^2 - 2x - 9}{6x^2} \quad = -12x^3 \quad \text{(Urdhva)} \]

But the original coefficient of \(x^3\) is 13.

\[ \therefore \text{the difference} = 13x^3 + 12x^2 = 25x^3 \]

\[ \frac{25x^3}{x^2} = 25x \quad (Q_1) \]

\[ Q_2 = 25x \]

Step 3:

\[ \frac{x^2 - 2x - 9}{6x^2 + 25x} \quad = -50x^2 \quad 54x^2 = -104x^2 \quad \text{(Tiryak)} \]

The original coefficient of \(x^4\) is 39.

\[ \therefore \text{the difference} = (39 + 104) \quad x^2 = 143x^2 \]

\[ \frac{143x^2}{x^2} \quad = 143 \quad (Q_1) \]

\[ Q_3 = 143 \]

Step 4:

\[ \frac{x^2 - 2x - 9}{6x^2 + 25x + 143} \quad \cdot (-286 - 225) \quad -511x \quad \text{(Tiryak)} \]

The original coefficient of \(x\) is 37.

\[ \therefore \text{the difference} = (37 + 511) \quad 548x \quad (R_1) \]

\[ R_1 - 548x \]

Step 5:

\[ \frac{x^2 - 2x - 9}{6x^2 + 25x + 143} \quad -1287 \quad \text{(Urdhva)} \]

\[ Q_1 \quad Q_2 \quad Q_3 \]
Vedic Mathematics

The original absolute value is 45
\[\therefore \text{the difference} = 45 + 1287 = 1332 \quad (R_2) \]
\[R_2 = 1332\]
\[Q = Q_1 + Q_2 + Q_3 = 6x^2 + 25x + 143\]
\[R = R_1 + R_2 = 548x + 1332\]

Comparison of all the methods

Example 18: \[
\begin{align*}
7x^{10} + 26x^9 + 53x^8 + 56x^7 + 43x^6 + 40x^5 + 41x^4 + 38x^3 + 19x^2 + 8x + 5
\end{align*}
\]
\[
\begin{align*}
x^9 + 3x^4 + 5x^3 + 3x^2 + x + 1
\end{align*}
\]

Method 1 \hspace{1cm} \text{Argumental Division – (Urdhva Tiryak)}

Step 1: \[
\begin{align*}
7x^{10} = 7x^3 \quad (Q_1)
\end{align*}
\]
(x^{10})

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend
\[3x^4 + 5x^3 + 3x^2 + x + 1\]
with the successive quotients

Step 2: \[
\begin{align*}
x^9 + 3x^4 + 5x^3 + 3x^2 + x + 1
\end{align*}
\]
\[7x^5\]
\[Q_1\]
\[= 21x^9 \quad \text{(Urdhva)}\]

But the original coefficient of \(x^9\) is 26

\[\cdot \quad \text{The difference} = 26x^9 - 21x^9 = 5x^9, \quad \frac{5x^9}{x^9} = (Q_2) \quad Q_2 = 5x^3\]

Step 3 \[
\begin{align*}
x^3 + 3x^2 + 5x^2 + 3x^2 + x + 1
\end{align*}
\]
\[7x^4 + 5x^4\]
\[Q_1 \quad Q_2\]
\[= 15x^8 + 35x^8 = 50x^8 \quad \text{(Tiryak)}\]

But the original coefficient of \(x^8\) is 50

\[\cdot \quad \text{The difference} = 53x^8 - 50x^8 = \frac{3x^8}{x^8} = 3x^4 \quad (Q_3) \quad Q_3 = 3x^3\]

Step 4: \[
\begin{align*}
x^4 + 3x^4 + 5x^3 + 3x^3 + x + 1
\end{align*}
\]
\[7x^7 + 5x^4 + 3x^3\]
\[Q_1 \quad Q_2 \quad Q_3\]
\[= 9x^7 + 21x^7 + 25x^7 = 55x^7 \quad \text{(Tiryak)}\]

The original coefficient of \(x^7\) is 56

The difference = 56x^7 - 55x^7 = x^7

\[\frac{x^7}{x^7} = x^2 \quad (Q_4) \quad Q_4 = x^3\]

Step 5: \[
\begin{align*}
x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1
\end{align*}
\]
\[7x^8 + 5x^4 + 3x^3 + x\]
\[Q_1 \quad Q_2 \quad Q_3 \quad Q_4\]
\[= 3x^6 + 7x^6 + 15x^6 + 15x^6 = 40x^6 \quad \text{(Tiryak)}\]

The original coefficient of \(x^6\) is 43
Vedic Mathematics

The difference \( 43x^6 - 40x^6 = 3x^6, \frac{3x^6}{x^5} = 3x \) (Q₃) \( \boxed{Q₃ = 3x} \)

Step 6: \( x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1 \)
\( \frac{7x^5}{Q₁} + \frac{5x^4}{Q₂} + \frac{3x^3}{Q₃} + \frac{x^2}{Q₄} + \frac{x}{Q₅} + \frac{1}{Q₆} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original coefficient of \( x^5 \) is 40

\( ∴ \text{The difference} = 40x^5 - 35x^5 - 5x^5, \frac{5x^5}{x^5} = 5 \) (Q₀) \( \boxed{Q₀ = 5} \)

Step 7: \( x^4 + 3x^3 + 5x^2 + 3x + 1 \)
\( \frac{7x^4}{Q₁} + \frac{5x^3}{Q₂} + \frac{3x^2}{Q₃} + \frac{x}{Q₄} + \frac{1}{Q₅} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original coefficient of \( x^4 \) is also 41

\( ∴ \text{Difference} = 0 \)
\( \boxed{R₁ = 0} \)

Step 8: \( x^3 + 3x^2 + 5x^1 + 3x^1 + x + 1 \)
\( \frac{7x^3}{Q₁} + \frac{5x^2}{Q₂} + \frac{3x^1}{Q₃} + \frac{x}{Q₄} + \frac{1}{Q₅} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original coefficient of \( x^3 \) is also 38

\( ∴ \text{Difference} = 0 \)
\( \boxed{R₂ = 0} \)

Step 9: \( x^2 + 3x^2 + 5x^1 + 3x^1 + x + 1 \)
\( \frac{7x^2}{Q₁} + \frac{5x^1}{Q₂} + \frac{3x^1}{Q₃} + \frac{x}{Q₄} + \frac{1}{Q₅} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original coefficient of \( x^2 \) is also 19

\( ∴ \text{Difference} = 0 \)
\( \boxed{R₃ = 0} \)

Step 10: \( x^1 + 3x^1 + 5x^1 + 3x^1 + x + 1 \)
\( \frac{7x^1}{Q₁} + \frac{5x^1}{Q₂} + \frac{3x^1}{Q₃} + \frac{x}{Q₄} + \frac{1}{Q₅} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original coefficient of \( x \) is also 8

\( ∴ \text{The difference} = 0 \)

Step 11: \( x^1 + 3x^1 + 5x^1 + 3x^1 + x + 1 \)
(Absolute)
\( \frac{7x^1}{Q₁} + \frac{5x^1}{Q₂} + \frac{3x^1}{Q₃} + \frac{x}{Q₄} + \frac{1}{Q₅} \)
\( Q₁Q₂Q₃Q₄Q₅Q₆ \)

The original absolute value is also 5

\( ∴ \text{Difference} = 0 \)

Quotient = \( Q₁ + Q₂ + Q₃ + Q₄ + Q₅ + Q₆ = 7x^3 + 5x^4 + 3x^3 + x^2 + 3x + 5 \)
Remainder = \( R₁ + R₂ + R₃ + R₄ + R₅ = 0 \)
Method 2

Straight Division

\[
\begin{align*}
D_1 & \quad D_2 & \quad D_3 & \quad D_4D_5 \\
3x^4+5x^3+3x^2+x+1 & \quad 7x^{10}+26x^9+53x^8+56x^7+43x^6+40x^5 & \quad + 41x^4+38x^3+19x^2+8x+5 \\
\end{align*}
\]

\[
x^5 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
\]

\[
7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5 = 0
\]

Step 1:

\[
\frac{7x^{10}}{x^5} = 7x^5 (Q_1), \quad R_1 = 0 \quad \boxed{Q_1 = 7x^5}
\]

\[
(x^{10})
\]

Step 2:

\[
ID_1 = 26x^9 - \left( \frac{D_1}{3x} \right) = 26x^9 - 21x^9 = \frac{5x^9}{x^5} = 5x^4 \quad R_2 = 0 \quad \boxed{Q_2 = 5x^4}
\]

\[
(x^9)
\]

Step 3:

\[
ID_2 = 53x^8 - \left( \frac{D_1 + D_2}{7x^4} \right) = 53x^8 - 50x^4 = \frac{3x^8}{x^4} = 3x^4 \quad R_3 = 0 \quad \boxed{Q_3 = 3x^4}
\]

\[
(x^8)
\]

Step 4:

\[
ID_3 = 56x^7 - \left( \frac{D_1 + D_2 + D_3}{x^7} \right) = 56x^7 - (9x^7 + 21x^7 + 25x^7)
\]

\[
= 56x^7 - 55x^7 = \frac{x^7}{x^7} = x^2 \quad R_4 = 0 \quad \boxed{Q_4 = x^2}
\]

\[
(x^7)
\]

Step 5:

\[
ID_4 = 43x^6 - \left( \frac{D_1 + D_2 + D_3}{5x^3} \right) = 43x^6 - 40x^6 = 3x^6
\]

\[
= \frac{3x^6}{x^3} = 3x \quad R_5 = 0 \quad \boxed{Q_5 = 3x}
\]

\[
(x^6)
\]

Step 6:

\[
ID_5 = 40x^5 - \left( \frac{D_1 + D_2 + D_3 + D_4}{x^5} \right) = 40x^5 - 35x^5 = 5x^5
\]

\[
= \frac{5x^5}{x^5} = 5 \quad R_6 = 0 \quad \boxed{Q_6 = 5}
\]

\[
(x^5)
\]
Step 7. Remainder = \( 41x^4 + 38x^3 + 19x^2 + 8x + 5 \) 

\[
\begin{array}{cccc}
3x^5 & 5x^4 & 3x^2 & x \\
5x^4 & 3x^3 & x & \downarrow \\
3x^2 & x & \downarrow \\
5x^3 & 3x^2 & x & \downarrow \\
5x & 3x^2 & x & \downarrow \\
\end{array}
\]

\[
= 41x^4 + 38x^3 + 19x^2 - 8x + 5 - 41x^4 - 38x^3 - 19x^2 - 8x - 5 = 0
\]

Quotient = \( 7x^3 + 5x^2 + 3x^2 + x^2 + 3x + 5 \), Remainder = 0

**Method 3 Paravartya Yojana:**

\[
\begin{array}{ccccccc}
x^3 & +3x^2+5x^2+3x^2+x+1 & x^{10} & x^9 & x^8 & x^7 & x^6 & x \\
-3 & -5 & -3 & -1 & -1 & 7 & 26 & 53 & 56 & 43 & 40 & 41 & 38 & 19 & 8 & 5 \\
& & -21 & 35 & -21 & -7 & 7 & & & & & & & & & \\
& & -3 & 5 & & 3 & -1 & -1 & & & & & & & & & \\
\end{array}
\]

Quotient = \( 7x^3 + 5x^2 + 3x^3 + x^2 + 3x + 5 \) Remainder = 0

**Example 19:**

\[
\frac{8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6}{7x^2 + 2x - 1}
\]

**Step 1:**

\[
\frac{8x^5}{7x^2} = \frac{8x^3}{7} (Q_1)
\]

\[
Q_1 = \frac{8x^3}{7}
\]

Carrying out the Urddhva Tiryak multiplication with the remaining part of the divisor \( 2x + 1 \) with the successive quotients

**Step 2:**

\[
7x^2 + 2x + 1
\]

\[
= \frac{16x^4}{7} (Urdhva)
\]

\[
\frac{8x^3}{7} Q_1
\]
Vedic Mathematics

\[
\frac{47x^4}{7} = \frac{47x^4}{49x^2} = \frac{47x^2}{49}
\]

The original coefficient of \(x^4\) is 9

\[\therefore \text{Difference} = 9x^4 - 16x^4 = \frac{47x^4}{7}\]

**Step 3:**

\[
7x^2 + 2x + 1
\]

\[
(x^3) \quad \frac{8x^3}{7} + \frac{94x^3}{49} \quad (\text{Tiryak})
\]

\[
\frac{8x^3}{7} + \frac{94x^3}{49} = \frac{56 + 94}{49}x^3 = \frac{150}{49}x^3
\]

But the original coefficient of \(x^3\) is 7

\[\therefore \text{The difference} = 7x^3 - \frac{150x^3}{49} = \frac{343x^3 - 150x^3}{49} = \frac{193x^3}{49}\]

\[\frac{193x^3}{49} = \frac{193x}{343} \quad (Q_3)
\]

**Step 4:**

\[
7x^2 + 2x + 1
\]

\[
(x^2) \quad \frac{8}{7}x^2 + \frac{47}{49}x + \frac{193}{343}x
\]

\[
\frac{8}{7}x^2 + \frac{47}{49}x + \frac{193}{343}x
\]

Q_1 \quad Q_2 \quad Q_3

But the original coefficient of \(x^2\) is 3

\[\therefore \text{the difference} = 3x^2 - \frac{715}{343}x^2 = \frac{314}{343}x^2\]

\[\frac{314}{343}x^2 = \frac{314}{2401} \quad (Q_4)
\]

**Step 5:**

\[
7x^2 + 2x + 1
\]

\[
(x) \quad \frac{8}{3}x^2 + \frac{47}{343}x^2 + \frac{193}{343}x + \frac{314}{2401}x
\]

Q_1 \quad Q_2 \quad Q_3 \quad Q_4

But the original coefficient of \(x\) is 5
Vedic Mathematics

\[
\text{Difference} = 5x - \frac{1979}{2401}x = \frac{10026}{2401}x \quad (R_1)
\]

\[
R_1 = \frac{10026}{2401}x
\]

Step 6: \( 7x^2 + 2x + 1 \)

(Absolute term)

\[
\begin{array}{cccc}
\frac{8}{3} & \frac{47}{49} & \frac{193}{343} & \frac{314}{2401} \\
Q_1 & Q_2 & Q_3 & Q_4
\end{array}
\]

But the original absolute term is 6

\[
\text{Difference} = 6 - \frac{314}{2401} = \frac{14092}{2401}
\]

\[
a = \frac{14092}{2401}
\]

\[
\therefore \text{Quotient, } Q = Q_1 + Q_2 + Q_3 + Q_4 = \frac{8}{7}x^3 + \frac{47}{49}x^2 + \frac{193}{343}x + \frac{314}{2401}
\]

Remainder, \( R = R_1 + R_2 = \frac{10026}{2401}x + \frac{14092}{2401} \)

\section*{Division of Polynomial using urdhva Tiryak :}

(Problems from Swami's text) Page 79

Problem 1: \[ \frac{x^4 - 3x^3 + 7x^2 + 5x + 7}{x - 4} \]

Step 1: \[ x^4 + x^3 \quad (Q_1) \quad Q_1(x) \]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend \(-4\) with the successive quotients.

Step 2: \( x - 4 \)

\( (x^3) \)

\[
\begin{array}{c}
\frac{-4x^3}{x^3} \\
Q_1
\end{array}
\]

The original coefficient of \( x^3 \) is \(-3\)

\[
\therefore \text{Difference} = -3x^3 + 4x^2 = x^2; x^3 + x = x^2
\]

\[
Q_2 = x^2
\]

Step 3 \( x - 4 \)

\( (x^2) \)

\[
\begin{array}{c}
-4x^2 \\
Q_1 \quad Q_2
\end{array}
\]

The original coefficient of \( x^2 \) is 7

\[
\therefore \text{Difference} = 7x^2 + 4x^2 = 11x^2, 11x^2 + x = 11x
\]

\[
Q_2 = 11x
\]
Vedic Mathematics

Step 4:
\[
\frac{x - 4}{x^3 + x^2 + 11x} = -44x \quad \text{The original coefficient of } x \text{ is 5}
\]
\[
\begin{array}{c|c|c|c}
& Q_1 & Q_2 & Q_3 \\
\hline
Q_1 & & & \\
Q_2 & & & \\
Q_3 & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{Difference} = 5x + 44x = 49x \quad ; \quad 49x + x = 49 \quad Q_4 = 49
\]

Step 5:
\[
\frac{x - 4}{x^3 + x^2 + 11x + 49} = -196 \quad \text{(Urdhva)}
\]
\[
\begin{array}{c|c|c|c|c}
& Q_1 & Q_2 & Q_3 & Q_4 \\
\hline
Q_1 & & & & \\
Q_2 & & & & \\
Q_3 & & & & \\
Q_4 & & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{But the absolute value is 7}
\]
\[
\Rightarrow \quad \text{Difference} = 7 + 196 = 203
\]
\[
\Rightarrow \quad \text{Quotient} = x^3 + x^2 + 11x + 49 \quad ; \quad \text{Remainder} = 203
\]

Problem 2:
\[
\frac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{x^2 - 2x - 9}
\]

Step 1:
\[
6x^4 + x^2 = 6x^2 \quad (Q_1) \quad Q_1 = 6x^2
\]
\[
(x^4)
\]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend – 2x – 9 with the successive quotients.

Step 2:
\[
\frac{x^2 - 2x - 9}{x^2} = -12x^3 \quad \text{(Urdhva)}
\]
\[
\begin{array}{c|c|c|c|c}
& Q_1 & & & \\
\hline
Q_1 & & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{The original coefficient of } x^3 \text{ is 13}
\]
\[
\Rightarrow \quad \text{Difference} = 13x^2 + 12x^3 = 25x^3 \quad , \quad 25x^3 + x^2 = 25x \quad (Q_2) \quad Q_2 = 25x
\]

Step 3:
\[
\frac{x^2 - 2x - 9}{6x^2 + 25x} = -50x^2 - 54x^2 = -104x^2 \quad \text{(Tiryak)}
\]
\[
\begin{array}{c|c|c|c|c}
& Q_1 & Q_2 & & \\
\hline
Q_1 & & & & \\
Q_2 & & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{The original coefficient of } x \text{ is 39}
\]
\[
\Rightarrow \quad \text{Difference} = 39x^2 + 104x^2 = 143x^2 \quad , \quad 143x^2 + x^2 = 143 \quad (Q_3) \quad Q_3 = 143
\]

Step 4:
\[
\frac{x^2 - 2x - 9}{6x^2 + 25x + 143} = -286x - 225x = -511x \quad \text{(Tiryak)}
\]
\[
\begin{array}{c|c|c|c|c}
& Q_1 & Q_2 & Q_3 & \\
\hline
Q_1 & & & & \\
Q_2 & & & & \\
Q_3 & & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{The original coefficient of } x \text{ is 37}
\]
\[
\Rightarrow \quad \text{Difference} = 37x + 511x = 548x \quad (R_1) \quad R_1 = 548x
\]

Step 5:
\[
\frac{x^2 - 2x - 9}{6x^2 + 25x + 143} = -1287 \quad \text{(Urdhva)}
\]
\[
\begin{array}{c|c|c|c|c}
& Q_1 & Q_2 & Q_3 & \\
\hline
Q_1 & & & & \\
Q_2 & & & & \\
Q_3 & & & & \\
\end{array}
\]
\[
\Rightarrow \quad \text{The original absolute value is 45}
\]
\[
\Rightarrow \quad \text{Difference} = 45 + 1287 = 1332 \quad R_2 = 1332
\]
\[
\Rightarrow \quad \text{Quotient} = 6x^2 + 25x + 143 \quad , \quad \text{Remainder} = 548x + 1332
\]
Vedic Mathematics

Problem 3 \[ \frac{x^4 - 4x^2 + 12x - 9}{x^2 - 2x + 3} \]

Step 1: \[ \frac{x^4}{x^2} = x^2 \quad (Q_1) \quad Q_1 = x^2 \]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend \(-2x + 3\) with the successive quotients

Step 2: \[ \frac{x^2 - 2x + 3}{x^2} \quad = -2x^3 \quad \text{The original coefficient of } x^3 \text{ is 0} \]
\[ \therefore \text{ Difference } = 0x^3 + 2x^3 = 2x^3 \]
\[ 2x^3 = 2x \quad (Q_2) \quad Q_2 = 2x \]

Step 3: \[ \frac{x^2 - 2x + 3}{x^2} \quad = -4x^2 + 3x^2 = -x^2 \quad \text{(Tiryak)} \]
\[ \therefore \text{ Difference } = -4x^2 + x^2 = -3x^2 \]
\[ -3x^2 = -3 \quad (Q_1) \quad Q_1 = -3 \]

Step 4: \[ \frac{x^2 - 2x + 3}{x} \quad = 6x + 6x = 12x \quad \text{(Tiryak)} \]
\[ \therefore \text{ Difference } = 0 \quad (R_1) \]

Step 5: \[ \frac{x^2 - 2x + 3}{x^2 + 2x - 3} \quad = -9 \quad \text{(Urdhva)} \]
\[ \therefore \text{ Difference } = 0 \quad (R_2) \]

Quotient = \[ x^1 + 2x - 3 \]; Remainder = 0

Problem 4 \[ \frac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{3x^3 + 2x + 9} \]

Step 1: \[ \frac{6x^4}{3x^3} = 2x^2 \quad (Q_1) \quad Q_1 = 2x^2 \]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend \(2x + 9\) with the successive quotients

Step 2: \[ \frac{3x^2 + 2x + 9}{2x^2} \quad = 4x^3 \quad \text{(Urdhva)} \]
\[ \therefore \text{ Difference } = 13x^3 - 4x^3 = 9x^3 \]
\[ 9x^3 = 3x \quad (Q_2) \quad Q_2 = 3x \]
Vedic Mathematics

Step 3:

$$3x^2 + \frac{2x}{2x^2} + \frac{9}{3x} = 6x^2 + 18x^2 = 24x^2$$

The original coefficient of $$x^2$$ is 39

$$\therefore$$ Difference $$= 39x^2 - 24x^2 = 15x^2$$

$$\frac{15x^2}{3x^2} = 5\ (Q_3)$$

$$Q_3 = 5$$

Step 4:

$$3x^2 + \frac{2x}{x} + \frac{9}{2x^2 + 3x + 5} = 10x + 27x = 37x$$

The original coefficient of $$x$$ is 37

$$\therefore$$ Difference $$= 0 \ (R_1)$$

Step 5:

$$3x^2 + \frac{2x}{(Absolute)} + \frac{9}{2x^2 + 3x + 5} = 45$$

The original absolute value is 45

$$Q_1 \ Q_2 \ Q_3$$

$$\therefore$$ Difference $$= 0 \ (R_2)$$

Quotient $$= 2x^2 + 3x + 5$$, Remainder $$= 0$$

Problem 5:

$$\frac{16x^4 + 36x^2 + 81}{4x^2 + 6x + 9}$$

Step 1:

$$\frac{16x^4}{4x^2} = 4x^2 \ (Q_1)$$

$$Q_1 = 4x^2$$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend $$6x + 9$$ with the successive quotients

Step 2:

$$\frac{4x^2 + 6x + 9}{4x^2} = 24x^3 \ (Urdhva)$$

The original coefficient of $$x^3$$ is 0

$$\therefore$$ Difference $$= -24x^3$$

$$\frac{-24x^3}{4x^2} = -6x \ (Q_2)$$

$$Q_2 = -6x$$

Step 3:

$$\frac{4x^2 + 6x + 9}{4x^2} = -36x^2 + 36x^2 = 0x^2 \ (Tiryak)$$

The original coefficient of $$x^2$$ is 36

$$\therefore$$ Difference $$= 36x^2 - 0x^2 = 36x^2$$

$$\frac{36x^2}{4x^2} = 9 \ (Q_3)$$

$$Q_3 = 9$$

Step 4:

$$\frac{4x^2 + 6x + 9}{(x)} = 54x - 54x = 0x \ (Tiryak)$$

The original coefficient of $$x$$ is 0

$$\therefore$$ Difference $$= 0x \ (R_1)$$

Step 5:

$$\frac{4x^2 + 6x + 9}{(Absolute)} = 81$$

The original absolute value is also 81

$$\frac{4x^2 - 6x + 9}{4x^2} = 81 - 81 = 0 \ (R_2)$$

$$\therefore$$ Difference $$= 81 - 81 = 0$$

Quotient $$= 4x^2 - 6x + 9$$, Remainder $$= 0$$
Problem 6  \[ \frac{-2x^5 - 7x^4 + 2x^3 + 18x^2 - 3x - 8}{x^3 - 2x^2 + 0.x + 1} \]

Step 1: \[ \frac{-2x^5}{x^3} = -2x^2 \quad (Q_1) \quad Q_1 = -2x^2 \]

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend

2x^2 + 0x + 1 with the successive quotients

Step 2: \[ \frac{x^3 - 2x^2 + 0x + 1}{-2x^2} = 4x^4 \quad (Urdhva) \]

\[ \frac{-2x^2}{Q_1} \]

.. Difference = -7x^4 - 4x^4 = -11x^4

\[ -11x^4 = -11x \quad (Q_2) \quad Q_2 = -11x \]

Step 3: \[ \frac{x^3 - 2x^2 + 0x + 1}{-11x} = 22x^3 \quad (Tiryak) \]

\[ \frac{-2x^2}{Q_1} \quad \frac{-11x}{Q_2} \]

.. Difference = 2x^3 - 22x^3 = -20x^3

\[ -20x^3 = -20 \quad (Q_3) \quad Q_3 = -20 \]

Step 4: \[ \frac{x^3 - 2x^2 + 0x + 1}{40x^2 - 2x^2} = 38x^2 \quad (Tiryak) \]

\[ \frac{-40x^2}{Q_1} \quad \frac{-2x^2}{Q_2} \quad \frac{-11x - 20}{Q_3} \]

The original coefficient of x^2 is 18

\[ \text{Difference} = 18x^2 \quad 38x^2 \quad 20x^2 \quad (R_1) \]

Step 5: \[ \frac{x^2 + 0x + 1}{-3x + 11x - 8x} = 11x \quad (Tiryak) \]

\[ \frac{-2x^2}{Q_1} \quad \frac{11x}{Q_2} \quad \frac{20}{Q_3} \]

.. Difference = -3x + 11x - 8x \quad (R_2)

Step 6: \[ \frac{x^2 - 2x^2 + 0x + 1}{-20} = -20 \quad (Tiryak) \]

\[ \frac{-2x^2}{Q_1} \quad \frac{-11x}{Q_2} \quad \frac{-20}{Q_3} \]

The original absolute value is -8

.. Difference = -8 + 20 = 12 \quad (R_3)

Quotient = -2x^2 - 11x - 20

Remainder = -20x^2 + 8x + 12
CHAPTER - VI

POLYNOMIAL DIVISION USING STRAIGHT DIVISION METHOD:

(a) for single variable

Using the same procedure as used for numbers, the straight division process can be applied for polynomials also

Problem 1 Consider an example $5x^2 + 2x + 1 + 3x + 2$

The divisor $3x + 2$ is partitioned to form the part divisor (PD) $3x$ and dwajanka 'D' as 2. The Dhvajanka has one term 2, so the dividend is also partitioned by taking one term from the right which is designated as remainder region, following the usual rules of the partition as in the number division

$$
\begin{array}{c|ccc}
(D) & 5x^2 & 2x & + 1 \\
2 & \hline
(PD) & 3x & 0 & + 0 \\
& & & \\
& \frac{5x}{3} & -\frac{4}{9} & + \frac{17}{9} \\
Q_1 & Q_2
\end{array}
$$

The division can be carried out in two ways

(1) For zero intermediate remainder

(2) For non-zero intermediate remainder

(a) For zero intermediate remainder

Step I: The first term of the dividend is divided by $3x$ to get $Q_1$ (quotient) i.e., $\frac{5x^2}{3x} = \frac{5x}{3} (Q_1)$ and the remainder is zero. The remainder is placed between the first and the second terms, below the dividend
Step 2: The next intermediate dividend (ID) is $0 + 2x$. The Urdhva multiplication of Dhvajanka with $Q_1$ is first subtracted from this ID and the result is then divided by the PD, $3x$ to obtain the next term in the quotient ($Q_2$)

\[
i.e., \quad 2x - \left( \begin{array}{c} \frac{2}{3} \\ \frac{5x}{3} \end{array} \right) = 2x - \frac{10x}{3} = \frac{-4x}{3}
\]

\[Q_1\]

\[
\left( \frac{-4x}{3} \right) + 3x = \left( \frac{-4x}{3} \right) \left( \frac{1}{3x} \right) = \frac{-4}{9} (Q_2)
\]

The remainder $R_2$ is zero.

Step 3: Now one enters the remainder region. This has the new ID $01$ from which the Urdhva multiplication of Dhvajanka with $Q_2$ is subtracted to obtain the final remainder

\[
01 - \left( \begin{array}{c} D \\ 2 \\ \frac{-4}{9} \end{array} \right) = 1 - \left( \frac{-8}{9} \right) = \frac{17}{9}
\]

**Quotient** $= \frac{5x}{3} - \frac{4}{9}$, **Remainder** $= \frac{17}{9}$

### Vedic Method 1:

<table>
<thead>
<tr>
<th>2</th>
<th>$5x^2 + 2x$</th>
<th>+ 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x$</td>
<td>0</td>
<td>0 (R₁)</td>
</tr>
<tr>
<td>(5x)</td>
<td>-4</td>
<td>17 (R₂)</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>$Q_2$</td>
<td></td>
</tr>
</tbody>
</table>

### Current Method:

\[
3x + 2 \quad 5x^2 + 2x + 1 \left( \frac{5x}{3} - \frac{4}{9} \right)
\]

\[
5x^2 + \frac{10x}{3} (\text{(-)}) (\text{(-)})
\]

\[
\frac{-4x}{3} + 1
\]

\[
\frac{-4x}{3} - \frac{8}{9}
\]

\[
\frac{17}{9}
\]
It can be proved that quotient multiplied by the total divisor when added to the final remainder gives the dividend.

Quotient is \( \frac{5x - 4}{3} - \frac{9}{9} \)

Divisor is \( \frac{3x + 2}{5x^2 + 2x - 8} \)

Remainder is \( + \frac{17}{9} \)

\[
\frac{5x^2 + 2x + 1}{\text{Dividend}}
\]

It is shown that the dividend = (quotient) (divisor) + remainder. = \( q \times d + r = \text{Div} \)

Vedic Method I is valid for any value of \( x \)

There is another method in which one need not aim at zero remainder as the intermediate stage (Thus fractions can be avoided )

(b) For non-zero intermediate remainder.

In this method also the partition rules are followed in the same manner

Step1: The first term of the dividend is divided by 3x i.e., \( (5x^2) + (3x) \) to obtain the first term of the quotient as \( x (Q_1) \) and the remainder \( 2x^2 (R_1) \).

Step2: Now the intermediate dividend is \( ID \ 2x^2 + 2x \)

The Urdhva multiplication of \( D \) and \( Q_1 \) is subtracted from \( ID \) and the result is divided by \( PD \). i.e.,

\[
2x^2 + 2x - \begin{bmatrix}
2 \\
\hline
x
\end{bmatrix} = 2x^2 + 2x - 2x = 2x^2
\]

Q_1

The second degree term is reduced to first degree term as follows

The result \( 2x^2 \) is written as (2x) (x) = (2x) (10) = 20x (as \( x = 10 \))

\[
20x + 3x = 6, \quad 2x
\]

Q_2 \ R_2
Step 3: One enters into the remainder region by taking the new ID as \(2x + 1\). From this the Urdhva multiplication of \(D\) and \(Q_2\) is subtracted to obtain the remainder.

\[D\]

\[2x + 1 - 2x + 1 - 12 = 2x - 11\]

Quotient = \(x + 6\)
Remainder = \(2x - 11\)
This method is valid only for \(x = 10\)

Verifying the division using
Dividend = (Divisor) (Quotient) + Remainder

\[(\text{Divisor}) = 3x + 2 \quad \text{(Quotient)} = x + 6\]

\[3x^2 + 20x + 12\]
Remainder = \(2x - 11\)

\[x^2 + 22x + 1 \quad (20x = 2x^2 \text{ for } x = 10)\]

\[5x^2 + 2x + 1\]

\[\frac{5x^2 + 2x + 1}{3x + 2} = \frac{2x^2 + 22x + 1}{3x + 2} \quad (\text{When } x = 10)\]

In this method, the quotient when multiplied with divisor along with the addition of remainder does not directly give the dividend. But in case of \(x\) value being considered as 10, it can be shown that the value so obtained can be equated to the dividend with following reading of the terms.

The result of multiplication after adding the remainder = \(3x^2 + 22x + 1\)

When we consider the second digit to be carried over to the next term, the result is \(5x^2 + 2x + 1\). This is justified because \(22x\) can be treated as \(2x + 20x\), where \(20x\) can be considered as \(2x^2\) (\(x = 10\)). Further we can say that the dividend \(5x^2 + 2x + 1 = 521\) (\(x = 10\)). and \(3x^2 + 22x + 1 = 521\)

The result obtained in non-zero intermediate remainder method is

\[3x^2 + 22x + 1 = 300 + 220 + 1 \text{ (} x = 10 \text{)}\]

\[= (300 + 200) + 20 + 1\]

\[= 500 + 20 + 1\]

\[= 5x^2 + 2x + 1\]
This method is also valid for any value of $x$, provided we re-write the expressions suitably to give the original dividend.

For example one can work-out for $x = 1, 2, 3 \ldots$ etc

(C) \[ x = 1 \quad \text{given} \quad 5x^2 + 2x + 1 + 3x + 2 \]

\[
\begin{array}{c|cc|c}
2 & 5x^2 & + & 2x & + & 1 \\
3x & 2x^2 & & & & \\
\hline
x & + & 0 & 2x + 1
\end{array}
\]

(1) \[ \frac{5x^2}{3x} = x \quad (Q_1), \quad 2x^2 \quad (R_1) \]

(2) \[ 2x^2 + 2x - \left( \frac{2}{x} \right) = 2x^2 + 2x - 2x = 2x^2 = (2x)(x) = 2x \quad (x=1) \]

\[ \frac{2x}{3x} \quad 0 \quad (Q_2), \quad 2x \quad (R_2) \]

(3) \[ 2x + 1 - \left( \frac{2}{0} \right) = 2x + 1 \quad \text{Remainder} \]

Quotient $= x$, Remainder $= 2x + 1$

Quotient $\times$ Divisor $= x(3x+2) = 3x^2 + 2x$

Remainder $= 2x + 1$

$Q \times \text{divisor} + R = 3x^2 + 4x + 1$

On comparison with the dividend $4x$ can be written as $2x + 2x$, and $2x$ can be written as $2x^2$ as $x = 1$.

$3x^2 + 4x + 1 = 3x^2 + 2x + 2x + 1$

$= (3x^2 + 2x^2) + 2x + 1$ (since $x = 1$; $2x^2 = 2x$)

$= 5x^2 + 2x + 1$

Also: \[ 5x^2 + 2x + 1 = 5 + 2 + 1 = 8 \] and \[ 3x^2 + 4x + 1 = 3 + 4 + 1 = 8 \]

Divisor $= 3x + 2 = 5$

$8 / 5 = 1 \quad Q, \quad 3 \quad R$

$Q = x = 1$

$R = 2x + 1 = 3$
Vedic Mathematics

(d) Base $x = 2$ :

$$
\begin{array}{c|c|c}
   +2 & 5x^2 + 2x & +1 \\
  3x \downarrow & 2x^2 & \downarrow x \\
   & x + 1 & \downarrow x - 1
\end{array}
$$

1) $5x^2 + 3x = x \ (Q_1), \ 2x^2 \ (R_1)$

2) $2x^2 + 2x = \begin{pmatrix} 2 \\ x \end{pmatrix} = 2x^2 + 2x - 2x - 2x^2 - (2x)(x)\cdot x = (2x)(2)(x-2) \div 4x \ (\ddot{x} = 2)$

$4x + 3x = 1 \ (Q_2), \ x \ (R_2)$

3) $x + 1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = x + 1 - 2 = x - 1$ Remainder

Quotient = $x + 1$, Remainder = $x - 1$

(Quotient) (Divisor) = $x + 1$

$$
\begin{align*}
\frac{3x + 2}{3x^2 + 5x + 2} \\
\text{Remainder} = \frac{x - 1}{3x^2 + 6x + 1}
\end{align*}
$$

On comparison with the dividend $6x$ can be written as $2x + 4x$ and $4x$ as $2x^2 (x = 2)$

$3x^2 + 6x + 1 = 3x^2 + (4x + 2x) + 1$

$= (3x^2 + 2x^2) + 2x + 1$

$= 5x^2 + 2x + 1$

Also $5x^2 + 2x + 1 = 5*4 + 2*2 + 1 = 25$

$3x + 2 = 3*2 + 2 = 8$

$25 \div 8 = 3 \ Q, \ 1 \ R$

Quotient = $x + 1 = 2 + 1 = 3 \ (Q)$

Remainder = $x - 1 = 2 - 1 = 1 \ (R)$
Vedic Mathematics

(e) Base x = 3

When x = 3 the division has to be carried out in the same way and one has to take care that in the reduction x = 3 has to be considered as followed:

\[
\begin{array}{c|ccc}
3x & 2 & 5x^2 + 2x & + 1 \\
\hline
& 2x^2 & & \\
\hline
& x & + 1 & 3x - 1
\end{array}
\]

(1) \[ 5x^2 + 3x = (x) \quad , \quad 2x^2 (R_1) \]

\[ (Q_1) \quad (R_1) \]

(2) \[ 2x^2 + 2x = 2x(x + 1) = 2x(3 + 1) = 8x - \begin{pmatrix} 2 \\ x \end{pmatrix} = 8x - 2x = 6x \]

\[ 6x + 3x = 1 \quad , \quad 3x \]

\[ (Q_2) \quad (R_2) \]

(3) \[ 3x + 1 - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3x + 1 - 2 = 3x - 1 \quad \text{Remainder.} \]

Verifying the Division

(Quotient) (quotient) = \[ 3x + 2 \]

\[ x + 1 \]

\[ \frac{3x^2 + 5x + 2}{3x - 1} \]

\[ 3x^2 + 8x + 1 \]

\[ \text{i.e., } \frac{5x^2 + 2x + 1}{3x + 2} = \frac{3x^2 + 8x + 1}{3x + 2} \quad (\text{for } x = 3) \]

On comparison of \[ 3x^2 + 8x + 1 \] with the dividend \[ 5x^2 + 2x + 1 \], it can be seen that 8x can be written as \[ 2x + 6x \] and again 6x can be written as \[ 2x^2 \] (For \( x = 3 \))

\[ 3x^2 + 8x + 1 = 3x^2 + (6x + 2x) + 1 \]

\[ = (3x^2 + 2x^2) + 2x + 1 \]

\[ = 5x^2 + 2x + 1 \]
Vedic Mathematics

Also \(5x^2 + 2x + 1 = (5)(3^2) + (2)(3) + 1\)
\[= 45 + 6 + 1 = 52\]
\[3x + 2 = (3)(3) + 2 = 9 + 2 = 11\]

\[
\begin{array}{c}
11 ) 52 ( 4 \text{ (Q)} \\
\underline{44} \text{ (R)}
\end{array}
\]

Quotient \(= x + 1 = 3 + 1 = 4\)
Remainder \(= 3x - 1 = (3)(3) - 1 = 9 - 1 = 8\)

**Problem 2:** Consider \(3x^2 + 7x + 1 + 2x + 5\)

**CURRENT METHOD**

\[2x + 5 \quad 3x^2 + 7x + 1 \quad (3x - 1) \]
\[
\begin{array}{c}
3x^2 + 15x \\
\underline{(-) (2)}
\end{array}
\]
\[
\begin{array}{c}
x + 1 \\
\underline{2}
\end{array}
\]
\[
\begin{array}{c}
x - 5 \\
\underline{4}
\end{array}
\]
\[
\begin{array}{c}
+ + \\
\underline{9}
\end{array}
\]

\[Q = \frac{3x}{2} - \frac{1}{4} \quad R = \frac{9}{4}\]

This is valid for all values of \(x\), \(x = 1, 2, 3, 10\)

**Step 1:**

\[2x) 3x^2 (\frac{3x}{2}) \quad \text{(Q1)}\]

\[
\begin{array}{c}
3x^2 \\
\underline{0}
\end{array}
\]

\[R_1\]

**Step 2:**

\[0 + 7x - \left(\frac{3x}{2}\right) \quad \text{(Q2)}\]

\[
\begin{array}{c}
7x - \frac{15x}{2} \\
\underline{\left(\frac{3x}{2}\right)}
\end{array}
\]

\[Q = Q_1 + Q_2 = \left(\frac{3x}{2} - \frac{1}{4}\right) \text{ and } R = \frac{9}{4}\]

**VEDIC METHOD**

(a) **DIVISION WITH ZERO REMAINDER**

\[D_1\]

\[
\begin{array}{c}
5 \\
\underline{3x^2 + 7x + 1}
\end{array}
\]

\[
\begin{array}{c}
2x \\
\underline{0}
\end{array}
\]

\[Q_1 \quad Q_2 \]

\[
\begin{array}{c}
3x \\
\underline{-1}
\end{array}
\]

\[\frac{9}{4}\]

Remainder

**Step 3:**

\[
D_1\]

\[
\begin{array}{c}
5 \\
\underline{3x^2 + 7x + 1}
\end{array}
\]

\[
\begin{array}{c}
0 + 1 - \left(\frac{5}{-4}\right) \\
\underline{Q_2}
\end{array}
\]

\[
\begin{array}{c}
1 - \left(-\frac{5}{4}\right) = 1 + \frac{5}{4}
\end{array}
\]

\[Q = Q_1 + Q_2 = \left(\frac{3x}{2} - \frac{1}{4}\right) \quad \text{and } R = \frac{9}{4}\]
(b) Base \( x = 10 \) (non-zero remainder)

\[
\begin{array}{c|c|c|c}
D_1 & 3x^2 + 7x & + & 1 \\
5 & x^2 & & \\
2x & R_1 & 0 & \\
\hline
x & + & 6 & -2x - 9 \\
Q_1 & Q_2 & & \\
\end{array}
\]

Step 1:

\[2x \left( 3x^2 \right) \Rightarrow 2x^3 \quad Q_1\]

\[\frac{(-)}{x^2} \quad R_1\]

\[\begin{align*}
(x^2 + 7x) - & \quad \begin{pmatrix} 5 \\ \uparrow \\ x \end{pmatrix} \\
& = x^2 + 7x - 5x = x^2 + 2x = x(x + 2) \\
& = x(10 + 2) = 12x \quad [x = 10 \text{ (Base)}]
\end{align*}\]

Step 2:

\[2x \left( 12x \right) \Rightarrow 12x^2 \quad (Q_2)\]

\[\frac{12x}{0} \quad (R_2)\]

\[\begin{align*}
D_1 & \quad \begin{pmatrix} 5 \\ \uparrow \end{pmatrix} \\
0 + 1 & \quad \begin{pmatrix} \downarrow \ \\ 6 \end{pmatrix} \\
& = 1 - 30 = -29 \text{ Remainder} \\
Q &= Q_1 + Q_2 = x + 6 & R &= -2x - 9
\end{align*}\]

From non-zero remainder procedure we get quotient as \( x + 6 \) and remainder \(-2x - 9\).

Quotient \((Q) \times \text{Divisor} (D) + \text{Remainder} (R) = \text{Dividend} (\text{Div})\)

Here \((x + 6) \times (x + 5) + ( -2x - 9) = 2x^2 + 15x + 21 \quad \therefore 15x = 10x + 5x\)

and \(10x = x^2\) also \(21 = 20 + 1 = 2x + 1\)

When the base \( x = 10 \), this expression can be written as \((2+1) x^2 + (5+2) x + 1\)

\[3x^2 + 7x + 1\]

Carrying out the last digit to the next immediate left with its status gives the dividend used.
Vedic Mathematics

Division

(c) Base x = 1

\[
\begin{array}{c|ccc}
D_1 & 5 & 3x^2 + 7x & + 1 \\
2x & x^2 & x & \\
\hline
& R_1 & R_2 \\
& x & + 1 & 1 \\
Q_1 & Q_2 & \\
\end{array}
\]

Step 1

\[
2x \) 3x^2 ( x \quad (Q_1)
\]

\[
\frac{2x^2}{x^2} \quad (R_1)
\]

\[
\frac{(x^2 + 7x) - \left( \begin{array}{c}
5 \\
\uparrow \\
x \\
\end{array} \right)}{x} = x^2 + 7x - 5x = x^2 + 2x = x(x + 2) = 3x
\]

\[
Q_1 \quad [x = 1 \quad \text{(Base)}]
\]

Step 2:

\[
2x \) 3x ( 1 \quad (Q_2)
\]

\[
\frac{x}{2x} \quad (R_2)
\]

\[
\frac{x + 1 - \left( \begin{array}{c}
5 \\
\uparrow \\
1 \\
\end{array} \right)}{1} = x + 1 - 5 = x - 4 \quad \text{Remainder.}
\]

Q = Q_1 + Q_2 = x + 1 and R = (x - 4)

When x = 1, the quotient is x + 1 and the remainder is x - 4

Verifying: Q x D + R = Div

\[
\begin{align*}
Q &= x + 1 \\
D &= 2x + 5 \\
Q \times D &= 2x^2 + 7x + 1 \\
R &= x - 4 \\
\text{Div} &= 2x^2 + 8x + 1 \quad \text{This can be written as } (2 + 1)x^2 + 7x + 1 = 3x^2 + 7x + 1 \quad \text{(Since } 8x = 7x + 1 \text{) and } (1x = 1x^2 \text{ when) } x = 1
\end{align*}
\]
(d) Base \( x = 2 \)

\[
\begin{array}{c|c|c|c}
D_1 & 3x^2 + 7x + 1 \\
5 & \hline \\
2x & x^2 \\
& R_1 \\
& 0 \\
& R_2 \\
x & + \\
Q_1 & + 2 \\
& -9 \\
Q_2 & R \\
\end{array}
\]

**Step 1**

\[2x \times 3x^2(x + Q_1)\]

\[
\begin{array}{c|c}
5 & \hline \\
2x^2 & x^2 \\
& (R_1)
\end{array}
\]

\[(x^2 + 7x) - \begin{bmatrix} 5 \\ x \end{bmatrix} = x^2 + 7x - 5x = x^2 + 2x = x(x + 2) = x(2 + 2) = 4x\]

\[x = 2 \text{ (Base)}\]

**Step 2:**

\[2x \times 4x(2 + Q_2)\]

\[
\begin{array}{c|c}
5 & \hline \\
0 & \end{array}
\]

\[
\begin{array}{c|c}
D_1 & \hline \\
5 & \hline \\
0 & \end{array}
\]

\[(0 + 1) - \begin{bmatrix} 5 \\ 2 \end{bmatrix} = 1 - 10 = -9 \text{ Remainder}\]

\[Q = Q_1 + Q_2 = (x + 2) \text{ and } R = -9\]

The quotient is \( x + 2 \) remainder is \(-9\)

Verifying: \( Q \times D + R = \text{Div} \)

\[
\begin{align*}
Q &= x + 2 \\
D &= 2x + 5 \\
Q \times D &= 2x^4 + 9x + 10 \\
R &= -9 \\
9x &= 7x + 2x \\
\frac{2x^2 + 9x + 1}{2x^2 + 9x + 1} &= 3x^2 + 7x + 1 \\
(\because 2x = x^2 \text{ When } x = 2)
\end{align*}
\]
(e) Base $x = 3$

$$
\begin{array}{c|c|c}
D_1 & 3x^2 + 7x & + 1 \\
 2x & R_1 & x \\
 x + 2 & Q_1 & Q_2 \\
\hline
(x-9) & R_2 & \\
\end{array}
$$

Step 1: $2x)\ 3x^2(x \quad (Q_1)$

$$
\begin{array}{c}
\underline{2x^2} \\
\underline{x^2} \quad (R_1)
\end{array}
$$

$$(x^2 + 7x) - \begin{pmatrix} \uparrow \\ x \end{pmatrix} = x^2 + 7x - 5x = x^2 + 2x = x(x+2)$$

$$(3+2), \ x=3 \quad \text{(Base)}$$

$$Q_1 = 5x$$

Step 2: $2x)\ 5x (2 \quad (Q_2)$

$$
\begin{array}{c}
\underline{4x} \\
\underline{x} \quad (R_2)
\end{array}
$$

$$(x + 1) - \begin{pmatrix} \uparrow \\ 2 \end{pmatrix} = x + 1 - 10 = (x - 9)$$

$$Q = Q_1 + Q_2 = x + 2 \quad \text{and} \quad R = x - 9$$

This needs to be worked out for each value of $x$ separately.

The quotient is $x + 2$ remainder is $x - 9$

Verifying: $Q \times D + R = \text{Div}$

$$
\begin{align*}
Q &= x + 2 \\
D &= 2x + 5 \\
Q \times D &= 2x^2 + 9x + 10 \\
R &= \frac{x - 9}{2x^2 + 10x + 1} = (\because 3x = x^2 \text{ When } x = 3) \\
&= 3x^2 + 7x + 1
\end{align*}
$$
Problem 3: \(8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6 + 7x^2 + 2x + 1\).

(A) Zero Remainder method

Let us consider the Dividend as \(8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6\)

The divisor as \(7x^2 + 2x + 1\) and the division is carried out digit by digit in this method with zero remainder.

Step 1: The Divisor is split into two parts, the Dhwajanka \((D)(2x + 1)\) part divisor \((PD)\) \(7x^2\).

Step 2: As there are two terms in \(Dhwajanka\) a line of partition is drawn after counting two terms from the last in the Dividend, indicating the remained region.

The divided is \(8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6\)

Step 3: To write down the data as follow

<table>
<thead>
<tr>
<th>Quotient Region</th>
<th>Remainder Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x + 1)</td>
<td>(5x + 6)</td>
</tr>
<tr>
<td>(D)</td>
<td>(\frac{8x^3}{7} + \frac{47x^2}{49} + \frac{193x}{343} + \frac{314}{2401})</td>
</tr>
<tr>
<td>(7x^2)</td>
<td>(\frac{10026x}{2401} + \frac{14092}{2401})</td>
</tr>
<tr>
<td>(Q_1)</td>
<td>(Q_2)</td>
</tr>
</tbody>
</table>

Step 4: Divide \(8x^3\) by \(7x^2\) to get zero remainder when the quotient \(Q_1\) is \(\frac{8x^3}{7}\) (\(Q_1\)).

Step 5: The in terminate dividend is \(0 + 9x^4\) (digit by digit division) \((Q_2)\)

subtract from this the Urdhva product \(\left(\frac{D_1}{Q_1}\right) = \left(\frac{2x}{8x^3}\right) = \frac{16x^4}{7}\)

i.e., \(9x^4 - \frac{16x^4}{7} = \frac{47x^4}{7}\) and dividing by \(7x^2\)

one gets \(\frac{47}{49}x^2\) as \(Q_2\)

The remainder is zero.
Step 6: The next divisor is \((Q_2)\) \(0 + 7x^3 = 7x^3\)

Subtract from \(7x^3\) the product as the \(\begin{array}{c}
D_1 & D_2 \\
2x & 1 \\
8x & 47x^2 \\
7 & 49 \\
Q_1 & Q_2 \\
\end{array}\)

\[\begin{align*}
\frac{150x^3}{49} &= \text{Tiryak product} \\
\frac{193x^3}{49} &= \frac{193x^3}{49} = Q_3
\end{align*}\]

To get \(7x^3 - \frac{150x^3}{49} = \frac{193x^3}{49} = \frac{193x^3}{49} = Q_3\)

This is to be divided by \(7x^2\) to get \(Q_3\)

to get \(Q_3 = \frac{193x}{343}\), the remainder is zero

Step 7: The next Tiryak product part dividend is \(0 + 3x^2 = 3x^2\)

From this subtract

\(\begin{array}{c}
D_1 & D_2 \\
2x & 1 \\
47x^2 & 193x \\
\frac{49}{343} & Q_3 \\
\end{array}\)

\[\begin{align*}
\frac{386x^2 + 47x^2}{343} &= \frac{715x^2}{343} \\
\frac{314x^2}{343} &= \frac{314x^2}{343}
\end{align*}\]

This is to be divided by \(7x^2\) to get \(Q_4 = \frac{314}{343} \times \frac{x^2}{7x^2} = \frac{314}{2401} = Q_4\). The remainder is zero.

\(5x + 6 = 5x + 6\)

Step 8: The working enters into the remainder region. The terms in the remainder region are \(0 + 5x + 6 = 5x + 6\). To obtain the remainder, Subtract from this the Tiryak product

\[\begin{array}{c}
D_1 & D_2 \\
2x & 1 \\
193x & 314 \\
\frac{343}{2401} & Q_3 \\
\frac{314}{2401} & Q_4 \\
\end{array}\]

i.e. \(5x + 6\)
Vedic Mathematics

Division

\[5x + 6 - \frac{1979x}{2401} \cdot \frac{314}{2401} = \frac{10026x}{2401} + \frac{14092}{2401}\]

Quotient = \[\frac{8x^3}{7} + \frac{47x^2}{49} + \frac{193x}{343} + \frac{314}{2401}\]
\[Q_1 + Q_2 + Q_3 + Q_4\]

Remainder = \[\frac{10026x}{2401} + \frac{14092}{2401}\]
\[R_1 + R_2\]

It can be proved that quotient × divisor + remainder = Dividend. This working is based on zero remainder at every stage of finding out the quotients

(A) Vedic Method - I (Working for zero remainder-any base x)

<table>
<thead>
<tr>
<th>2x + 1</th>
<th>[8x^3 + 9x^4 + 7x^3 + 3x^2 + ]</th>
<th>5x + 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x^2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[8x^3 + 47x^2 + 193x + 314]</td>
<td>[\frac{10026x}{2401} + \frac{14092}{2401}]</td>
<td></td>
</tr>
<tr>
<td>[7 + 49 + 343 + 2401]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

II But one need not aim at zero remainders during the working of the quotients. But the quotients can be converted to zero remainder at every stage of working, so that it is valid for any base

This is as follows

(B) Vedic Method – II (x = 10)

<table>
<thead>
<tr>
<th>2x + 1</th>
<th>[8x^3 + 9x^4 + 7x^3 + 3x^2 + ]</th>
<th>5x + 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x^2</td>
<td>(x^5)</td>
<td>(3x^4)</td>
</tr>
<tr>
<td>(R_1)</td>
<td>(R_2)</td>
<td>(R_3)</td>
</tr>
<tr>
<td>(x^2 + 2x^2 + 4x + 4)</td>
<td>(43x + 2)</td>
<td></td>
</tr>
<tr>
<td>(Q_1)</td>
<td>(Q_2)</td>
<td>(Q_3)</td>
</tr>
</tbody>
</table>

After the preliminary partitioning as given earlier

Step I : Dividing \[8x^3 + 7x^2 = x^{\frac{3}{2}} + x^{\frac{1}{2}}\]
\[(Q_1) (R_4)\]
Step 2: The next intermediate dividend is $R_1 + 9x^4 = x^5 + 9x^4$ with $x = 10$ we can simplify this as follow:

$$x^5 + 9x^4 - \begin{pmatrix} 2x \\ x^3 \end{pmatrix} = x^5 + 9x^4 - 2x^4 = x^5 + 7x^4 = x^4 (x + 7) = 17x^4$$

$$17x^4 - 7x^2 = \begin{pmatrix} 2x^2 \\ 3x^4 \end{pmatrix}$$

(Q2) (R2)

Step 3: The next intermediate dividend is $3x^4 + 7x^3$ On simplifying this further we get

$$3x^4 + 7x^3 - \begin{pmatrix} 2x \\ 1 \\ x^3 \\ 2x^2 \end{pmatrix} = 3x^4 + 7x^3 - (4x^3 + x^3) = 3x^4 + 2x^2 = x^2 (3x + 2) = 32x^2$$

32x^2 + 7x^2 = 4x , 4x^3

(Q3) (R3)

Step 4: The next intermediate dividend is $4x^3 + 3x^2$ This is further simplified as

$$4x^3 + 3x^2 - \begin{pmatrix} 2x \\ 2x^2 \\ 4x \end{pmatrix} = 4x^3 + 3x^2 - (8x^2 + 2x^2) = 4x^3 - 7x^2 = x^2 (4x - 7) = 33x^2$$

33x^2 - 7x^2 = 4x , 5x^2

(Q4) (R4)

Step 5: The next intermediate dividend in the remainder region is $5x^2 + 5x$. This is simplified as

$$5x^2 + 5x - \begin{pmatrix} 2x \\ 4x \end{pmatrix} = 5x^2 + 5x - 12x = 5x^2 - 7x = x (5x - 7) = 43x$$

No division is required as this is in the remainder region

Step 6: The next term in the remainder region is 6 which is simplified as

$$6 - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = 6 - 4 = 2$$
Vedic Mathematics

Quotient = \( x^3 + 2x^2 + 4x + 4 \)
Remainder = \( 43x + 2 \)

Verification: Quotient x Divisor + remainder = Dividend

Quotient = \( x^3 + 2x^2 + 4x + 4 \)
Divisor = \( 0 + 7x^2 + 2x + 1 \)
\[
\begin{array}{c}
7x^3 + 16x^4 + 33x^3 + 38x^2 + 12x + 4 \\
\underline{43x + 2} \\
7x^3 + 16x^4 + 33x^3 + 38x^2 + 55x + 6
\end{array}
\]

We can show that the quotient set so obtained will also give the same dividend as the original, when the frames is multiplied by the divisor \( 7x^3 + 2x + 1 \) and the remainder is added to the result. In order to get the final dividend, a reorientation of the terms taking \( x \) as 10 is necessary as shown below.

The quotient is \( x^3 + 2x^2 + 4x + 4 \)
Divisor is \( 7x^2 + 2x + 1 \)

Multiplication of these two results in \( 7x^3 + 16x^4 + 33x^3 + 38x^2 + 12x + 4 \)
To this the remainder \( (43x + 2) \) when added gives
\( 7x^3 + 16x^4 + 33x^3 + 38x^2 + 55x + 6 \)

When \( x = 10 \)

(1) \( 55x = (50 + 5)x \)
\[
50x = 5x^2
\]
\[
\therefore 55x = 5x^2 + 5x
\]

(2) \( 38x^2 + 5x^2 = 43x^2 \)
\[
43x^2 = (40 + 3)x^2
\]
\[
40x^2 = 4x^3
\]
\[
\therefore 43x^2 = 4x^3 + 3x^2
\]

(3) \( 4x^3 \) When added to \( 33x^3 \) results in \( 37x^3 \)

\[
37x^3 = (30 + 7)x^3
\]
\[
30x^3 = 3x^4
\]
\[
\therefore 37x^3 = 3x^4 + 7x^3
\]
(4) This $3x^4$ when added $16x^4$ to becomes $19x^4$

$$19x^4 = (10 + 9)x^4$$
$$10x^4 = x^5$$

$$19x^4 = x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6$$

(5) This $x^5$ when added to $7x^3$ becomes $8x^5$

$\therefore$ The dividend is $8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6$

To deduce the results of (A) from the results of (B).

The division using the straight division method is first carried out to obtain the quotients and the remainder. The results are given in (B). This method is simpler to workout from which the zero remainder quotients can be deduced. The procedure is as follows (successive deduction method)

Step 1: $Q_1 (B)$ is $x^3$

$$Q_1 (A) = \frac{8x^5}{7x^2} = \frac{8}{7}x^3$$

$\therefore$ $Q_1 (A) = \frac{8}{7}Q_1 (B)$

This 8 is Co-efficient of $x^5$ the first term of the dividend and 7 is Co-efficient of part divisor

Step 2: To obtain $Q_2 (A)$ from $Q_2 (B)$ one has to consider the working details of B and also the results obtained in the (A) prior to Q.

For example (a) $Q_2 (B) = 2x^2$

(b) $7x^2 \times 2x^2 + 3x^4 + (x^3 \times 2x) = 2x^4 - x^5$

(PD) $(Q_2 \ B)$ $(R_2 \ B)$ $(Q_1 \ B \times D_1)$ $(R_1 \ B)$

$$19x^4 \times x^3 = x^4 (19x)$$

$9x^4 \times x = 10$

(c) $\frac{9x^4}{7x^2}$ [This is also written as $\frac{63x^4}{49}$] $\frac{9x^2}{7}$

(In comparison with method A)

(d) Consider $(2x) \left( \frac{x^4}{Q_1 \ A} \right)$

$$\frac{16x^4}{7}$$
(e) Divide this by $7x^2$, the part divisor

$$\frac{16x^4}{7} = \frac{16x^4}{49x^2} = \frac{16}{49} x^2$$

(f) Subtracting this value from the value in (c)

$$\frac{63x^2}{49} \text{ to get } (Q_2 \ A)$$

i.e., \( \frac{63x^2}{49} - \frac{16x^2}{49} = \frac{47}{49} x^2 \)

This is the procedure for the other co-efficient in (A).

Step 3: From $Q_3 \ B = 4x$, to deduce $Q_3 \ A$

$$(P \ D) (Q_3 B) + R_3 B + \begin{pmatrix} D_1 & D_2 \\ Q_1 B & Q_2 B \end{pmatrix} - R_2 B - \begin{pmatrix} 2x^2 & 1 \\ 8 & 47 \end{pmatrix} \begin{pmatrix} 7 \\ 49 \end{pmatrix}$$

This belongs to (A)

The substitution of values

$$2x \quad 1$$

$$(7x^3)(4x) + 4x^3 + \begin{array}{c} 2x^2 \end{array} - 3x^4 = \left( \frac{94 + 56}{49} \right) x^3$$

$$28x^3 + 4x^3 + 5x^3 - 3x^4 = \left( \frac{94 + 56}{49} \right) x^3$$

Since $x^3 (37 - 3x) = 7x^3$

$$= \left( \frac{37x^3 - 3x^4}{49} \right) \frac{150}{49} x^3 = 7x^3 - \frac{150}{49} x^3$$

$$= \left( \frac{343 - 150}{49} \right) x^3 = \frac{193}{49} x^3$$

This is to be divided by $7x^2$ (PD) which gives $\frac{193}{343} x$ for $Q_3 \ A$
Vedic Mathematics

Division

Step 4: From $Q_4 \cdot B = 4$, to obtain $Q_4 \cdot A$

\[(P.D) (Q_4 \cdot B) + R_4 \cdot B + \begin{pmatrix} D_1 & D_2 \\ Q_2 B & Q_3 B \end{pmatrix} - R_3 \cdot B - \begin{pmatrix} 2x & 1 \\ \frac{47}{x^2} & \frac{193}{x^2} \end{pmatrix} = (7x^2) \cdot 4 + 5x^2 + \begin{pmatrix} 2x & 1 \\ 2x^2 & 4x \end{pmatrix} - 4x^3 - \left( \frac{386 + 329}{343} x^2 \right)\]

\[= (33x^2 + 10x^2 - 4x^3) - \left( \frac{715}{343} x^2 \right)\]

\[= x^2 (43 - 4x) - \left( \frac{715}{343} x^2 \right) (\text{Since } 4x = 40)\]

\[= 3x^2 - \frac{715}{343} x^2\]

\[= \frac{1029 - 715}{343} x^2\]

\[= \frac{314}{343} x^2 \text{ This divided by } 7x^2 \text{ gives } \frac{314}{2401} Q_4(A)\]

Step 5: To deduce the corresponding remainder of (A) from (B)

1 part of remainder  \[= 43x + \begin{pmatrix} 2x \\ 4x \end{pmatrix} = 43x + \begin{pmatrix} 193 \\ 341 \end{pmatrix} x + \frac{314}{2401} Q_4(A)\]

\[= (43x + 12x - 5x^2) - \left( \frac{628}{2401} + \frac{193}{343} \right) x\]

\[= (55x - 5x^2) - \left( \frac{628+1351}{2401} \right) x\]
\[ = 5x(11 - x) - \frac{1979}{2401} x \quad \text{(since } x = 10) \]

\[ = 5x - \frac{1979}{2401} x = \left( \frac{12005 - 1979}{2401} \right) x \]

\[ = \frac{10026}{2401} x \quad \text{R}_1(A) \]

II Part of remainder = 2 + \[
\begin{array}{c}
1 \\
4
\end{array}
\] - \[
\begin{array}{c}
1 \\
\frac{314}{2401}
\end{array}
\]
\[ = 6 - \frac{314}{2401} \]

\[ - \frac{14406 - 314}{2401} \]

\[ - \frac{14092}{2401} \quad \text{R}_2(A) \]
CURRENT METHOD

\[ 7x^2 + 2x + 1 \]
\[ \overline{8x^3 + 9x^4 + 7x^3 + 3x^2 + 5x + 6} \]
\[ \overline{8x^3 + \frac{16}{7}x^4 + \frac{8}{7}x^3} \]
\[ \overline{(-) (-) (-)} \]
\[ \overline{\frac{47}{7}x^4 + \frac{41}{7}x^3 + 3x^2} \]
\[ \overline{\frac{47}{7}x^4 + \frac{94}{49}x^3 + \frac{47}{49}x^2} \]
\[ \overline{(-) (-) (-)} \]
\[ \overline{\frac{193}{49}x^3 + \frac{100}{49}x^2 + 5x} \]
\[ \overline{\frac{193}{49}x^3 + \frac{386}{343}x^2 + \frac{193}{343}x} \]
\[ \overline{(-) (-) (-)} \]
\[ \overline{\frac{314}{343}x^2 + \frac{772}{343}x + 6} \]
\[ \overline{\frac{314}{343}x^2 + \frac{628}{2401}x + \frac{314}{2401}} \]
\[ \overline{(-) (-) (-)} \]
\[ \overline{\frac{10026}{2401}x + \frac{14092}{2401}} \]

Problem 4:

Consider \((8x^3 - 9x^4) + 7x^3 - 3x^2 + 5x + 2) + (7x^2 + 2x + 1)\)

(a) Zero Remainder method:

<table>
<thead>
<tr>
<th>(D_1)</th>
<th>(D_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x + 1</td>
<td>8x^3 - 9x^4 + 7x^3 - 3x^2</td>
</tr>
<tr>
<td>7x^2</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>445</td>
</tr>
<tr>
<td>7</td>
<td>343</td>
</tr>
</tbody>
</table>

\(Q_1\) | \(Q_2\) | \(Q_3\) | \(Q_4\) | \(R_1\) | \(R_2\)
(1) \( 8x^3 + 7x^2 = \frac{8}{7} x^2 \) (Q₁)

(2) \(-9x^4 - \left(\begin{array}{c} D_1 \\ 2x \\ \frac{8}{7} x^2 \\ Q_1 \end{array}\right) = -9x^4 - \frac{16}{7} x^4 = -\frac{63-16}{7} x^4 = -\frac{79}{7} x^4 \) (Q₂)

\[-\frac{79}{7} x^4 + 7x^2 = -\frac{79}{49} x^2 \] (Q₂)

(3) \(7x^3 - \left(\begin{array}{c} 2x \\ \frac{8}{7} x \\ \frac{79}{49} x^2 \\ Q_1 \end{array}\right) = 7x^3 - \left(\frac{102}{49} x^3\right) = \frac{445}{49} x^3 \) (Q₃)

\[\frac{445}{49} x^3 + 7x^2 = \frac{445}{343} x \] (Q₃)

(4) \(-3x^2 - \left(\begin{array}{c} D_1 \\ 2x \\ \frac{79}{49} x^2 \\ Q_2 \end{array}\right) = -3x^2 - \left(\frac{890}{343} x - \frac{79}{49} x^2\right) \]

\[= \left(-3 - \frac{890}{343} \right) x^2 = -\frac{1366}{343} x^2 \] (Q₄)

\[\frac{1366}{343} x^2 + 7x^2 = -\frac{1366}{2401} \] (Q₄)

(5) \(5x - \left(\begin{array}{c} 2x \\ \frac{445}{343} x \\ -\frac{1366}{2401} \end{array}\right) = 5x - \left(\frac{-2732}{2401} x + \frac{445}{343} x\right) \]

\[= \left(\frac{11622}{2401} \right) x \) (R₁)
Vedic Mathematics

(6) \[
2 - \begin{pmatrix}
1 \\
\frac{1366}{2401}
\end{pmatrix} = 2 + \frac{1366}{2401} = \frac{6168}{2401} (R_2)
\]

Quotient = \(\frac{8}{7} x^3 - \frac{79}{49} x^2 + \frac{445}{343} x - \frac{1366}{2401}\), Remainder = \(\frac{11622}{2401} x + \frac{6168}{2401}\)

(b) Current Method:

\[
7x^3 + 2x + 1 \mid 8x^5 - 9x^4 + 7x^3 - 3x^2 + 5x + 2
\]

\[
\begin{align*}
&\quad 8x^5 + \frac{16}{7} x^4 + \frac{8}{7} x^3 \\
&\quad (-) \quad (-) \quad (-)
\end{align*}
\]

\[
\begin{align*}
&\quad - \frac{79}{7} x^4 + \frac{41}{7} x^3 - 3x^2 \\
&\quad - \frac{79}{7} x^4 - \frac{158}{49} x^3 - \frac{79}{49} x^2 \\
&\quad (+) \quad (+) \quad (+)
\end{align*}
\]

\[
\begin{align*}
&\quad \frac{445}{49} x^3 - \frac{68}{49} x^2 + 5x \\
&\quad \frac{445}{49} x^3 + \frac{890}{343} x^2 + \frac{445}{343} x \\
&\quad (-) \quad (-) \quad (-)
\end{align*}
\]

\[
\begin{align*}
&\quad - \frac{1366}{343} x^2 + \frac{1270}{343} x + 2 \\
&\quad - \frac{1366}{343} x^2 - \frac{2732}{2401} x - \frac{1366}{2401} \\
&\quad (+) \quad (+) \quad (+)
\end{align*}
\]

\[
\frac{11622}{2401} x + \frac{6168}{2401}
\]
(C) With remainder and 10 base

\[ \begin{array}{c|c|c|c|c|c|}
2x + 1 & 8x^4 - 9x^4 + 7x^3 - 3x^2 & + 5x + 2 \\
7x^2 & x^4(R_1) & 6x^4(R_2) & 5x^4(R_3) & 2x^4(R_4) \\
1x^3 + 1x^2 + 9x + 4 & 9x - 2 \\
\end{array} \]

(1) \( 8x^2 + 7x^2 = 1.x^4, 1.x^3 \)

\[ Q_1 \quad R_1 \]

(2) \[
x^3 + \overline{9}x^4 \begin{bmatrix} 2x \\ 1.x^3 \end{bmatrix} = x^3 + \overline{9}x^4 - 2x^4 = x^3 + \overline{1}x^4 = x^3(x + \overline{1} \overline{1}) = \overline{1}x^4
\]

\[ \overline{1}x^4 + 7x^2 = \overline{1}.x^2, 6x^4 \]

\[ Q_2 \quad R_2 \]

(3) \[
6x^4 + 7x^3 - \begin{bmatrix} 2x \\ x^1 \end{bmatrix} = 6x^4 + 7x^3 - \overline{1}x^3 = 6x^4 + 8x^3 = x^3(6x + 8) = \overline{6}8x^3
\]

\[ 68x^3 + 7x^2 = 9x, 5x^3 \]

\[ Q_3 \quad R_3 \]

(4) \[
5x^3 + \overline{3}x^2 - \begin{bmatrix} 2x \\ \overline{1}x^2 \end{bmatrix} = 5x^3 + \overline{3}x^2 - 17x^2
\]

\[ = 5x^3 - 20x^2 = x^2(5x - 20) = 30x^2 \]

\[ 30x^2 + 7x^2 = 4, 2x^2 \]

\[ Q_4 \quad R_4 \]

(5) \[
2x^2 + 5x + 2 - \begin{bmatrix} 2x \\ 9x \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix}
\]

\[ = 2x^2 + 5x + 2 - 17x - 4
\]

\[ - 2x^2 - 12x - 2 - 2x(\overline{6}) - 2
\]

\[ \overline{8}x + 2 \]
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Quotient  = 1 \cdot x^3 + \frac{1}{3} x^2 + 9x + 4
= 0 \cdot x^3 + 9x^2 + 9x + 4

Remainder = 8x - 2

Problem 5:

Consider \((5x^4 + 3x^3 + 2x^2 + x + 2) + (3x^2 + x + 4)\)

(a) Working for zero Remainder.

<table>
<thead>
<tr>
<th>x + 4</th>
<th>5x^4 + 3x^3 + 2x^2 + x + 2</th>
<th>x + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x^2</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>\frac{5}{3} x^2 + \frac{4}{9} x - \frac{46}{27}</td>
<td>\frac{25}{27} x + \frac{238}{27}</td>
<td></td>
</tr>
</tbody>
</table>

1. \[5x^4 + 3x^2 = \frac{5}{3} x^2\]

2. \[3x^3 - \left( x \begin{array}{c} \uparrow \\ \frac{5}{3} x^2 \end{array} \right) = 3x^2 - \frac{5}{3} x^3 = \frac{4}{3} x^3\]

\[\frac{4}{3} x^3 + 3x^2 = \frac{4}{9} x\]

3. \[2x^2 - \left( x \begin{array}{c} + 4 \\ \frac{5}{3} x^2 + \frac{4}{9} x \end{array} \right) = 2x^2 - \frac{4}{9} x^2 - \frac{20}{3} x^2 = -\frac{46}{9} x^2\]

\[-\frac{46}{9} x^2 + 3x^2 = -\frac{46}{27}\]
(4) \[ \text{Remainder} = x + 2 - \frac{46}{9} \]
\[ = x + 2 + \frac{46}{27}x - \frac{16}{9}x + \frac{184}{27} \]
\[ = \frac{25}{27}x + \frac{238}{27} \]
\[ \text{Quotient} = \frac{5}{3}x^3 + \frac{4}{9}x - \frac{46}{27} \]

(b) \[ 3x^2 + x + 4 \] \[ 5x^4 + 3x^3 + 2x^2 + x + 2 \]
\[ \frac{5x^4 + \frac{5}{3}x^3 + \frac{20}{3}x^2}{3} \]
\[ \frac{(-)}{(-)} \]
\[ \frac{4}{3}x^3 - \frac{14}{3}x^2 + x \]
\[ \frac{4}{3}x^3 + \frac{4}{9}x^2 + \frac{16}{9}x \]
\[ \frac{(-)}{(-)} \]
\[ -\frac{46}{9}x^2 - \frac{7}{9}x + 2 \]
\[ -\frac{46}{9}x^2 - \frac{46}{27}x - \frac{184}{27} \]
\[ (+) \]
\[ \frac{25}{27}x + \frac{238}{27} \]

(C) \[ \text{Working for base } x = 10 \]

<table>
<thead>
<tr>
<th>x+4</th>
<th>5x^4 + 3x^3 + 2x^2 +</th>
<th>x + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x^2</td>
<td>2x^4</td>
<td>x^3</td>
</tr>
<tr>
<td>x^2 + 7x + 0</td>
<td>14x + 6</td>
<td>Q_1 Q_2 Q_3</td>
</tr>
</tbody>
</table>
Vedic Mathematics

1) \(5x^4 + 3x^3 = x^2, 2x^4\)
   \[\text{Q}_1, \text{R}_1\]

2) \(2x^4 + 3x^3 - x^3 = 2x^4 + 2x^3 = 2x^3 (x+1) = 22x^3 (\therefore x = 10)\)
   \[22x^3 + 3x^2 = 7x, x^3\]
   \[\text{Q}_2, \text{R}_2\]

3) \(x^3 + 2x^2 - \)
   \[\text{Verification}\]
   \[\text{Q} = x^2 + 7x\]
   \[\text{D} = 3x^2 + x + 4\]
   \[\text{Remainder} = \frac{3x^4 + 22x^3 + 11x^2 + 28x}{3x^4 + 2x^3 + 11x^2 + 11x + 2}\]
   \[= x^2 + x + 2 - 28x - 0 = x^2 - 27x + 2\]
   \[= x(x-27) + 2 = 17x + 2\]
   \[\text{Quotient} = x^2 + 7x,\]
   \[\text{Remainder} = -17x + 2\]

(4) \(x^2 + x + 2 - \)
   \[\text{Verification}\]
   \[\text{Q} = x^2 + 7x\]
   \[\text{D} = 3x^2 + x + 4\]
   \[\text{Remainder} = \frac{3x^4 + 22x^3 + 11x^2 + 28x}{3x^4 + 2x^3 + 11x^2 + 11x + 2}\]
   \[= x^2 + x + 2 - 28x - 0 = x^2 - 27x + 2\]
   \[= x(x-27) + 2 = 17x + 2\]
   \[\text{Quotient} = x^2 + 7x,\]
   \[\text{Remainder} = -17x + 2\]

\[= 5x^4 + 3x^3 + 2x^2 + x + 2\]
\[= \text{Dividend}\]
(d) Working for base \( x = 2 \)

\[
\begin{array}{ccc|ccc}
\text{x + 4} & 5x^4 + 3x^3 + 2x^2 & +x + 2 \\
\hline
3x^2 & 2x^4 & 0 & 2x^2 \\
& R_1 & R_2 & R_3 \\
\hline
1x^2 + 2x - 2 & 2x^2 - 5x + 10 \\
Q_1 & Q_2 & Q_3
\end{array}
\]

\( (1) \) \( 5x^4 + 3x^2 = 1x^2, 2x^4 \)

\[
Q_1 \ R_1
\]

\( (2) \) \( 2x^4 + 3x^3 - \left( \begin{array}{c} x \\ x^2 \end{array} \right) = 2x^4 + 2x^3 = 2x^3 (x + 1) = 6x^3 
\]

\[
6x^3 + 3x^2 = 2x, 0 \\
Q_2 \ R_2
\]

\( (3) \) \( 2x^2 - \left( \begin{array}{c} x \\ x^2 \ 2x \end{array} \right) = 2x^4 - 6x^2 = -4x^2 
\]

\[
-4x^2 + 3x^2 = -2, 2x^2 \\
Q_3 \ R_3
\]

\( (4) \) \( 2x^2 + x + 2 - \left( \begin{array}{c} x^4 \\ 2x^2 - 2 \end{array} \right) - \left( \begin{array}{c} 4 \\ -2 \end{array} \right) 
\]

\[
= 2x^2 + x + 2 - 6x + 8 \\
= 2x^2 - 5x + 10 
\]

\[ \therefore \text{Quotient} = x^2 + 2x - 2 \\
\text{Remainder} = 2x^2 - 5x + 10 \]
Verifying for base $x = 2$:

$$Q = x^2 + 2x - 2$$

$$\text{Divisor} = \frac{3x^2 + x + 4}{3x^4 + 7x^3 + 0x^2 + 6x - 8}$$

$$\phantom{3x^4+7x^3+0x^2+6x-8} \overline{2x^2 - 5x + 10}$$

$$3x^4 + 5x^3 + 2x^2 + x + 2$$

$$= 3x^4 + (4 + 3)x^3 + 2x^2 + x + 2$$

$$= 3x^4 + (2x + 3)x^3 + 2x^2 + x + 2$$

$$= 3x^4 + 2x^4 + 3x^3 + 2x^2 + x + 2$$

$$= 5x^4 + 3x^3 + 2x^2 + x + 2$$

= Dividend

\[ (0) \quad \text{Working for base } x = 3 \]

\[ \begin{array}{c|c|c|c}
   x+4 & 5x^4 + 3x^3 + 2x^2 + & x + 2 \\
   3x^2 & 2x^4 & \text{\(2x^2\)} & \text{\(5x^2\)} \\
   \hline
   x^2 + 2x & +0 & Q_1 & Q_2 \text{ \(-1\)} & Q_3 \\
   \hline
   -19x + 2 & \end{array} \]

(1) \hspace{1em} 5x^3 + 3x^2 = x^2 \cdot 2x^4 \quad Q_1 \text{ } R_1

(2) \hspace{1em} 2x^4 + 3x^3 = x^3 (2x + 3) = 9x^2 - \begin{pmatrix} x \\ \uparrow \\
   x^2 \end{pmatrix} = 9x^3 - x^3

8x^3 + 3x^2 = 2x \cdot 2x^3 \quad Q_2 \text{ } R_2
(3) \(2x^3 + 2x^2 = 2x^2 (x+1) = 8x^2 - \begin{pmatrix} x & 4 \\ x^2 & 2x \end{pmatrix} = 8x^2 - 6x^2 = 2x^2\)

\[2x^2 + 3x^2 = 0, 2x^2\]

\[Q_3, R_3\]

(4) \(2x^2 + x + 2 - \begin{pmatrix} x & 4 \\ 2x & 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = 2x^2 + x + 2 - 8x = 2x^2 - 7x + 2\)

\[= x (2x - 7) + 2 = -19x + 2\]

(5) \(5x^2 + x + 2 - \begin{pmatrix} x & +4 \\ 2x & -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = 5x^2 + x + 2 + x - 8x + 4\)

\[= 5x^2 - 6x + 6\]

\[= x (5x - 6) + 6 = 9x + 6\]

Verifying for base \(x = 3\)

Quotient \(= x^2 + 2x - 1\)

Division \(= 3x^2 + x + 4\)

\[\frac{3x^4 + 7x^3 + 3x^2 + 7x + 4}{9x + 6}\]

Remainder \(= 3x^4 + 7x^3 + 3x^2 + 16x + \frac{2}{15x} = 5x^4 + 3x^3 + 2x^2 x + 2\)

Given is \(x\) But calculated is \(+16x\)

\[5x^2 = \frac{15x}{8x^2}\]

Calculated difference \(15x = 5x^2 (x=3)\)

After the becomes \(3x^2 + 5x^2 = 8x^2\)

But given is \(2x^2\)

\[\frac{2x^2}{6x^2}\]

\[\text{Difference} 6x^2 = 2x^3 (x = 3)\]

Now the Calculated value becomes \(7x^3 + 2x^3 = 9x^3\)

But the given value is \(3x^3\)

\[\therefore \text{Difference is} = 6x^3\]

\[6x^3 = 2x^4 (x=3)\]

Now the calculated value \(3x^4 + 2x^4 = 5x^4\)

And the given is \(5x^4\)
\[ \text{Dividend} = 5x^4 + 3x^3 + 2x^2 + x + 2 \]
\[ \text{i.e., } 
3x^4 + 7x^3 + 3x^2 + 16x + 2 = 3x^4 + 7x^3 + 3x^2 + (15 + 1)x + 2 
\]
\[ = 3x^4 + 7x^3 + 3x^2 + (5x + 1)x + 2 \quad (15 = 5x \text{ when } x = 3) 
\]
\[ = 3x^4 + 7x^3 + (3x^2 + 5x) + x + 2 
\]
\[ = 3x^4 + 7x^3 + 8x^2 + x + 2 
\]
\[ = 3x^4 + 7x^3 + (6 + 2)x^2 + x + 2 
\]
\[ = 3x^4 + 7x^3 + (2x + 2)x^2 + x + 2 \quad (6 = 2x \text{ when } x = 2) 
\]
\[ = 3x^4 + (7x^3 + 2x^2) + 2x^2 + x + 2 
\]
\[ = 3x^4 + 9x^3 + 2x^2 + x + 2 
\]
\[ = 3x^4 + (6 + 3)x^3 + 2x^2 + x + 2 
\]
\[ = 3x^4 + 2x^4 + 3x^3 + 2x^2 + x + 2 \quad (6x^3 = 2x^4 \text{ when } x = 3) 
\]
\[ = 5x^4 + 3x^3 + 2x^2 + x + 2 
\]
(b) **Extension of Evaluation of Quotients and Remainders**

In the present investigation, the authors have used the straight division method as implied by swamiji where in a partition method is applied and the remainder is derived. The same method can be applied to work out the division continuously to write the result in the form of quotients and also to get the remainders at every stage of the division. The results stand the test of division at any stage of division

\[(2 + 3x + 5x^2 + 3x^3) \div (2 - x + 3x^2)\]

The details of the work by the authors are given below

\[
\begin{array}{cccccccc}
-x + 3x^2 & 2 & + & 3x & + & 5x^2 & + & 3x^3 & 0x^4 & 0x^5 & 0x^6 \\
2 & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 + 2x & 2x^2 & - & 1x^3 & - & 13x^4 & - & 7x^5 & + & 71x^6 \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\
\end{array}
\]

Terms in the Remainder region are \(5x^2 + 3x^3\)

The absolute remainder is calculated as

\[0 + 5x^2 + 3x^3 - \left( \frac{-x + 3x^2}{1 + 2x} \right) = 5x^2 + 3x^3 + 2x^2 - 3x^2 - 6x^3\]

\[R = 4x^2 - 3x^3\]

**Verification I with R**

- **Divisor** = \(2 - x + 3x^2\)
- **Quotient** = \(\frac{1 + 2x + 0}{2 + 3x + x^2 + 6x^3}\)
- **R_1** = \(\frac{4x^2 - 3x^3}{4x^2 - 3x^3}\)
- **Dividend** = \(2 + 3x + 5x^2 + 3x^3\)

**Verification II**

Continuation of the Division in the Remainder region term by term i.e., (a) \(5x^1\) (b) \(3x^2\)
a) \[5x^2 \div \left( \begin{array}{c} -x \\ 1 \\ 2x \end{array} \right) \cdot \left( \begin{array}{c} 3x^2 \\ 2x \end{array} \right) = 4x^2 + 2 = 2x^2 \] 
When the term \(4x^2\) is again divided by the P.D then the quotient is \(2x^2\)

b) \[3x^3 \div \left( \begin{array}{c} -x \\ 2x \end{array} \right) \cdot \left( \begin{array}{c} 3x^2 \\ 2x \end{array} \right) = -x^3 + 2 = -\frac{1}{2}x^3 \] 
This is the next quotient.

Division is stopped and the absolute remainder is evaluated as

\[
\text{Remainder } R_2 = 0 \cdot \left( \begin{array}{c} -x \\ 2x^2 \end{array} \right) + 3x^2 \]

\[
= -\frac{1}{2}x^4 - 6x^4 + \frac{3}{2}x^3 = -\frac{13}{2}x^4 + \frac{3}{2}x^3
\]

one can verify at this stage also

**Verification with R_2**

\[
\text{Divisor } = 2 - x + 3x^2
\]

\[
\text{Quotient} = 1 + 2x + 2x^2 - \frac{1}{2}x^3
\]

\[
Q \times \text{Divisor} = 2 + 3x + 5x^2 + 3x^3 + \frac{13}{2}x^4 - \frac{3}{2}x^3
\]

\[
R_2 = -\frac{13}{2}x^4 + \frac{3}{2}x^3
\]

Thus the dividend (original) is obtained

**Verification III:** It is further proceeded to obtain three more quotients (Q_5, Q_6 and Q_7) starting with zero dividend terms in the reminder region

\[
0x^4 \div \left( \begin{array}{c} -x \\ 2x^2 \end{array} \right) \cdot \left( \begin{array}{c} +3x^2 \\ -\frac{1}{2}x^3 \end{array} \right) = -\frac{1}{2}x^4 - 6x^4 = -\frac{13}{2}x^4
\]

\[
-\frac{13}{2}x^4 + 2 = -\frac{13}{4}x^4 \] 

\[Q_5\]
Vedic Mathematics

\[ 0x^5 - \frac{1}{x} - \frac{13}{4} x^4 - \frac{3}{2} x^3 = -\frac{7}{4} x^3 \]

\[ -\frac{7}{4} x^4 + 2 = -\frac{7}{4} x^4 \]

\[ 0x^6 - \frac{3}{4} x^5 + \frac{3}{8} x^4 - \frac{7}{8} x^6 + \frac{39}{4} x^6 = \frac{71}{8} x^6 \]

\[ \frac{71}{8} x^6 + 2 = \frac{71}{16} x^6 \]

At this stage again the absolute remainder is calculated

\[ 3x^2 \]

Remainder \( R_1 = 0 - \left[ -\frac{7}{x} \times \frac{71}{16} x^6 \right] \frac{71}{16} x^6 \]

\[ \frac{71}{16} x^7 + \frac{21}{8} x^8 + \frac{213}{16} x^9 \]

\[ R_1 \cdot \frac{113}{16} x^7 + \frac{213}{16} x^9 \]

**Verification.** At the 3rd stage

\[ Q_1 = 1 + 2x + 2x^2 - \frac{1}{2} x^3 - \frac{13}{4} x^4 - \frac{7}{8} x^5 + \frac{71}{16} x^6 \]

\[ \text{Divisor} = 2 - x + 3x^2 + 0 + 0 + 0 + 0 \]

\[ Q \times \text{Divisor} = 2 + 3x + 5x^2 + 3x^3 + 0x^4 + 0x^5 + 0x^6 + \frac{113}{16} x^7 + \frac{213}{16} x^8 \]

Remainder \( R_1 \)

\[ \frac{113}{16} x^7 + \frac{213}{16} x^8 \]

\[ 2 + 3x + 5x^2 + 3x^3 \]

One can proceed still further to get the quotients and absolute remainder as per ones own choice.
(c) **Division of Bipolynomials Straight Division Method:**

The usual straight division method developed by Swamiji by partitioning the divisor into Dhvajanka and part divisor is also extendable to Bipolynomials. The method described here is exactly on the basis of the method described by Swamiji and the details are given below with one example, where in all the different steps to obtain the final quotient and the remainder are clearly shown.

**Problem 1:**

The divisor is \(5+7x+4y\)

The dividend is \(5+2x+4x^2+5x^3+3y+7xy+8x^2y+5y^2+8xy^2+6y^3\) (both the powers of \(x\) and \(y\) are taken in the ascending order)

A partition is shown in the divisor with \(7x+4y\) as the dhvajanka and \(5\) as the part divisor. In accordance with this the dividend is partitioned at \(8xy^2\), indicating that from \(5\) through \(5y^3\) depicts the quotient region, whereas the two terms \(8xy^2+6y^3\) come under remainder region.

In brief, the following details are worked out.

1) The quotients are obtained from the quotient region by applying straight division method. While doing so, it may be necessary that the terms of the dividend (this includes also the remainder region) may get modified.

2) The quotients are also to be sorted out from the remainder region.

3) The remainders are obtained under two different categories

   a) From the quotient (modified quotient) region.

   b) From the modified quotients in the remainder region as a unit which is clearly indicated in the following working details.
Step 1:

In the first instance, the table below shows the problem and partitions drawn in the divisor and dividend.

<table>
<thead>
<tr>
<th>Quotient Region</th>
<th>Remainder Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x + 4y</td>
<td>8xy^2 + 6y^3</td>
</tr>
<tr>
<td>5 + 2x + 4x^2 + 5x^3 + 3y + 7xy + 8x^2y + 5y^2</td>
<td>-248/25xy^2 - 116/25y^3</td>
</tr>
<tr>
<td>7x^2 - 77/5x^3 - 4y + 4xy - 44/5x^2y + 4/5y^2</td>
<td>-203/25xy^2</td>
</tr>
<tr>
<td>7/5xy - 434/25x^2y</td>
<td>251/25xy^2 + 34/25y^3</td>
</tr>
<tr>
<td>11x^2 - 52/5x + 62/5xy + 454/25x^2y + 29/25y^2</td>
<td>251/125xy^2 + 34/125y^3</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

Here 5 acts as the part divisor (PD) and 7x + 4y as the Dhwajanka and is used in multiplication. In the straight division the partition in the dividend is shown by counting the same number of terms from the right end of the dividend towards left equivalent to the number of terms in the Dhwajanka.

Step 2:

The division is carried out term by term of the dividend by obtaining the corresponding quotients through the formation of new dividends with the help of Dhwajanka. These new dividends are then divided by the part divisor (PD) to obtain the final quotients.

The first term of the dividend 5 is to be divided by PD, 5. The result is 1 shown exactly below 5 in the answer line as Q₁.

The Intermediate dividend (ID) is 0+2x = 2x. ID is converted into the new dividend through working in the following way.
2x - \left( \frac{7x}{1} \right) = -5x

The new dividend \(-5x\) has to be divided by PD, 5 to get the corresponding quotient \(Q_2\)

\(-5x + 5 = -x (Q_2)\).

Step 3:

The next ID is \(0 + 4x^2 = 4x^2\)

\[4x^2 - \left( \frac{\frac{7x}{1}}{\frac{4y}{-x}} \right) = 11x^2 - 4y\]

Now \(11x^2\) is to be divided by 5 to represent the quotient \(Q_1\) under \(x^2\) term

\[11x^2 + 5 = \frac{11}{5} x^2 (Q_1)\]

\(-4y\) actually has to be added to the term 3y of the dividend thus the y-term gets modified to \(-y\) (i.e., \(-4y + 3y = -y\))

This is the modified dividend to be considered for division under the term \(y\)

The placement is shown in the table concerned with the problem

Step 4:

Let us consider the \(x^3\)-term

\[0 + 5x^3 = 5x^3\] as ID. The new dividend is \[5x^3 - \left( \frac{\frac{7x}{1}}{\frac{4y}{-x}} \right) = \frac{52}{5} x^3 + 4xy\]

Thus the term, \(x^3\) now is \(\frac{52}{5} x^3\)

This is divided by 5 to get the corresponding quotient \(Q_4 = \frac{52}{25} x^3\)

The \(4xy\) is now to be added to the term \(7xy\) to get the modified dividend.
Step 5:

The modified y-term is \(-y\) (step 3). The corresponding new dividend obtained is

\[-y - \left( \frac{7x}{11x^2} \right) = -y + \frac{364}{25} x^4 - \frac{44}{5} x^2 y\]

As \(y\) is not further simplified, it can be divided by 5 to get the corresponding quotient \((-y/5)\) as \((Q_5)\).

There is no \(x^4\)-term in the dividend so it can be taken to represent the remainder \((R_1) = \frac{364x^4}{25}\)

The result \(-\frac{44}{5} x^2 y\) is to be added to the corresponding term \(8x^2 y\) of the dividend which is finally modified to (step 7).

Step 6:

Consider the modified \(xy\)-term \(7xy + 4xy = 11xy\)

The new dividend of the modified term is

\[11xy - \left( \frac{52}{25} x^3 \right) = - \frac{62}{5} xy + \frac{208}{25} x^3 y\]

\(xy\) is term divided by 5 to get the quotient \((62/25)xy\) \((Q_6)\).

The term \(\frac{208}{25} x^3 y\) can be taken to be the remainder \(R_2\) as \(x^3 y\)-term is not in the dividend.

Step 7:

The \(x^2 y\)-term is modified as

\[8x^2 y - \frac{44}{5} x^2 y = - \frac{4}{5} x^2 y\]
The new dividend is further simplified as

\[
\frac{4}{5} x^2 y - \left( \frac{7x}{y} \times \frac{4y}{62xy} \right) = -\frac{454}{25} x^2 y + \frac{4}{5} y
\]

The simplified term of \( x^2 y \) with the addition is \((-454/25) x^2 y\). This is divided by 5 to get the corresponding quotient, \((-454/125)x^2 y\) \( (Q_7) \)

The term \((4/5)y^2\) can be simplified with the corresponding \( y^2 \)-terms of the dividend

**Step 8:** The modified \( y^2 \)-term is \(5y^2 + \frac{4}{5} y^2 = \frac{29}{5} y^2\)

The new dividend is

\[
\frac{29}{5} y^2 - \left( \frac{7x}{25} \times \frac{4y}{62xy} \right) = \frac{29}{5} y^2 + \frac{3178}{125} x^2 y - \frac{248}{25} xy^2
\]

\(\frac{29}{5} y^2 \) can be divided by 5 to get the corresponding quotient \(\frac{29}{25} y^2 \) \( (Q_8) \)

The term \((-248/25) xy^2\) is added to the corresponding \( 8xy^2 \) term of the dividend and shown in the remainder region.

The term \((3178/125) x^3 y\) is the remainder \( R_2 \) as \( x^3 y \) term is not present in the dividend

**Step 9: Remainder region**

Consider the modified \( xy^2 \)-term \( 8xy^2 - \frac{248}{25} xy^2 = -\frac{48}{25} xy^2 \)

The new dividend is

\[
\frac{48}{25} xy^2 - \left( \frac{7x}{125} \times \frac{4y}{62xy} \right) = -\frac{251}{25} xy^2 + \frac{1816}{125} y^2
\]

This result of \( xy^2 \)-term is divided by 5, the quotient \((-251/125)xy^2\) \( (Q_9) \) is worked out from the terms of the remainder region. The term \((1816/125) x^2 y^2\) is to be considered as the remainder \( R_4 \) as the dividend has no such term
Step 10:

The new dividend is concerned with \( y^3 \)-term

\[
6 y^3 - \left( \frac{7x}{25} \cdot \frac{4y}{25} - \frac{251xy^2}{125} \right) = 6y^3 - \frac{116y^3}{25} + \frac{1757x^2y^3}{125}
\]

There is an addition of \( y^3 \)-term by \( -\frac{116}{25} y^3 \), so that the simplified result is

\[
6y^3 - \frac{116}{25} y^3 = \frac{34}{25} y^3
\]

This is divided by 5 to get \( Q_{10} \) as \( \frac{24}{125} y^3 \)

In addition there is a term \( \frac{1757}{125} x^2y^2 \) which can be taken to be the remainder, \( R_3 \)

which when added to the similar term \( R_4 \), \( \frac{1816}{125} x^3y^2 \) results in \( \frac{3573}{125} x^3y^2 \)

At this stage, let us write down the results obtained as quotients and remainders

\[
\text{Quotient} = 1 - x + \frac{11}{5} x^2 - \frac{52}{25} x^3 - \frac{y}{5} + \frac{62}{25} xy - \frac{454}{125} x^2y + \frac{29}{25} y^2 - \frac{251}{125} xy^2 + \frac{34}{125} y^3
\]

\[
\text{Remainder} = \frac{364}{25} x^4 + \frac{208}{25} x y + \frac{3178}{125} y^2 + \frac{1816}{125} x^2y^2 + \frac{1757}{125} x^3y^2
\]

\( R_1, R_2, R_3 \) are those obtained from dividend terms in the quotient region where as \( R_4, R_5 \) are from remainder region.

Step 11:

To obtain the remaining remainders from the quotients in the modified remainder region, for example, the two modified quotients obtained in the remainder region are

\[
- \frac{251}{125} xy^2 + \frac{34}{125} y^3
\]
The remainders from the above two are obtained in the following manner

\[
\begin{bmatrix}
7x & 4y \\
-251x & 24y \\
125 & 125 \\
\end{bmatrix}
- \begin{bmatrix}
4y \\
34 \\
125 \\
\end{bmatrix}
\]

\[
\begin{array}{c}
1004x^3 - \frac{238}{125}x^2y - \frac{136}{125}y^4 \\
R_4 & R_7 & R_4 \\
\end{array}
\]

The remainders are from \(R_1\) to \(R_4\) put to together

Straight division method as applied to Binomials is tested by the rule (Quotient)(divisor) + Remainder is the given dividend.

**Problem 2:**

Another example is worked out as follows

\[
(2+4x+6x^2+4y+8xy+x^2y) + (2+x+x^2+2y+3xy)
\]

\[
x + x^2 + 2y + 3xy
\]

\[
2
\]

\[
\begin{array}{ccccccc}
\dfrac{0}{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{1}{2} & x & 0 & 0 & 0 & 0 & 0 \\
\dfrac{3}{2} & x^2 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{7}{4} & x^3 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{9}{4} & x^4 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{17}{4} & x^5 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{25}{4} & x^6 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{34}{4} & x^7 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

**Q1**

**Q2**

**Q3**

**Q4**

**Q5**

**Q6**

**Step 1:**

\[
2 + 2 = 1 \quad (Q_1)
\]

**Step 2:**

\[
x - \dfrac{2}{3}x = 3x
\]

\[
3x + 2 = \frac{3}{2}x \quad (Q_2)
\]

**Step 3:**

\[
6x^2 - \dfrac{x^2}{2x} = \frac{7}{2}x^2
\]

\[
-\dfrac{1}{x^2} + 2 \quad x^2 \quad (Q_3)
\]
2y + 2 = y(Q_4), \quad R_1 = - \frac{13}{4} x^3

Step 6: 8xy - \left( \begin{array}{c} x \\ 2x^2 \\ y \\ \end{array} \right) = xy - \frac{7}{4} x^4

xy + 2 = xy/2 (Q_5) \quad R_2 = (-7/4)x^4

\frac{17}{2} x^3 y + 2 = (-17/4)x^3 y (Q_6)

Step 7: Remaining Remainders

0 - \left( \begin{array}{c} x \\ \frac{7}{4} x^2 \\ y \\ \end{array} \right) - \left( \begin{array}{c} 2y \\ \frac{3}{2} y^2 \\ \frac{17}{4} x^2 y \\ \end{array} \right) = \frac{3}{2} x^{'y} - 2y^2 + \frac{17}{4} x^4 y - 4xy^2 + 7x^2 y^2 + \frac{51}{4} y^2

R_1 \quad R_4 \quad R_5 \quad R_6 \quad R_7 \quad R_8
Verification

Quotient \[ Q = 1 + \frac{2}{2}x + \frac{7}{2}x^2 + y + \frac{xy}{2} - \frac{17}{4}x^2y \]

Divisor \[ D = 2 + x + x^2 + 2y + 3xy + 0 \]

Q X D \[ = 2 + 4x + 6x^2 + 4y + 8xy + x^2y + \frac{13}{4}x^3 + \frac{7}{4}x^4 + \frac{3}{2}x^3y + 2y^2 - \frac{17}{4}x^4y \]
\[ + 4xy^2 - 7x^2y^2 - \frac{51}{4}x^4 y^2 \]

Remainder \[ = \frac{-13}{4}x^3 - \frac{7}{4}x^4 - \frac{3}{2}x^3y - 2y^2 + \frac{17}{4}x^4y - 4xy^2 + 7x^2y^2 + \frac{51}{4}x^3y \]

\[ Q \times D + R = 2 + 4x + 6x^2 + 4y + 8xy + x^2y \quad = \text{Dividend} \]

Problem 3:

Divide \( (3 + 4x + x^2 + 2x^3 + 2x^4 + 4y + 17xy + 12x^3y + 2x^3y + 10x^4y + 4y^2 + 7xy^2 + 20x^2y^2 + 9x^3y^2 + 5x^4y^2 + 4y^3 - 5xy^3 + x^2y^3 + 3x^3y^3 + y^4 - xy^4 - 3x^2y^4 - 4x^3y^4 - 2x^4y^4) \)

by \( (3 - 2x + 2x^2 + 4y + 2x^2y + y^2 + xy^2 + x^2y^2) \)
### Vedic Mathematics

#### Division

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<tr>
<th>-2x + 2x^2 + 4y + 2x^2y + y^2 + xy^2 + x'y^2</th>
<th>3 + 4x + x^2 + 2x^2 + 2x^4 + 4y + 17xy + 12x'y + 2x'y^3 + 10x'y + 4y^2 + 7xy^2 + 20x'y'</th>
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<tbody>
<tr>
<td>-4x'y^2 + 4x'y^2</td>
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\[R_1, R_2\]
1. \[ 3 + 3 = 1 \]

2. \[ 4x - \left( \begin{array}{c}
-2x \\
1
\end{array} \right) = 4x + 2x = 6x + 3 = 2x \]

3. \[ x^2 - \left( \begin{array}{c}
-2x \\
2x
\end{array} \right) = x^2 + 4x^2 - 2x^2 = 3x^2 + 3 = x^2 \]

4. \[ 2x^3 - \left( \begin{array}{ccc}
-2x & 2x^2 & 4y \\
1 & 2x & x^2
\end{array} \right) = 2x^3 + 2x^3 - 4y - 4x^3 \]
\[ = -2x^3 + 2x^3 - 4y = -4y + 0 
\]
\[ 0 \cdot x^3 + 3 = 0 \cdot x^3 \]

5. \[ 2x^4 - \left( \begin{array}{ccc}
-2x & 2x^2 & 4y & 2x^2y \\
1 & 2x & x^2 & 0
\end{array} \right) = 2x^4 - 2x^2y - 2x^4 - 8xy = 0 
\]
\[ x^4 - 2x^2y - 8xy 
\]
\[ \therefore 0 \cdot x^4 + 3 = 0 \cdot x^4 \]

6. \[ 0y - \left( \begin{array}{ccc}
-2x & 2x^2 & 4y & 2x^3y \\
1 & 2x & x^2 & 0
\end{array} \right) = -y^2 - 4x^3y - 4x^2y + 0.0y \]

7. \[ 9xy - \left( \begin{array}{ccc}
-2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 \\
1 & 2x & x^2 & 0 & 0 & 0
\end{array} \right) = 9xy - xy^2 - 2xy^2 - 2x^4y \]
\[ = 9xy - 3x^2y^2 - 2x^4y 
\]
\[ 9xy + 3 = 3xy \]

8. \[ 6x^2y - \left( \begin{array}{ccc}
-2x & 2x^3 & 4y & 2x^2y & y^2 & xy^2 \\
1 & 2x & x^2 & 0 & 0 & 0
\end{array} \right) 
\]
\[ = 6x^2y + 6x^2y - x^2y^2 - 2x^2y^2 - x^2y^2 = 12x^2y - 4x^2y^2 \]
\[ \therefore 12x^2y + 3 = 4x^3y \]

9. \[ -2x^3y - \left( \begin{array}{ccc}
-2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 \\
2x & x^4 & 0 & 0 & 0 & 3xy
\end{array} \right) 
\]
\[ = -2x^3y + 8x^3y - 2x^3y^2 - 6x^3y - x^3y^2 = 0x^3y - 3x^3y^2 \]
\[ \therefore 0 \cdot x^3y + 3 = 0 \cdot x^3y \]
10. \[ 8x^4y = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^3y & y^2 & xy^2 & x^3y^2 \\ x^2 & 0 & 0 & 0 & 3xy & 4x^3y & 0 \end{pmatrix} \]

\[ = 8x^4y - x^4y^2 - 8x^4y - 12xy^2 = 0x^4y - x^4y^2 - 12xy^2 \quad \therefore 0x^4y + 3 = 0x^4y \]

11. \[ 3y^2 = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & 0 & 3xy & 4x^2y & 0 & 0 \end{pmatrix} \]

\[ = 3y^2 - 16x^2y^2 - 6x^2y^2 \quad \therefore 3y^2 + 3 = y^2 \]

12. \[ -8xy^2 = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & 3xy & 4x^3y & 0 & 0 & y^2 \end{pmatrix} \]

\[ = -8xy^2 + 2xy^2 - 3xy^3 - 8x^4y^2 = -6xy^2 - 3xy^3 - 8x^4y^2 \]

\[ - 6xy^2 + 3 = -2xy^2 \]

13. \[ 0x^3y^2 = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^3y & y^2 & xy^2 & x^3y^2 \\ 0 & 3xy & 4x^2y & 0 & 0 & y^2 & -2xy^2 \end{pmatrix} \]

\[ = 0x^3y^2 - 4x^3y^2 - 2x^3y^2 - 3x^3y^2 - 4x^3y^2 = -6x^3y^2 - 7x^3y^2 \]

\[ \therefore 6x^3y^2 + 3 = -2x^3y^2 \]

14. \[ 0 = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 3xy & 4x^2y & 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 \end{pmatrix} \]

\[ = -4x^2y^2 - 3x^2y^2 + 4x^2y^2 - 4x^2y^2 - 4y^2 = -7x^2y^2 - 4y^2 + 0x^2y^2 \]

15. \[ -4x^4y^2 = \begin{pmatrix} -2x & 2x^2 & 4y & 2x^3y & y^2 & xy^2 & x^3y^2 \\ 4x^2y & 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 & 0 \end{pmatrix} \]

\[ = -4x^4y^2 - 4x^4y^2 + 8xy^2 - 2x^2y^3 = 0x^4y^2 - 4x^4y^2 + 8xy^2 - 2x^2y^3 \]

\[ \therefore 0x^4y^2 + 3 = 0x^4y^2 \]
16 \[ 0 y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0y^3 + 8x^2y^3 - y^4 + 4x^4y^3 \quad \therefore 0y^3 + 3 = 0y^3 \]

17 \[ 0 xy^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0xy^3 - xy^4 + 2xy^4 + 4x^4y^3 = 0xy^3 + xy^4 + 4x^4y^3 \quad \therefore 0xy^3 + 3 = 0xy^3 \]

18 \[ 0x^2y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0x^2y^3 - x^2y^4 + 2x^2y^4 + 2x^3y^4 = 0x^2y^3 + 3x^3y^4 \quad \therefore 0x^2y^3 + 3 = 0x^3y \]

19 \[ 0x^3y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0x^3y^3 + 2x^3y^4 \quad \therefore 0x^3y^3 + 3 = 0x^3y^4 \]

20 \[ 0y^4 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y^2 & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0y^4 + 2x^4y^4 \quad .0y^4 + 3 = 0y^4 \]

21 \[ 0xy^4 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy & -2x^2y^2 & 0 & 0 \end{pmatrix} = 0xy^4 - 0 \quad \therefore 0xy^4 + 3 = 0xy \]

The remaining Quotients are Zeros

Final Quotient = \(1 + 2x + x^2 + 3xy + 4x^2y + y^2 - 2xy^2 - 2x^2y^2\)

Final Remainder = 0
### Straight Division Method for three Variables

Divide \((5 + 2x + 3y + 4z + 2xy + 3xz + 6y^2 + 7z^2 + 2x^2 + 3y^2 + 4y^2 + 9y^2 + 5z^2 + 5x^2 + 4x^2)\)

by \((5 + 7x + 4y + 2z)\)

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<tr>
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<td>(\frac{10}{125})</td>
<td>(\frac{10}{125})</td>
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</tr>
</tbody>
</table>

#### \(O_{17}\)

<table>
<thead>
<tr>
<th>(O_{17})</th>
<th>(O_{18})</th>
<th>(O_{19})</th>
<th>(O_{20})</th>
<th>(O_{21})</th>
<th>(O_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{114}{125})</td>
<td>(\frac{114}{125})</td>
<td>(\frac{114}{125})</td>
<td>(\frac{114}{125})</td>
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</table>

#### \(R_{23}\)

<table>
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<tr>
<th>(R_{23})</th>
<th>(R_{24})</th>
<th>(R_{25})</th>
<th>(R_{26})</th>
<th>(R_{27})</th>
<th>(R_{28})</th>
</tr>
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<tbody>
<tr>
<td>(\frac{106}{125})</td>
<td>(\frac{106}{125})</td>
<td>(\frac{106}{125})</td>
<td>(\frac{106}{125})</td>
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</tbody>
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### Continuation

<table>
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<tr>
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<th>( a_{10} )</th>
<th>( a_{11} )</th>
<th>( a_{12} )</th>
<th>( a_{13} )</th>
<th>( a_{14} )</th>
<th>( a_{15} )</th>
<th>( a_{16} )</th>
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<th>( a_{18} )</th>
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<th>( a_{20} )</th>
<th>( a_{21} )</th>
<th>( a_{22} )</th>
<th>( a_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-14 ( \frac{3}{5} )</td>
<td>-136 ( \frac{9}{25} )</td>
<td>-63 ( \frac{3}{25} )</td>
<td>1796 ( \frac{3}{25} )</td>
<td>-202 ( \frac{3}{25} )</td>
<td>264 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
<td>126 ( \frac{3}{25} )</td>
</tr>
<tr>
<td>-14 ( \frac{3}{5} )</td>
<td>-63 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
<td>3022 ( \frac{3}{25} )</td>
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</tr>
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</tr>
</tbody>
</table>

### Continuation of Quotient Line

<table>
<thead>
<tr>
<th>( q_{17} )</th>
<th>( q_{18} )</th>
<th>( q_{19} )</th>
<th>( q_{20} )</th>
<th>( q_{21} )</th>
<th>( q_{22} )</th>
<th>( q_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-138 ( \frac{9}{125} )</td>
<td>-228 ( \frac{9}{125} )</td>
<td>-104 ( \frac{9}{125} )</td>
<td>-216 ( \frac{9}{125} )</td>
<td>-91 ( \frac{9}{125} )</td>
<td>-30061 ( \frac{9}{625} )</td>
<td>-51 ( \frac{9}{625} )</td>
</tr>
<tr>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
</tr>
<tr>
<td>+ ( \frac{456}{125} ) ( \frac{9}{125} )</td>
<td>-448 ( \frac{9}{125} )</td>
<td>-108 ( \frac{9}{125} )</td>
<td>-256 ( \frac{9}{125} )</td>
<td>-128 ( \frac{9}{125} )</td>
<td>-128 ( \frac{9}{125} )</td>
<td>-128 ( \frac{9}{125} )</td>
</tr>
<tr>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
<td>( \frac{9}{125} )</td>
</tr>
</tbody>
</table>

**Excess Remainders:**

\[
\begin{align*}
\alpha_{18} & = 104 \frac{9}{125} \\
\alpha_{19} & = 128 \frac{9}{125} \\
\alpha_{20} & = 104 \frac{9}{125} \\
\alpha_{21} & = 256 \frac{9}{125} \\
\alpha_{22} & = 128 \frac{9}{125} \\
\alpha_{23} & = 128 \frac{9}{125} \\
\end{align*}
\]
Working details of problem. 4 Page No.

(1) \( \frac{5}{5} = 1 \) Q₁

(2) \[ 2x - \left( \begin{array}{c} 7x \\ 1 \end{array} \right) = -\frac{5x}{5} = -x \) (Q₂)

(3) \[ 3y - \left( \begin{array}{ccc} 7x & 4y \\ 1 & -x \end{array} \right) = 3y + 7x^2 - 4y = y + 7x^2, \quad y + 5 = \frac{-y}{5} \] (Q₃)

(4) \[ 4z - \left( \begin{array}{ccc} 7x & 4y & 2z \\ 1 & -x & -\frac{y}{5} \end{array} \right) = 4z - 2z + \frac{7xy}{5} + 4xy = 2z + \frac{27}{5} xy,\ 2z + 5 = \frac{2z}{5} \] (Q₄)

\* Div

(5) \[ \left( \frac{27xy}{5} + 2xy \right) = \frac{37xy}{5} - \left( \begin{array}{ccc} 7x & 4y & 2z \\ -x & -\frac{y}{5} & -\frac{2z}{5} \end{array} \right) = \frac{37xy}{5} + 2xz - \frac{14x^2}{5} + \frac{4y^2}{5} \cdot \frac{37xy}{25} \] (Q₅)

\* Div \* \* Div

(6) \[ \left( 3xz + 2xz - \frac{14xz}{5} \right) = \frac{11xz}{5} - \left( \begin{array}{ccc} 7x & 4y & 2z \\ -y & 2z & \frac{37xy}{25} \end{array} \right) = \frac{11xz}{5} - \frac{259xy}{25} + \frac{2yz}{5} - \frac{8yz}{5} \]

\* Div \* \* Div

(7) \[ \left( 4yz + \frac{2yz}{5} - \frac{8yz}{5} \right) = \frac{14yz}{5} - \left( \begin{array}{ccc} 7x & 4y & 2z \\ 2z & \frac{37xy}{5} & \frac{11xz}{25} \end{array} \right) = \frac{14yz}{5} - \frac{77x^2y}{25} - \frac{4z^2}{5} - \frac{148xy^2}{25}, \]

\* \* Terms carried over to their following proper terms,

Div = Original Dividend term
\(* \) Div

(8) \( (5x^2 + 7x^2) = 12x^2 - \begin{pmatrix} 7x & 4y & 2z \\ 37xy & 11xz & 14yz \\ 25 & 25 & 25 \end{pmatrix} \) = \( 12x^2 - \frac{98xyz}{25} - \frac{74xyz}{25} - \frac{44xyz}{25} ; \frac{12x^2}{5} \) (Q8)

(R8) (R8) (R8)

\( \text{Div} * \)

(9) \( (6y^2 + 4y^2) = \frac{34y^2}{5} - \begin{pmatrix} 7x & 4y & 2z \\ 11xz & 14yz & 12x^2 \\ 25 & 25 & 25 \end{pmatrix} \) = \( \frac{34y^2}{5} - \frac{84y^2}{5} - \frac{22xz^2}{25} - \frac{56y^2z}{25} ; \frac{34y^2}{25} \) (Q9)

\( \text{Div} * \)

(10) \( (7z^2 + \frac{4z^2}{5}) = \frac{31z^2}{5} - \begin{pmatrix} 7x & 4y & 2z \\ 14yz & 12x^2 & 34y^2 \\ 25 & 25 & 25 \end{pmatrix} \) = \( \frac{31z^2}{5} - \frac{238xy^2}{25} - \frac{28yz^2}{25} - \frac{48x^2y}{5} ; \frac{31z^2}{25} \) (Q10)

\( \text{Div} * \)

(11) \( \left( 2x^2y - \frac{259x^2y}{25} - \frac{48x^2y}{5} \right) - \begin{pmatrix} 7x & 4y & 2z \\ 12x^2 & 34y^2 & 31z^2 \\ 25 & 25 & 25 \end{pmatrix} \) = \( \frac{-449x^2y}{25} - \frac{217x^2z}{25} - \frac{24x^2z}{5} - \frac{136y^3}{25} \)

\( \therefore \frac{-449x^2y}{125} \) (Q11)

\( \text{Div} * \)

(12) \( \left( 3x^2z - \frac{77x^2y}{25} - \frac{24x^2y}{5} \right) = -\frac{122x^2z}{25} - \begin{pmatrix} 7x & 4y & 2z \\ 34y^2 & 31z^2 - \frac{449x^2y}{25} \end{pmatrix} \) = \( -\frac{122x^2z}{25} + \frac{3143x^2y}{125} - \frac{68y^2z}{25} - \frac{124yz^2}{25} \)

(R8)

\( \therefore \frac{-122x^2z}{125} \) (Q12)
Vedic Mathematics

\begin{align*}
\text{Div} \quad * & \quad * \\
(13) \quad \left(4y^2x - \frac{148xy^2}{25} - \frac{238xy^2}{25}\right) = -\frac{286xy^2}{25} & \quad \begin{pmatrix}
7x & 4y & 2z \\
31x^2 & -449x^2y & -122x^2z \\
25 & 125 & 125
\end{pmatrix} \\
\quad \quad - \frac{286xy^2}{25} & + \frac{62x^2}{25} + \frac{854x^2z}{125} + \frac{1796x^3y^2}{25} & \quad \text{Q(13)}
\end{align*}

\text{(R_5)}

\begin{align*}
\text{Div} \quad * & \quad * \\
(14) \quad \left(9y^3x - \frac{56y^2z}{25} - \frac{68y^2z}{25}\right) = \frac{101y^3z}{25} & \quad \begin{pmatrix}
7x & 4y & 2z \\
449x^2y & -122x^2z & -286xy^2 \\
125 & 125 & 125
\end{pmatrix} \\
\quad \quad \frac{101y^3z}{25} & + \frac{2002x^2y^2}{125} + \frac{898x^2yz}{125} + \frac{488x^2yz}{125} & \quad \text{Q(14)}
\end{align*}

\text{R_6} \quad \text{R_7}

\begin{align*}
\text{Div} \quad * & \quad * \\
(15) \quad \left(5x^2x - \frac{22x^2z}{25} - \frac{217x^2z}{25}\right) = -\frac{114x^3z}{25} & \quad \begin{pmatrix}
7x & 4y & 2z \\
-122x^2z & -286xy^2 & -101y^2z \\
125 & 125 & 125
\end{pmatrix} \\
\quad \quad -\frac{114xz^2}{25} & - \frac{707x^2zy^2}{125} + \frac{244x^2z^2}{125} + \frac{1144xy^3}{125} & \quad \text{Q(15)}
\end{align*}

\text{(R_4)} \quad \text{(R_5)}

\begin{align*}
\text{Div} \quad * & \quad * \\
(16) \quad \left(4yzx - \frac{28yz^2}{25} - \frac{124yz^2}{25}\right) = \frac{52yz^2}{25} & \quad \begin{pmatrix}
7x & 4y & 2z \\
-286xy^2 & 101y^2z & -114xz^2 \\
125 & 125 & 125
\end{pmatrix} \\
\quad \quad \frac{52yz^2}{25} & - \frac{798x^2z^2}{125} + \frac{572xy^2z}{125} - \frac{404y^3z}{125} & \quad \text{Q(16)}
\end{align*}

\text{(R_{10})} \quad \text{(R_{11})}
\[
\begin{align*}
(17) & \quad \left( 6x^3 - \frac{84x^2}{5} \right) = \frac{-54x^3}{25} - \begin{pmatrix} 7x & 4y & 2z \\ 101y^2z & -114x^2z & -52yz^2 \\ \frac{125}{125} & \frac{125}{125} & \frac{125}{125} \end{pmatrix} \\
\quad & \quad \therefore \frac{-54x^3}{25} \quad \text{(Q17)} \\
(R_{12}) & \quad (R_{13}) \\
(18) & \quad \left( 8y^3 - \frac{136y^2}{25} \right) = \frac{64y^3}{25} - \begin{pmatrix} 7x & 4y & 2z \\ -114x^2z & -52yz^2 & 54x^3 \\ \frac{125}{125} & \frac{125}{125} & \frac{25}{25} \end{pmatrix} \\
\quad & \quad \frac{64y^3}{25} + \frac{378x^2}{25} + \frac{228x^2y^2}{125} + \frac{208y^2z^2}{125} \quad \therefore \frac{64y^3}{125} \quad \text{(Q18)} \\
(R_{13}) & \quad (R_{14}) \\
(19) & \quad \left( 3z^3 - \frac{62x^3}{25} \right) = \frac{13x^3}{25} - \begin{pmatrix} 7x & 4y & 2z \\ -52yz^2 & -54x^3 & 64y^3 \\ \frac{125}{125} & \frac{25}{25} & \frac{125}{125} \end{pmatrix} \\
\quad & \quad \frac{13x^3}{25} + \frac{448xy^2}{125} + \frac{104yz^2}{125} + \frac{216x^2y^2}{25} \quad \therefore \frac{13x^3}{125} \quad \text{(Q19)} \\
(R_{17}) & \quad (R_{16}) \quad (R_{18}) \\
(20) & \quad \left( 5x^2y^2 + \frac{1796x^2y^2}{125} + \frac{2002x^2y^3}{125} \right) = \frac{4423x^2y^3}{125} - \begin{pmatrix} 7x & 4y & 2z \\ -54x^3 & 64y^3 + 13x^3 \\ \frac{25}{25} & \frac{125}{125} & \frac{125}{125} \end{pmatrix} \\
\quad & \quad \frac{4423x^2y^3}{125} + \frac{108x^2z}{25} + \frac{91zx^3}{125} + \frac{256y^6}{125} \quad \therefore \frac{4423x^2y^3}{625} \quad \text{(Q20)} \\
(R_{19}) & \quad (R_{20}) \quad (R_{21})
\end{align*}
\]
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Division

\[
\text{Div} \quad \left( \frac{3y^2z^2}{125} + \frac{202y^2z^2}{125} \right) = \frac{381y^2z^2}{125} - \frac{7x}{125} \quad \frac{4y}{125} \quad \frac{2z}{125}
\]

\[
\frac{381y^2z^2}{125} - \frac{30961x^3y^1}{125} - \frac{128y^2z}{125} - \frac{52y^2z}{125} = \frac{381y^2z^2}{125} \quad (Q_{21})
\]

\[
\text{Div} \quad \left( \frac{4x^2z^2}{125} + \frac{244x^2z^2}{125} + \frac{798x^2z^2}{125} \right) = \frac{1542x^2z^2}{125} - \frac{7x}{125} \quad \frac{4y}{125} \quad \frac{2z}{125}
\]

\[
\frac{1542x^2z^2}{125} - \frac{2667xy^2z^2}{125} - \frac{26x^4}{125} - \frac{17692x^2y}{125} = \frac{1542x^2z^2}{125} \quad (Q_{22})
\]

\[
0 - \begin{pmatrix} 7x & 4y & 2z \\ \frac{4423x^2y^2}{625} & \frac{381y^2z^2}{625} & \frac{1542x^2z^2}{625} \end{pmatrix} - \frac{10794x^2z^2}{625} - \frac{8846xy^2z^2}{625} - \frac{1524y^2z^2}{625}
\]

\[
(R_{10}) \quad (R_{2x}) \quad (R_{2n})
\]

\[
0 - \begin{pmatrix} 4y & 2z \\ \frac{381y^2z^2}{625} & \frac{1542x^2z^2}{625} \end{pmatrix} = -\frac{762y^2z^2}{625} - \frac{6168x^2yz^2}{625}
\]

\[
(R_{31}) \quad (R_{32})
\]

\[
0 - \begin{pmatrix} 2z \\ \frac{1542x^2z^2}{625} \end{pmatrix} = -\frac{3084}{625} \quad (R_{33})
\]
(e) **Argumental Division applied to Bipolynomials:**

An attempt is made to describe the methods of division of Bipolynomials by Bipolynomial using the array display of the terms of both dividend and divisor separately. The result is explained in the array display but the choice of the terms is at our disposal.

The simple relation between the dividend, divisor and the quotient is taken as (in the form of arrays)

<table>
<thead>
<tr>
<th>Dividend</th>
<th>Divisor</th>
<th>Quotient (Designated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>x^2 x^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>x x^2 x^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>x x^2 x^3</td>
</tr>
<tr>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>y</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>y^2</td>
<td>...</td>
<td>y y^2</td>
</tr>
<tr>
<td>y^3</td>
<td>...</td>
<td>y^3</td>
</tr>
</tbody>
</table>

\[
\text{Divisor} \times \text{Quotient} = \text{Dividend}
\]

Quotient consists of terms derived from remainders also

We have adopted for the term remainder concept to any term that doesn't belong to the given dividend form.

Now the procedure is to collect the absolute term, co-efficient of x, x^2, x^3... e.t.c y, xy, x^2y, x^3y ... e.t.c., y^2, y^3, y^4, y^5, y^6... e.t.c. Similarly the other powers.

In the actual multiplication of the given divisor and the designated quotient one has to equate the absolute term with the absolute term of the dividend, and co-efficient s of the similar powers of the product terms with those of the dividend terms. Thus the designated terms a_0, a_1, ... could be evaluated.

1 represents only absolute term
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An example is worked out and the full details are given below

\[ \text{Dividend} \]
\[ 5 + 2x + 4x^2 + 5x^3 + 3y^2 + 7xy + 8x^2y + 5y^3 + 8xy^2 + 6y^4 \]

<table>
<thead>
<tr>
<th>1</th>
<th>x</th>
<th>x^2</th>
<th>x^3</th>
<th>x^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>y</td>
<td>3</td>
<td>7</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>y^2</td>
<td>5</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y^3</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \text{Divisor} \]
\[ 5 + 7x + 4y \]

<table>
<thead>
<tr>
<th>1</th>
<th>x</th>
<th>x^2</th>
<th>x^3</th>
<th>x^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \text{Quotient} \]
\[ 1 x x^2 x^3 \]

\[ \text{1} a_0 \ a_1 \ a_2 \ a_3 \]
\[ y \ b_0 \ b_1 \ b_2 \ b_3 \]
\[ y^2 \ c_0 \ c_1 \ c_2 \ c_3 \]
\[ y^3 \ d_0 \ d_1 \ d_2 \ d_3 \]

(1) \text{Constant}
\[ 5a_0 = 5 \Rightarrow a_0 = 1 \]

(2) \text{Coeff of } x (a_1):
\[ x = k \ x \ x \ k \]
\[ 5a_1 + 7a_0 = 2 \]
\[ \text{or} \quad 5a_1 + 7 = 2 \Rightarrow a_1 = -1 \]

(3) \text{Coeff of } x^2 (a_2):
\[ x^2 = x. x, k. x^2 \ \text{Vice-versa} \]
\[ 5a_2 + 7a_1 = 4 \]
\[ \Rightarrow 5a_2 \cdot 7 = 4 \quad a_2 = \frac{11}{5} \]

(4) \text{Coeff of } x^3 (a_3):
\[ x^3 = x. x^2, k. x^3 \]
\[ 7a_2 + 5a_3 = 5 \]
\[ \frac{77}{5} + 5a_3 = 5 \]
\[ 5a_3 = 5 - \frac{77}{5} = \frac{25 - 77}{5} = \frac{-52}{5} \quad a_3 = \frac{-52}{25} \]
(5) \text{Coeff of } y (b_0) \\
y = k.y, y.k \\
5b_0 + 4a_0 = 3 \\
or \quad \frac{5b_0 + 4}{4} = 3 \\

(6) \text{Coeff of } xy (b_1) \\
xy = k.xy, x.y \text{ vice-versa} \\
5b_1 + 4a_1 + 7b_0 = 7 \\
5b_1 \cdot 4 \cdot \frac{7}{5} = 7 \\
5b_1 - \frac{27}{5} = 7 \\
5b_1 = 7 + \frac{27}{5} = \frac{62}{5} \\
\therefore \quad b_1 = \frac{62}{5} \\

(7) \text{Coeff of } x^2 y (b_2) \\
x^2y = x.xy, x^2y, k.x^2y \text{ vice-versa} \\
4a_2 + 7b_1 + 0.b_0 + 5b_2 = 8 \\
44 + 434 + \frac{5b_2}{25} = 8 \\
654 + 5b_2 = 8 \\
25 \\
5b_2 = 8 - \frac{654}{25} \\
200 - 654 - 454 \quad \therefore \quad b_2 = \frac{-454}{125} \\
25 \quad 125
(8) \( \text{Coeff } x^3y(y_0) \)
\[ x^3y = x.x^2y, x^2xy, x^3y, kx^2y, \ \text{vice-versa} \]
\[ x^3y = x.x^2y, x^2xy, x^3y \ldots \]
\[ y, x^3, kx^2y \]
\[ \therefore 5b_3 + 7b_2 + 0 + 0 + 4a_1 = 0 \]
\[ 5b_3 + 7\left( -\frac{454}{125} \right) + 4\left( -\frac{52}{25} \right) = 0 \]
\[ 5b_3 = \frac{3178}{125} + \frac{208}{25} \]
\[ 5b_3 = \frac{3178 + 1040}{625} = \frac{4218}{625} \]
\[ \therefore b_3 = \frac{4218}{625} \]

(9) \( \text{Coeff } y^2(c_0) \):
\[ y^2 = k.y^2, y.y \ \text{vice-versa} \]
\[ 5c_0 + 4b_0 = 5 \Rightarrow 5c_0 - \frac{4}{5} = 5 \]
\[ 5c_0 = 5 + \frac{4}{5} \Rightarrow \quad \therefore c_0 = \frac{29}{25} \]

(10) \( \text{Coeff of } xy^2(c_1) \)
\[ xy^2 = k.xy^2, x^2y, xy \ \text{vice-versa} \]
\[ 5c_1 + 7c_0 + 4b_1 = 8 \]
\[ 5c_1 + 7\left( \frac{29}{25} \right) + 4\left( \frac{62}{25} \right) = 8 \]
\[ 5c_1 = 8 - \frac{451}{25} = \frac{200 - 451}{25} = -\frac{251}{25} \]
\[ \therefore c_1 = -\frac{251}{125} \]

(11) \( \text{Coeff of } x^3y^2(c_2) \)
\[ x^3y^2 = kx^3y^2, x^2y^2, x.xy^2, xyxy, x^2yy \ \text{vice-versa} \]
\[ 5c_2 + 7c_1 - 4b_2 = 0 \]
\[ 5c_2 - \frac{1757}{125} - \frac{1816}{125} = 0 \]
(12) \textbf{Coeff. of } x^3y^2 (c_2) \\
\begin{align*}
x^3y^2 &= x (x^3y^2), x^2(xy^2), x'(y^2), k(x^3y^2) \text{ vice-versa} \\
7c_2 + 4b_1 + 5c_3 &= 0 \\
&= \frac{25011}{625} + \frac{16872}{625} + 5c_3 = 0 \\
5c_3 &= -\frac{41883}{625} \\
\therefore \ c_3 &= -\frac{41883}{3125}
\end{align*}

(13) \textbf{Coeff. } y^3 (d_0): \\
y^3 = y,y',y''y \\
\begin{align*}
5d_0 + 4c &= 6 \\
5d_0 + \frac{116}{25} &= 6 \\
5d_0 &= 6 - \frac{116}{25} = \frac{150 - 116}{25} = \frac{34}{25} \\
\therefore \ d_0 &= \frac{34}{125}
\end{align*}

(14) \textbf{Coeff. of } xy^3 (d_1): \\
\begin{align*}
&xy^3 = k(xy'), x(y^3), xy'(y^2), xy^2(y) \text{ vice-versa} \\
5d_1 + 7d_0 + 4c_1 &= 0 \\
5d_1 + \frac{238}{125} - \frac{1004}{125} &= 0 \\
5d_1 &= +\frac{766}{125} \\
\therefore \ d_1 &= +\frac{766}{625}
\end{align*}
\[ \text{Coeff. of } x^2y^3 (d_2) \]
\[ x^2y^3 = k(x'y')^3, x'(y'), x(xy'^2), xy(x'y'), x^2y^3(y'), x^2y^3(y) \text{ vice-versa} \]

\[
\begin{align*}
5d_2 + 0 + 0 + 0 + 7(d_1) + 0 + 0 + 0 + 0 + 0 + 4c_2 &= 0 \\
5d_2 + 7(d_1 + 4c_2) &= 0 \\
5d_2 + \frac{7(766)}{625} + \frac{4(3573)}{625} &= 0 \\
5d_2 &= -\frac{19654}{625} \\
\]

\[
\begin{align*}
d_2 &= -\frac{19654}{3125} \\
\end{align*}
\]

(16) \[ \text{Coeff. of } x^3y^2 (d_3) : \]
\[ x^3y^2 = k(x'y')^2, x'(y^2), x^2(2y'), x^2y^2(y'), x^3y^2(y), x^3y^2(y) \text{ vice-versa} \]

\[
\begin{align*}
5d_3 + 0 + 0 + 0 + 0 + 0 + 7d_2 + 0 + 0 + 4c_3 + 0 + 0 &= 0 \\
5d_3 + 7d_2 + 4c_3 &= 0 \\
5d_3 + 7\left(\frac{19654}{3125}\right) + 4\left(\frac{41883}{3125}\right) &= 0 \\
5d_3 &= -\frac{305110}{15625} \\
\]

\[
\begin{align*}
d_3 &= -\frac{305110}{15625} \\
\end{align*}
\]

Since \[ \frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} \]
\[ \Rightarrow \text{Dividend} = \text{Quotient} \times \text{Divisor} \]
the given problem by using \[ D = Q \times \text{Divisor} \] is written as follows
\[
\begin{array}{c|cccc}
& 1 & x & x^2 & x^3 \\
\hline
1 & & & & \\
y & \frac{1}{5} & \frac{62}{5} & \frac{454}{125} & \frac{4218}{625} \\
y^2 & \frac{29}{25} & \frac{251}{125} & \frac{3573}{625} & \frac{41883}{3125} \\
y^3 & \frac{34}{125} & \frac{766}{625} & \frac{19654}{3125} & \frac{305110}{15625} \\
\end{array}
\]
The same problem is worked out by the authors, using the general straight division method by writing down the problem in linear form and also considering partition, part divisor and Dhawajanka. A comparison of this with the values obtained by the Argumental division method is as follows.

**Authors Straight Division**

Quotients:

- \( Q_1 = 1 \)
- \( Q_2 = -x \)
- \( Q_3 = \frac{11x^2}{5} \)
- \( Q_4 = \frac{52x^3}{25} \)
- \( Q_5 = \frac{y}{5} \)
- \( Q_6 = \frac{62xy}{25} \)
- \( Q_7 = \frac{454x^2y}{125} \)
- \( Q_8 = \frac{29y^2}{25} \)
- \( Q_9 = \frac{251xy^2}{125} \)
- \( Q_{10} = \frac{34y^3}{125} \)

\[ \begin{align*}
1) & \quad R_1 = \frac{364}{25} x^4 \\
2) & \quad R_2 + R_3 = \frac{4218}{125} x^3y \\
   & \text{(This becomes quotient (Q_{11}) when divided by P.D.5)} \\
3) & \quad R_4 + R_5 = \frac{3573}{125} x^2y^2 \\
   & \text{(But when this is divided by P.D.5 one gets the quotient (Q_{12}))} \\
4) & \quad \text{One will get this value of it is still extended to get quotient term x^3y^2} \\
5) & \quad R_6 + R_7 = \frac{766}{125} xy^3 \\
   & \text{(This when divided by 5 gives the quotient term Q_{14}.)} \\
6) & \quad \text{One has to still extend to get quotient terms x^3y^2 and x^2y^3} \\
7) & \quad R_8 = \frac{136}{125} y^4
\end{align*} \]

**Argumental Division Method**

Quotients:

- \( Q_1 = 1 \)
- \( Q_2 = -x \)
- \( Q_3 = \frac{11x^2}{5} \)
- \( Q_4 = \frac{52x^3}{25} \)
- \( Q_5 = \frac{y}{5} \)
- \( Q_6 = \frac{62xy}{25} \)
- \( Q_7 = \frac{454x^2y}{125} \)
- \( Q_8 = \frac{29y^2}{25} \)
- \( Q_9 = \frac{251xy^2}{125} \)
- \( Q_{10} = \frac{34y^3}{125} \)

\[ \begin{align*}
\text{Not extended to that power (x^4)}
   & \quad Q_{11} = \frac{4218}{625} x^3y \\
\text{Not extended to that power (y^4)}
   & \quad Q_{12} = \frac{3573}{625} x^2y^2 \\
   & \quad Q_{13} = \frac{4883}{3125} x^3y^2 \\
   & \quad Q_{14} = \frac{766}{625} xy^3 \\
   & \quad Q_{15} = \frac{19654}{3125} x^2y^3 \\
   & \quad Q_{16} = \frac{3051103}{15625} x^3y^3
\end{align*} \]
A crucial difference between two is observed as follows.

In the I Method, (i.e. Straight Division) Verification can be carried at every stage of the division by considering the general rule, divisor \times quotient + remainder = dividend.

Whereas in the II Method (i.e., Argumental Method, procedure given by British authors) describing the arrangement of dividend, divisor and the result in the form of arrays where in the remainder concept has not been included. It is found that the above verification rule is not directly applicable unless one has the idea of remainders and also one extends the calculations, to explain the excess terms in the process of verification.
(f) **Successive division of Remainders:**

A comparison of the results of the two methods (viz., Straight Division and Argumental method) leading to some interesting results of few extensions in quotients and remainders. In order to study such difference we extended the division of the remainders successively treating the set of remainders as new dividends by the divisor to obtain the new quotients and remainders using St Division method.

It is interesting to note that division by the part divisor of any remainder gives the corresponding quotient and such successive divisions are attempted to on observe the new quotients and remainders.

This process of division is unending and one can stop the working at a particular choice of the powers of polynomials. The successive division is also compute programmed for problem.

Considering the remainders as the dividend one can continue the division this results in quotients and remainders. The process is further continued with the new set of remainders to get the author set of remainders one may continue. This process to get the required quotient or remainders at the choice of the individual. Here the authors have done for three sets of remainders for the further division. (I, II, III)

The remainders, which are obtained by St division by authors

\[
\begin{align*}
R_1 &= \frac{364}{25}x^4 \\
R_2 &= \frac{208x^3y}{25} + \frac{3178x^3y}{125} = \frac{1040}{125} + \frac{3178}{125} = \frac{4218x^3y}{125} \\
R_3 &= \frac{1816x^2y^2}{125} + \frac{1757x^2y^2}{125} = \frac{3573x^2y^2}{125} \\
R_4 &= \frac{1004xy^3}{125} - \frac{238xy^3}{125} = \frac{766xy^3}{125} \\
R_5 &= \frac{136y^4}{125}
\end{align*}
\]
Vedic Mathematics

Division

<table>
<thead>
<tr>
<th>7x + 4y</th>
<th>[ \frac{364x^4}{25} + \frac{4218x^3y}{125} + \frac{3573x^2y^2}{125} ]</th>
<th>[ \frac{766xy^3}{125} - \frac{136y^4}{125} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>[ \frac{364x^4}{125} + \frac{4218x^3y}{625} + \frac{3573x^2y^2}{625} ]</td>
<td>[ \frac{766xy^3}{625} - \frac{136y^4}{625} ]</td>
</tr>
</tbody>
</table>

(1) \( \frac{364x^4}{25} + 5 = \frac{364x^4}{125} \) (Q_i)

(2) \( \frac{4218x^3y}{125} = \frac{7x}{364x^4} \) \( \frac{4218x^3y}{125} = \frac{2548x^3}{125} \)

\( \frac{4218x^3y}{125} + 5 = \frac{4218x^3y}{625} \) (Q_i) \( \) & \( \frac{2548x^3}{125} \) (R_i)

(3) \( \frac{3573x^2y^2}{125} = \frac{4y}{364x^4} \) \( \frac{3573x^2y^2}{125} = \frac{29526x^4y}{125} - \frac{1456x^4y}{125} \)

\( \frac{3573x^2y^2}{125} + 5 = \frac{3573x^2y^2}{625} \) (Q_i) \( \) & \( \frac{36806x^4y}{625} \) (R_i)

(4) \( \frac{766xy^3}{125} = \frac{4y}{4218x^3y} + \frac{3573x^2y^2}{625} \)

\( \frac{766xy^3}{125} - \frac{2501x^2y}{625} - \frac{16872x^4y^2}{625} = \frac{766xy^3}{125} - \frac{41883x^3y^2}{625} \)

\( \frac{766xy^3}{625} \) (Q_i) \( \) & \( \frac{41883x^3y^2}{625} \) (R_i)
Vedic Mathematics

\[ \begin{align*}
(5) \quad & \frac{136y^4}{125} \frac{7x}{3573x^2y^2} \frac{4y}{766xy^3} = -\frac{136y^4}{125} \frac{5362x^2y^3}{625} + \frac{14292x^2y^3}{625} \\
& \frac{136y^4}{125} \frac{19654x^2y^3}{625} \\
& \frac{-136y^4}{625} (Q_1') \text{ & } \frac{19654x^2y^3}{625} (R_1') \\
(6) \quad & 0 - \frac{766xy^3}{625} \frac{-136}{625} \frac{4y}{y^4} = \frac{136y^4}{625} \\
& \frac{952}{625} xy^4 - \frac{3064}{625} xy^4 + \frac{544}{625} y^3 \\
& \frac{2112xy^4}{625} + \frac{544}{625} y^3 \\
& R_3' \quad R_4' \\
& \therefore \quad Q' = Q_1' + Q_2' + Q_3' + Q_4' + Q_5' = \frac{364x^4}{125} + \frac{4218x^3y}{625} + \frac{3573x^2y^2}{625} + \frac{766xy^3}{625} + \frac{136y^4}{625} \\
& R' = R_1' + R_2' + R_3' + R_4' + R_5' + R_6' = \frac{2548x^5}{125} - \frac{36806x^4y}{625} - \frac{41883x^3y^2}{625} - \frac{19654x^2y^3}{625} - \frac{2112xy^4}{625} + \frac{544}{625} y^3 \\
\text{II} \quad \text{In order to obtain further quotients, } R_1, R_2, \text{ Are to the dividend with the divisor which is shown as under} \\
\begin{array}{c|cc|cc|cc}
7x + 4y & \frac{2548x^5}{125} & \frac{36806x^4y}{625} & \frac{41883x^3y^2}{625} & \frac{19654x^2y^3}{625} & \frac{-2112xy^4}{625} + \frac{544}{625} y^3 \\
\hline
5 & & & & & \\
\hline
& 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\end{align*} \]
\[ \frac{36806x^4y}{625} - 2548x^3 = \frac{36806x^4y}{625} + \frac{17836x^6}{625} \]

\[ \therefore -\frac{36806}{625}x^4y + 5 = -\frac{36806x^4y}{3125}(Q_2) \text{ and } \frac{17836x^6}{625}(R_i') \]

\[ \frac{41883}{625}x^3y^2 - \frac{7x}{625} - \frac{2548x^3}{625} - \frac{36806}{3125}x^4 \]

\[ = \frac{41883x^3y^2}{625} - \frac{257642x^3y}{3125} + \frac{10192x^3y}{3125} = -\frac{41883x^3y^2}{625} + \frac{308602x^2y}{3125} \]

\[ \frac{41883}{625}x^3y^2 + 5 = \frac{41883}{625}x^3y^2 \]

\[ \therefore -\frac{41883x^3y^2}{3125}(Q_3') \text{ and } +\frac{308602x^2y}{3125}(R_2') \]

\[ \frac{-19654x^2y^3}{625} - \frac{7x}{3125} - \frac{36806x^4y}{3125} - \frac{41883x^3y^2}{3125} \]

\[ = -\frac{19654x^2y^3}{625} + \frac{293181x^2y^3}{3125} + \frac{147224x^2y^3}{3125} = -\frac{19654x^2y^3}{625} + \frac{440405x^4y^3}{3125} \]

\[ \frac{-19654}{625}x^2y^3 + 5 = \frac{-19654}{625}x^2y^3 \]

\[ \therefore -\frac{19654x^2y^3}{3125}(Q_4') \text{ and } \frac{440405x^4y^3}{3125}(R_i') \]

\[ \frac{-2112}{625}xy^4 - \frac{7x}{3125} - \frac{41883}{3125}x^3y^2 \]

\[ = -\frac{2112}{625}xy^4 + \frac{137578}{3125} + \frac{167532}{3125} \]

\[ \frac{-2112}{625}xy^4 + 5 = -\frac{2112}{3125}xy^4(Q'_5) + \frac{305110}{3125}x^4y^4(R_i') \]
Vedic Mathematics

\[ \begin{align*}
(6) & \quad \frac{544}{625}y^5 - \frac{7x}{3125}x^2y^3 + \frac{544}{3125}xy^5 = \frac{4y}{3125}x^2y^4 + \frac{78616}{3125}x^2y^4 \\
& = \frac{544}{625}y^5 + \frac{14784}{3125}x^2y^4 + \frac{544}{3125}xy^5 \\
& \quad + \frac{544}{625}y^5 + 5 = \frac{544}{3125}y^5(Q_6') + \frac{93400}{3125}x^2y^5(R')
\end{align*} \]

(7) Remainders

\[
\begin{array}{c|cc|c}
7x & 4y & 4y \\
\hline
0 & -2112 & \frac{544}{3125}y \\
& & \frac{544}{3125}y^4 \\
\hline
= & -\frac{3808}{3125}x^2y^4 + \frac{8448}{3125}xy^5 - \frac{2176}{3125}y^6 \\
& \frac{4640}{3125}x^2y^5(R'') - \frac{2176}{3125}y^6(R''')
\end{array}
\]

\[ \begin{align*}
\therefore Q &= Q_1' + Q_2' + Q_3' + Q_4' + Q_5' + Q_6' \quad \therefore Q &= Q_1' + Q_2' + Q_3' + Q_4' + Q_5' + Q_6' + Q_7' \\
R &= R_1 + Q_2 + R_3 + R_4 + R_5 + R_6 + R_7 \\
& = \frac{17836x^6}{625} + \frac{308602x^5y}{3125} + \frac{440405x^4y^2}{3125} + \frac{305110x^3y^3}{3125} + \frac{2112}{3125}xy^4 + \frac{544}{3125}y^5
\end{align*} \]

III

\[
\begin{array}{c|cccccc|c}
7x + 4y & \frac{17836x^6}{3125} & \frac{308602x^5y}{3125} & \frac{440405x^4y^2}{3125} & \frac{305110x^3y^3}{3125} & \frac{93400}{3125}x^2y^4 & \frac{4640}{3125}xy^5 & \frac{2176}{3125}y^6 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

<table>
<thead>
<tr>
<th>(Q_1)</th>
<th>(Q_2)</th>
<th>(Q_3)</th>
<th>(Q_4)</th>
<th>(Q_5)</th>
<th>(Q_6)</th>
<th>(Q_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(17836x^6)</td>
<td>(308602x^5y)</td>
<td>(440405x^4y^2)</td>
<td>(305110x^3y^3)</td>
<td>(93400)</td>
<td>(4640)</td>
<td>(2176)</td>
</tr>
<tr>
<td>(3125)</td>
<td>(15625)</td>
<td>(15625)</td>
<td>(15625)</td>
<td>(15625)</td>
<td>(15625)</td>
<td>(15625)</td>
</tr>
</tbody>
</table>
(1) \[ \frac{17836 x^4}{625} + 5 = \frac{17836}{3125} x^4 \ (Q_{3''}) \]

(2) \[
\begin{align*}
\frac{308602}{3125} x^3 y - \frac{7x}{17836} x^4 & = \frac{308602}{3125} x^3 y - \frac{124852}{3125} x^7, \\
\therefore \frac{308602}{3125} x^3 y + 5 & = \frac{308602}{15625} x^3 y \ (Q_{4''}), \frac{124852}{3125} x^7 \ (R_{4''})
\end{align*}
\]

(3) \[
\begin{align*}
\frac{440405}{3125} x^4 y^2 - \frac{7x}{17836} x^4 & = \frac{440405}{3125} x^4 y^2 - \frac{308602}{15625} x^3 y, \\
\therefore \frac{440405}{15625} x^4 y^2 + 5 & = \frac{440405}{15625} x^4 y^2 \ (Q_{5''}), -\frac{2516934}{3125} x^3 y \ (R_{5''})
\end{align*}
\]

(4) \[
\begin{align*}
\frac{305110}{3125} x^3 y^3 - \frac{7x}{308602} x^3 y & = \frac{305110}{15625} x^3 y^3 - \frac{1234408}{15625} x^3 y^2, \\
\therefore \frac{305110}{3125} x^3 y^3 + 5 & = \frac{305110}{15625} x^3 y^3 \ (Q_{6''}), -\frac{4317243}{15625} x^3 y^2 \ (R_{6''})
\end{align*}
\]

(5) \[
\begin{align*}
\frac{93400}{3125} x^2 y^4 - \frac{7x}{440405} x^2 y^2 & = \frac{93400}{15625} x^2 y^4 - \frac{3897390}{15625} x^2 y^3, \\
\therefore \frac{93400}{3125} x^2 y^4 + 5 & = \frac{93400}{15625} x^2 y^4 \ (Q_{7''}), -\frac{3897390}{15625} x^2 y^3 \ (R_{7''})
\end{align*}
\]

(6) \[
\begin{align*}
\frac{4640}{3125} x y^5 - \frac{7x}{305110} x^2 y^3 & = \frac{4640}{15625} x y^5 - \frac{1220440}{15625} x^2 y^4, \\
\therefore \frac{4640}{3125} x y^5 + 5 & = \frac{4640}{15625} x y^5 - \frac{653800}{15625} x^2 y^4
\end{align*}
\]
Vedic Mathematics

Division

\[ \therefore \frac{4640}{3125} x y^4 + 5 = \frac{4640}{15625} x y^4 (Q''_s) \cdot \frac{1874240}{15625} x y^4 (R''_s) \]

(7) \[ \frac{-2176}{3125} y^6 - \frac{7x}{15625} x^2 y^4 + \frac{4y}{15625} x^4 y^3 \]

\[ = \frac{-2176}{3125} y^6 - \frac{32480}{15625} x^2 y^4 - \frac{373600}{15625} x^2 y^3 \]

\[ \therefore \frac{-2176}{3125} y^6 + 5 = \frac{-2176}{15625} y^6 (Q''_s) \cdot \frac{406080}{15625} x^2 y^4 (R''_s) \]

Remainders

\[ \begin{array}{c|c|c}
7x & 4y & 4y \\
\hline
0 - \frac{4640}{15625} x y^4 & -\frac{2176}{15625} y^6 & -\frac{2176}{15625} y^6 \\
\hline
- + \frac{15232}{15625} x y^4 & -\frac{18560}{15625} x y^3 + \frac{8704}{15625} y^3 \\
\hline
- - \frac{3328}{15625} x y^4 (R''_s) + \frac{8704}{15625} y^3 (R''_s) \\
\hline
\end{array} \]

\[ Q'' = Q_1'' + Q_2'' + Q_3'' + Q_4'' + Q_5'' + Q_6'' + Q_7'' \]

\[ = \frac{17836}{3125} x^4 + \frac{308602}{15625} x^4 y + \frac{440405}{15625} x^4 y^3 + \frac{305110}{15625} x^4 y^3 + \frac{93400}{15625} x^4 y^3 + \frac{4640}{15625} x y^4 - \frac{21766}{15625} y^6 \]

\[ R'' = R_1'' + R_2'' + R_3'' + R_4'' + R_5'' + R_6'' + R_7'' + R_8'' \]

\[ R' = \frac{-124852}{3125} x^2 - \frac{251934}{3125} x^2 y - \frac{4317243}{15625} x^2 y^2 \]

\[ - \frac{3897390}{15625} x^2 y^3 - \frac{1874240}{15625} x^2 y^4 - \frac{4060802}{15625} x^2 y^5 \]

\[ - \frac{-3328}{15625} x y^6 + \frac{8704}{15625} y^7 \]

\[ \]
The straight division method as explained for Bipolynomials by British authors is as follows:

Consider dividend and divisor as sets of arrays. The terms which are beyond the dividend are treated as remainders. The working details are deducing the quotients of each row, similarly the remainder terms of each row. The following are the working details attempted by the authors for an example given in this book.

Consider \((5 + 2x + 4x^2 + 5x^3 + 3y + 7xy + 8x^2y + 5y^2 + 8xy^2 + 6y^3) + (5 + 7x + 4y)\)

The dividend is given below.

\[
\begin{array}{cccc}
1 & x & x^2 & x^3 \\
1 & 5 & 2 & 4 & 5 \\
y & 3 & 7 & 8 & 0 \\
y^2 & 5 & 8 & 0 & 0 \\
y^3 & 6 & 0 & 0 & 0 \\
\end{array}
\]

The divisor is

\[
\begin{array}{c}
1 & x \\
1 & 5 & 7 \\
y & 4 & 0 \\
\end{array}
\]

Applying straight division method, the part divisor (PD) is considered as 5 and the two Dhwajankas are \(7x\) and \(4y\)

\[
\begin{array}{cc}
D_1 & D_2 \\
7x & 4y \\
\end{array}
\]

The working details are given below in the form of steps for rows of the dividend

Row 1: \((\text{Const}, x, x^2, x^3)\) The four elements are represented as (1) (2) (3) and (4) by a diagram, exclusively for each digit

Row 1 now consists of

\[
\begin{array}{c}
(1) & (2) & (3) & (4) \\
5 \ldots & .2 \ldots & .4 \ldots & .5 \ldots \\
\end{array}
\]

The working details of division as follows
Vedic Mathematics

Division

Step 1 Consider (1) and divide it by 5 (P D)

\[ \underline{5 \ldots} + 5 = \underline{1 \ldots} \quad (Q_1) \]

Step 2 Consider (2) The working is to subtract the product of first Dhawajanka D₁ with Q₁ from (2) and the result is divided by 5 (P D)

\[ \begin{array}{c|c}
D₁ & 1 \\
\hline
0 & 7 \\
\end{array} \begin{array}{c}
1 \ldots \\
0 \\
\end{array} = 5 \times 9 (Q₁) \text{ and is represented as } \underline{1 \ldots} \quad (Q₂) \]

\[ Q₁ \]

Step 3 Consider (3). Then subtract the \( (x^2) \) cross multiplication represented by D₁, D₂, Q₁, Q₂.

\[ \begin{array}{c|c}
D₂ & 4 \\
\hline
0 & 7 \\
\end{array} \begin{array}{c}
1 -1 \ldots \\
\end{array} \div 5 \]

\[ Q₁, Q₂ \]

\[ [4 \ (7)] \div 5 = \frac{11}{5} x² \text{ and is represented as } \underline{11 \frac{1}{5}} \quad (Q₃) \]

Step 4 Consider (4)

\[ \begin{array}{c|c}
D₃ & 5 \\
\hline
0 & 7 \\
\end{array} \begin{array}{c}
1 -1 \frac{11}{5} \\
\end{array} + 5 \]

\[ = \frac{1}{5} \left( 5 \frac{77}{5} \right) = \frac{-52}{5} \div 5 = -\frac{52}{25} x^3 \text{ and is represented as } \underline{\ldots \ldots -\frac{52}{25}} \quad (Q₄) \]

For the remainders in the first row the same procedure is continued but is not to be divided by the part divisor.

Step 5 Remainder in \( x^4 \)

\[ \begin{array}{c|c}
D₄ & 0 \\
\hline
0 & 7 \\
\end{array} \begin{array}{c}
1 \ldots \ldots \frac{52}{25} \\
\end{array} = \frac{364}{25} \text{ represented as } \underline{\ldots \ldots -\frac{364}{25}} \quad R₁ \]

Row 2 \( (y, xy, x²y) \), \( x^3 y = 0 \) similar is the procedure for other rows second row consists of three elements and they are

\[ (1) \qquad (2) \qquad (3) \]

\[ 3 \quad 7 \quad 8 \]
Vedic Mathematics

Step 1: (y)  
Consider (1)

\[
\begin{array}{c}
\text{D} \\
3 \\
\hline
4 \\
\text{Q}
\end{array}
\quad \begin{array}{c}
\text{Q}_1
\end{array}
\]

\[(3 - 4) + 5 = -1 + 5 = \frac{-1y}{5}\text{ represented as } = \begin{array}{c}
\frac{-1}{5}
\end{array}\quad \text{Q}_3\]

Step 2: (xy)  
Consider (2)

\[
\begin{array}{c}
\text{D} \\
7 \\
\hline
4 \\
\text{Q}
\end{array}
\quad \begin{array}{c}
\text{Q}_2
\end{array}
\]

\[7 - \left( \frac{4y}{5} + \left( \frac{-1}{5} \right) \right) + 5 = \left\{ 7 + 4 + \frac{7}{5} \right\} + 5 = \frac{62}{25}\text{ xy and is represented as } = \begin{array}{c}
\frac{62}{25}
\end{array}\quad \text{Q}_1\]

Step 3: (x^2y)  
Consider (3)

\[
\begin{array}{c}
\text{D} \\
8 \\
\hline
4 \\
\text{Q}
\end{array}
\quad \begin{array}{c}
\text{Q}_3
\end{array}
\]

\[8 - \frac{11}{5} + 5 = \left( 8 - \left( \frac{4y}{5} + \left( \frac{11}{5} \right) \right) \right) + 5 = \frac{454}{125}\text{ x^2y and is represented as } = \begin{array}{c}
\frac{-454}{125}
\end{array}\quad \text{Q}_7\]

For the remainders in the second row the procedure is as follows

Step 4: (x^3y)

\[
\begin{array}{c}
\text{D} \\
0 \\
\hline
4 \\
\text{Q}
\end{array}
\quad \begin{array}{c}
\text{Q}_6
\end{array}
\]

\[0 - 7 + 5 = \left( 0 - \frac{52}{25} \right) + 5 = \frac{-454}{125}\quad \text{Q}_9\]
\[ = \left[ (7) \left( \begin{array}{c} -454 \\ 125 \end{array} \right) + (4) \left( \begin{array}{c} -52 \\ 25 \end{array} \right) \right] \]
\[ = \left[ \frac{3178}{125} + \frac{208}{25} \right] = \frac{4218}{125} \]
x^3\text{y and is represented as} \[ \frac{4218}{125} \]
R_2

Row 3. The elements in this row are
\[ \frac{5}{(1)} \quad \frac{8}{(2)} \]

Step 1 \((y^3)\) Consider (1)

\[ \begin{array}{ccc} D & Q \\ 5 & -4 & \frac{1}{5} + 5 \end{array} \]
\[ 5 - (4) \left( \frac{1}{5} \right) + 5 = \frac{29}{25} \text{y}^2 \text{ and is represented as} \]
\[ \frac{29}{25} \]
Q_3

Step 2 \((x^2)\) Consider (2)

\[ \begin{array}{ccc} D & Q \\ 8 & -4 & \frac{62}{25} \end{array} \]
\[ \frac{29}{25} \]
Q_6
\[ \frac{251}{125} \]

For the remainders in the third row the procedure is as follows
Vedic Mathematics

314

Division

Step 3: \((x^2 y^3)\)

\[
\begin{array}{c}
\begin{array}{c}
-7
\end{array}
\end{array}
\begin{array}{c}
-4
\end{array}
\]

\[
\begin{array}{c}
-454
\end{array}
\begin{array}{c}
125
\end{array}
\]

\[
\begin{array}{c}
-251
\end{array}
\begin{array}{c}
125
\end{array}
\]

\[
\begin{array}{c}
Q_7
\end{array}
\begin{array}{c}
Q_9
\end{array}
\]

\[
0 - (7) \left( \frac{251}{125} \right) - (4) \left( \frac{454}{125} \right) = \frac{1757}{125} + \frac{1816}{125} = \frac{3573}{125} \]

\(x^2 y^3\) and is represented as

\[
\begin{array}{c}
R_3
\end{array}
\begin{array}{c}
3573
\end{array}
\begin{array}{c}
125
\end{array}
\]

Row 4: the only element in row 4 is

\[
\begin{array}{c}
6
\end{array}
\]

Step 1 \((y^3)\)

\[
\begin{array}{c}
\begin{array}{c}
-7
\end{array}
\end{array}
\begin{array}{c}
-4
\end{array}
\]

\[
\begin{array}{c}
29
\end{array}
\begin{array}{c}
25
\end{array}
\]

\[
\begin{array}{c}
34
\end{array}
\begin{array}{c}
125
\end{array}
\]

\[
\begin{array}{c}
Q_9
\end{array}
\begin{array}{c}
Q_{10}
\end{array}
\]

\(6 + 5 \left( 6 - 4 \times \frac{29}{25} \right) + 5 = \frac{34}{125} y^3\) represented as

\[
\begin{array}{c}
R_{10}
\end{array}
\begin{array}{c}
34
\end{array}
\begin{array}{c}
125
\end{array}
\]

Fourth row remainder is worked out as follows

Step 2 \((xy^3)\)

\[
\begin{array}{c}
\begin{array}{c}
-7
\end{array}
\end{array}
\begin{array}{c}
-4
\end{array}
\]

\[
\begin{array}{c}
-34
\end{array}
\begin{array}{c}
125
\end{array}
\]

\[
\begin{array}{c}
Q_{10}
\end{array}
\begin{array}{c}
Q_{10}
\end{array}
\]

\[
= 0 - (7) \left( \frac{34}{125} \right) - (4) \left( \frac{251}{125} \right) = - \frac{238}{125} + \frac{1004}{125} = \frac{766}{125} \]

\(y^3 x\) and is represented as

\[
\begin{array}{c}
R_4
\end{array}
\begin{array}{c}
766
\end{array}
\begin{array}{c}
125
\end{array}
\]
The remainder $y^4$ is evaluated from row 5 as follows:

Row 5: Consider the element in row 5 as zero

**Step 1: ($y^4$)**

\[
\begin{array}{c}
\begin{array}{c}
\frac{136}{125}
\end{array}
\end{array}
\]

For all the other remainder the procedure is similar.

The features of all the above steps are represented as below:

<table>
<thead>
<tr>
<th>$D$</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5 2 4 5</td>
</tr>
<tr>
<td>4</td>
<td>3 7 8 0</td>
</tr>
<tr>
<td></td>
<td>5 8 0 0</td>
</tr>
<tr>
<td></td>
<td>6 0 0 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(PD)</th>
<th>5</th>
<th>1 -1 11 52</th>
<th>$\frac{364}{25}$ x^4 R_1</th>
<th>$\frac{4218}{125}$ x'y R_2</th>
<th>$\frac{3573}{125}$ x'y^2 R_3</th>
<th>$\frac{766}{125}$ x'y^3 R_4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Q_1 Q_2 Q_3 Q_4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 62 454</td>
<td>$\frac{1}{5}$ 1</td>
<td>$\frac{29}{5}$ 125</td>
<td>$\frac{34}{125}$</td>
<td>$\frac{136}{125}$ y^4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q_5 Q_6 Q_7</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>25 125</td>
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<tr>
<td></td>
<td></td>
<td>Q_8 Q_9</td>
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</tbody>
</table>
CHAPTER – VII
COMPUTER PROGRAMMING RESULTS

NIKHILAM

Division by Nikhilam Method

Enter the Dividend: 223

Enter the Divisor: 78

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

The Nikhilam Divisor: 22
Quotient Part is: 2,
Remainder Part is: 2, 3,
Results of Multiplication: 4, 4,

The Nikhilam Quotient is: 2,
The Nikhilam Remainder is: 6, 7,
Quotient in Ordinary form: 2
Remainder in Ordinary form: 67

Quotient: 2
Remainder: 67

Do you want to continue with another division(y/n): n
Division by Nikhilam Method

Enter the Dividend: 897356

Enter the Divisor: 721

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

The Nikhilam Quotient is: 9,
The Nikhilam Remainder is: 26, 83, 86,
Quotient in Ordinary form: 1240
Remainder in Ordinary form: 3316
Quotient Part is: 3,
Remainder Part is: 3, 1, 6,
Results of Multiplication: 6, 21, 27,

The Nikhilam Quotient is: 3,
The Nikhilam Remainder is: 9, 22, 33,
Quotient in Ordinary form: 1243
Remainder in Ordinary form: 1153
Quotient Part is: 1,
Remainder Part is: 1, 5, 3,
Results of Multiplication: 2, 7, 9,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 3, 12, 12,
Quotient in Ordinary form: 1244
Remainder in Ordinary form: 432

Quotient: 1244
Remainder: 432

Do you want to continue with another division(y/n): n
Division by Nikhilam Method

Enter the Dividend: 45679

Enter the Divisor: 99

Enter 0 if to ignore the decimal part
- Enter the number of digits in decimal part: 0

The Nikhilam Divisor: 1
Quotient Part is: 4, 5, 6, 7.
Remainder Part is: 9,
Results of Multiplication: 4,
9,
15,
22,

The Nikhilam Quotient is: 4, 9, 15, 22,
The Nikhilam Remainder is: 31,
Quotient in Ordinary form: 5072
Remainder in Ordinary form: 31

Quotient: 5072
Remainder: 31

Do you want to continue with another division(y/n): n

Division by Nikhilam Method

Enter the Dividend: 31589

Enter the Divisor: 7

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

The Nikhilam Quotient is: 4,
The Nikhilam Remainder is: 19,
Quotient in Ordinary form: 4510
Remainder in Ordinary form: 19
Quotient Part is: 1,
Remainder Part is: 9,
Results of Multiplication: 3,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 12,
Quotient in Ordinary form: 4511
Remainder in Ordinary form: 12
Quotient Part is: 1,
Remainder Part is: 2,
Results of Multiplication: 3,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 5,
Quotient in Ordinary form: 4512
Remainder in Ordinary form: 5

Quotient: 4512
Remainder: 5

Do you want to continue with another division(y/n): n

Division by Nikhilam Method

Enter the Dividend: 42567

Enter the Divisor: 8

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0
The Nikhilam Remainder is: 119,
Quotient in Ordinary form: 5306
Remainder in Ordinary form: 119
Quotient Part is: 1, 1,
Remainder Part is: 9,
Results of Multiplication: 2.

6,

The Nikhilam Quotient is: 1, 3,
The Nikhilam Remainder is: 15,
Quotient in Ordinary form: 5319
Remainder in Ordinary form: 15
Quotient Part is: 1,
Remainder Part is: 5,
Results of Multiplication: 2,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 7,
Quotient in Ordinary form: 5320
Remainder in Ordinary form: 7

Quotient: 5320
Remainder: 7

Do you want to continue with another division (y/n): n
REDUCTION

DIVISION PROCESS

The first number(DIVIDEND) is: 0.124

The second number(DIVISOR) is: 2122

The intermediate remainders are: 0012344

The final result is: 0.000058435

Remainder = 13

Do you want to have another calculation (Y/N)?

DIVISION PROCESS

The first number(DIVIDEND) is: 124

The second number(DIVISOR) is: 212.2

The intermediate remainders are: 012344

The final result is: 0.58435

Remainder = 1240

Do you want to have another calculation (Y/N)?
DIVISION PROCESS

The first number(DIVIDEND) is: 1 2 4

The second number(DIVISOR) is: 2 1 .2 2

The intermediate remainders are: 0 1 2 3 4 4

The final result is : 0 5 .8 4 3 5

Remainder = 1790

Do you want to have another calculation (Y/N)n

DIVISION PROCESS

The first number(DIVIDEND) is: 8 9 7 3 5 6

The second number(DIVISOR) is: 7 2 1

The intermediate remainders are: 0 1 3 4 5 8 9 4 6

The final result is : 1 2 4 4 .5 9 9 1 6

Remainder = 432

Do you want to have another calculation (Y/N)n
DIVISION PROCESS

The first number(DIVIDEND) is: 7 8

The second number(DIVISOR) is: 2 1 3 4 5

The intermediate remainders are: 0 1 3 5 7

The final result is : 0.0 0 3 6 5 4 2

Remainder = 7800

Do you want to have another calculation (Y/N)n

DIVISION PROCESS

The first number(DIVIDEND) is: 0.8 9 2 7 1 2 4

The second number(DIVISOR) is: 9 6 2 1 8 7 3 4

The intermediate remainders are: 0 0 8 6 10 14 23 21 24 31 32

The final result is : 0.0 0 0 0 0 0 0 0 9 2 7 7 9 4 7 8 9

Remainder = 692713

Do you want to have another calculation (Y/N)n
DIVISION PROCESS

The first number(DIVIDEND) is: 9 8 7 6 5

The second number(DIVISOR) is: 1 3 2 1

The intermediate remainders are: 0 2 3 4 4 4 3 3

The final result is . 7 4 . 7 6 5 3 2 9

Remainder = 1011

Do you want to have another calculation (Y/N)?
DIVISION PROCESS

The first number (DIVIDEND) is: 1 5 6 2 8

The second number (DIVISOR) is: 2 3 .4

The intermediate reminders are: 0 1 1 -1 0 1 0 0 1 -1 -1 0 0 -1

The final result is: 0 6 6 7 .8 6 3 2 4 7 8 6 8 4

do you want to have another calculation (Y/N) N

DIVISION PROCESS

The first number (DIVIDEND) is: 0 1 1

The second number (DIVISOR) is: 1 1 1

The intermediate reminders are: 0 0 0 0 0 0 0 0 0 0 0 0

The final result is: 0 .0 9 9 0 9 0 9 9 1 0

do you want to have another calculation (Y/N) N
DIVISION PROCESS

The first number (DIVIDEND) is: 0 0 0 .4 6 1 3 9 7

The second number (DIVISOR) is: 1 2 3 .4

The intermediate reminders are: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

The final result is: 0 .0 0 3 7 3 9 0 3 5 6 5 6 3 9 9 6 4

do you want to have another calculation (Y/N) N
PARVARTYA POLYNOMIALS

DIVISION

The Dividend is: $6 \cdot x^3 - 12 \cdot x^2 + 3 \cdot x^1 - 10$

The Divisor is: $2 \cdot x^1 - 5$

The Paravartya form: 5,

Intermediate Multiplicants in the Quotient part are: $30/2 \cdot x^2, 30/4 \cdot x^1$,

Quotient: $3 \cdot x^2 + 3/2 \cdot x^1 + 21/4$

Intermediate Multiplicants in the Remainder part are: $105/4 \cdot x^0$,

Remainder: $+65/4$

Do you want to continue with another calculation(y/n)?

DIVISION

The Dividend is: $6 \cdot x^5 + 2 \cdot x^4 + 5 \cdot x^3 + 1$

The Divisor is: $3 \cdot x^2 - 2 \cdot x^1 + 1$

The Paravartya form: 2, -1,

Intermediate Multiplicants in the Quotient part are: $12/3 \cdot x^4, -6/3 \cdot x^3, 36/9 \cdot x^3, -18/9 \cdot x^2, 378/81 \cdot x^2, -189/81 \cdot x^1$,

Quotient: $2 \cdot x^3 + 2 \cdot x^2 + 7/3 \cdot x^1 + 8/9$

Intermediate Multiplicants in the Remainder part are: $16/9 \cdot x^1, -8/9 \cdot x^0$,

Remainder: $-5/9 \cdot x^1 + 1/9$
Do you want to continue with another calculation(y/n)n

DIVISION

The Dividend is: 1 * x^3 -6 * x^2 +11 * x^1 -6
The Divisor is: 2 * x^1 -1

The Paravartya form: 1,

Intermediate Multiplicants in the Quotient part are: 1/2 * x^2, -11/4 * x^1,

Quotient: 1/2 * x^2 -11/4 * x^1 + 33/8

Intermediate Multiplicants in the Remainder part are: 33/8 * x^0,

Remainder: -15/8

Do you want to continue with another calculation(y/n)n
PARAVARTYA NUMERALS

Division by Paravartya Method

Enter the Dividend: 29429
Enter the Divisor: 1463

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

Paravartya form is: -4, -6, -3,
Quotient part is: 2, 9,
Remainder part is: 4, 2, 9,
Results of Multiplication with final quotient digits: -8, -12, -6,
-4, -6, -3.

Quotient in Vinculum form: 2, 1,
Remainder in Vinculum form: -12, -10, 6,
Quotient in Ordinary form: 21
Remainder in Vinculum form: -1294

Quotient: 20
Remainder: 169

Do you want to continue with another division(y/n): n

Division by Paravartya Method

Enter the Dividend: 25935
Enter the Divisor: 829

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

Paravartya form is: 2, -3, 1,
Quotient part is: 2, 5,
Remainder part is: 9, 3, 5,
Results of Multiplication with final quotient digits: 4, -6, 2, 18, -27, 9,

Quotient in Vinculum form: 2, 9,
Remainder in Vinculum form: 21, -22, 14,
Quotient in Ordinary form: 29
- Remainder in Vinculum form: 1894

Quotient: 31
Remainder: 236

Do you want to continue with another division(y'n): n

Division by Paravartya Method

Enter the Dividend: 101100
Enter the Divisor: 486

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

Paravartya form is: 0, 3, -2,
Quotient part is: 1, 0, 1,
Remainder part is: 1, 0, 0,
Results of Multiplication with final quotient digits. 0, 3, -2,
0, 0, 0,
0, 12, -8,

Quotient in Vinculum form: 1, 0, 4,
Remainder in Vinculum form: -1, 12, -8,
Quotient: 208
Remainder: 12
Do you want to continue with another division (y/n): n

Division by Paravartya Method

Enter the Dividend: 887356
Enter the Divisor: 721

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

Paravartya form is: 3, -2, -1,
Quotient part is: 8, 9, 7,
Remainder part is: 3, 5, 6,
Results of Multiplication with final quotient digits: 24, -16, -8,
99, -66, -33,
270, -180, -90,

Quotient in Vinculum form: 8, 33, 90,
Remainder in Vinculum form: 199, -208, -84,
Quotient in Ordinary form: 1220
Remainder in Vinculum form: 17736

Quotient: 1244
Remainder: 432

Do you want to continue with another division (y/n): n
ARGUMENTAL POLYNOMIALS

ARGUMENTAL DIVISION

The Dividend is: 24 * x^4 + 50 * x^3 + 35 * x^2 + 10 * x + 13
The Divisor is: 4 * x^1 + 1

The coefficient of the power of x=(maxpower of dividend-maxpower of divisor= 3)
, 6
The rest of the coefficients are in decreasing order.
11, 6, 1,

Quotient: + 6 * x^3 + 11 * x^2 + 6 * x^1 + 1

The individual remainders are: 0, 0, 0, 12,

Remainder: 12

Do you want to continue with another calculation(y/n)n

ARGUMENTAL DIVISION

The Dividend is: 10 * x^4 + 17 * x^3 + 20 * x^2 + 6 * x + 3
The Divisor is: 2 * x^2 + 3 * x + 3

The coefficient of the power of x=(maxpower of dividend-maxpower of divisor= 2)
, 5
The rest of the coefficients are in decreasing order.
1, 1,

Quotient: + 5 * x^2 + 1 * x + 1

The individual remainders are: 0, 0, 0, 0,

Remainder: 0

Do you want to continue with another calculation(y/n)n
ARGUMENTAL DIVISION

The Dividend is: \(2 \cdot x^{10} + 4 \cdot x^9 + 9 \cdot x^8 + 14 \cdot x^7 + 17 \cdot x^6 + 20 \cdot x^5 + 15 \cdot x^4 + 16 \cdot x^3 + 16 \cdot x^2 + 8 \cdot x^1 + 10\)

The Divisor is: \(2 \cdot x^5 + 2 \cdot x^4 + 3 \cdot x^3 + 1 \cdot x^2 + 2 \cdot x^1 + 3\)

The coefficient of the power of \(x=\text{(maxpower of dividend-maxpower of divisor)} = 5\)

\[1\]

The rest of the coefficients are in decreasing order.

1, 2, 3, 1, 1,

Quotient. \(+1 \cdot x^5 + 1 \cdot x^4 + 2 \cdot x^3 + 3 \cdot x^2 + 1 \cdot x^1 + 1\)

The individual remainders are: 0, 0, 0, 0, 0, 0, 4, 3, 7.

Remainder \(4 \cdot x^2 + 3 \cdot x^1 + 7\)

Do you want to continue with another calculation (y/n)? n
ARGUMENTAL NUMERALS

Division by Argument Method

Enter the Dividend: 109878548

Enter the Divisor: 6783

Intermediate Remainders are: 1, 4, 0, -1, -6, -5, -14, -128, -126.

Intermediate Quotients are: 0, 1, 7, -8, 0, -1.

Quotient: 16188

Remainder: 5514

Do you want to continue with another division(y/n): N

Division by Argument Method

Enter the Dividend: 89765

Enter the Divisor: 321

Intermediate Remainders are: 2, 1, -1, -12, -115.

Intermediate Quotients are: 2, 8, 0.

Quotient: 279

Remainder: 206

Do you want to continue with another division(y/n): N
Division by Argument Method

Enter the Dividend: 134289

Enter the Divisor: 2780

Intermediate Remainders are: 1, 1, 0, 64, 732, 7329.
Intermediate Quotients are: 0, 0, 6, -14.

Quotient: 48
Remainder: 1808

Do you want to continue with another division(y/n): N
STRAIGHT DIVISION 1 VARIABLE

STRAIGHT DIVISION

The Dividend is: $5 \cdot x^4 + 3 \cdot x^3 + 2 \cdot x^2 + 1 \cdot x^1 + 2$

The Divisor is: $3 \cdot x^2 + 1 \cdot x^1 + 4$

The part divisor: $3 \cdot x^2$

The Dhujanka part: $+1 \cdot x^1 + 4$

In the Quotient Region....

R 1 = 0, ID 1 = 0 + 3 \cdot x^3
R 2 = 0, ID 2 = 0 + 2 \cdot x^2
R 3 = 0, ID 3 = 0 + 2 \cdot x^2

Quotient: $5/3 \cdot x^2 + 4/9 \cdot x^1 - 48/27$

In Remainder Region.... R 4 = 0, ID 4 = 0 + 1 \cdot x^1
R 5 = 0, ID 5 = 0 + 1 \cdot x^1
R 6 = 0, ID 6 = 0 + 2 \cdot x^0

Remainder: $25/27 \cdot x^1 + 238/27$

Do you want to continue with another calculation(y/n)? N
STRAIGHT DIVISION

The Dividend is: 8 \cdot x^5 + 9 \cdot x^4 + 7 \cdot x^3 + 3 \cdot x^2 + 5 \cdot x^1 + 6
The Divisor is: 7 \cdot x^2 + 2 \cdot x^1 + 1

The part divisor: 7 \cdot x^2
The Dhwajanka part: +2 \cdot x^1 + 1

In the Quotient Region...
R 1 = 0, ID 1 = 0 + 9 \cdot x^4
R 2 = 0, ID 2 = 0 + 7 \cdot x^3
R 3 = 0, ID 3 = 0 + 7 \cdot x^3
R 4 = 0, ID 4 = 0 + 3 \cdot x^2
R 5 = 0, ID 5 = 0 + 3 \cdot x^2

Quotient. 8/7 \cdot x^3 + 47/49 \cdot x^2 + 193/343 \cdot x^1 + 314/2401

In Remainder Region....R 6 = 0, ID 6 = 0 + 5 \cdot x^1
R 7 = 0, ID 7 = 0 + 5 \cdot x^1
R 8 = 0, ID 8 = 0 + 6 \cdot x^0

Remainder 10028/2401 \cdot x^1 + 14092/2401

Do you want to continue with another calculation(y/n)? N
STRAIGHT DIVISION

The Dividend is: $8 \cdot x^5 - 9 \cdot x^4 + 7 \cdot x^3 - 3 \cdot x^2 + 5 \cdot x^1 + 2$

The Divisor is: $7 \cdot x^2 + 2 \cdot x^1 + 1$

The part divisor: $7 \cdot x^2$

The Dhawajaka part: $+2 \cdot x^1 + 1$

In the Quotient Region....
R 1 = 0, ID 1 = 0 + -9 \cdot x^4
R 2 = 0, ID 2 = 0 + 7 \cdot x^3
R 3 = 0, ID 3 = 0 + 7 \cdot x^3
R 4 = 0, ID 4 = 0 + -3 \cdot x^2
R 5 = 0, ID 5 = 0 + -3 \cdot x^2

Quotient: $8/7 \cdot x^3 - 79/49 \cdot x^2 - 445/343 \cdot x^1 - 1366/2401$

In Remainder Region....R 6 = 0, ID 6 = 0 + 5 \cdot x^1
R 7 = 0, ID 7 = 0 + 5 \cdot x^1
R 8 = 0, ID 8 = 0 + 2 \cdot x^0

Remainder: $17852/2401 \cdot x^1 + 6168/2401$

Do you want to continue with another calculation(y/n)N
STRAIGHT DIVISION

The Dividend is: \( 7 \cdot X^{10} + 26 \cdot X^9 + 53 \cdot X^8 + 56 \cdot X^7 + 43 \cdot X^6 + 40 \cdot x^5 + 41 \cdot \)
\( X^4 + 38 \cdot X^3 + 19 \cdot X^2 + 8 \cdot X + 5 \)

The Divisor is: \( 1 \cdot X^5 + 3 \cdot X^4 + 5 \cdot X^3 + 3 \cdot X^2 + 1 \cdot X + 1 \)

\[ R_{15} = 0, \text{ID} \ 15 = 0 + 40 \cdot x^5 \]

Quotient: \( 7 \cdot x^5 + 5 \cdot x^4 + 3 \cdot x^3 + 1 \cdot x^2 + 3 \cdot x + 1 \) + 5

In Remainder Region. R 16 = 0, ID 16 = 0 + 41 \cdot x^4

\[ R_{17} = 0, \text{ID} \ 17 = 0 + 41 \cdot x^4 \]
\[ R_{18} = 0, \text{ID} \ 18 = 0 + 41 \cdot x^4 \]
\[ R_{19} = 0, \text{ID} \ 19 = 0 + 41 \cdot x^4 \]
\[ R_{20} = 0, \text{ID} \ 20 = 0 + 41 \cdot x^4 \]
\[ R_{21} = 0, \text{ID} \ 21 = 0 + 38 \cdot x^3 \]
\[ R_{22} = 0, \text{ID} \ 22 = 0 + 38 \cdot x^3 \]
\[ R_{23} = 0, \text{ID} \ 23 = 0 + 38 \cdot x^3 \]
\[ R_{24} = 0, \text{ID} \ 24 = 0 + 38 \cdot x^3 \]
\[ R_{25} = 0, \text{ID} \ 25 = 0 + 19 \cdot x^2 \]
\[ R_{26} = 0, \text{ID} \ 26 = 0 + 19 \cdot x^2 \]
\[ R_{27} = 0, \text{ID} \ 27 = 0 + 19 \cdot x^2 \]
\[ R_{28} = 0, \text{ID} \ 28 = 0 + 8 \cdot x^1 \]
\[ R_{29} = 0, \text{ID} \ 29 = 0 + 8 \cdot x^1 \]
\[ R_{30} = 0, \text{ID} \ 30 = 0 + 5 \cdot x^0 \]

Remainder 0

Do you want to continue with another calculation (y/n)? N
STRAIGHT DIVISION 2 VARIABLES

STRAIGHT DIVISION

The Dividend is: 3 +4 * x^1 +1 * x^2 +2 * x^3 +2 * x^4 +4 * y^1 +17 * x^1 * y^1 +12 * x^2 * y^1 +2 * x^3 * y^1 +10 * x^4 * y^1 +4 * y^2 +7 * x^1 + y^2 +20 * x^2 * y^2 +9 * x^3 * y^2 +5 * x^4 * y^2 +4 * y^3 -5 * x^1 * y^3 +1 * x^2 * y^3 +3 * x^3 * y^3 +1 * y^4 -1 * x^1 * y^4 -1 * x^2 * x^2 * y^4 +4 * x^3 * y^4 -2 * x^4 * y^4

The Divisor is: 3 +2 * x^1 +2 * x^2 +4 * y^1 +2 * x^2 * y^1 +1 * y^2 +1 * x^1 * y^2 +2 * x^2 * y^2 +1 * x^1 * y^2

Quotient: + 1 + 2 * x^1 + 1 * x^2 + 3 * x^1 * y^1 + 4 * x^2 * y^1 + 1 * y^2 - 2 * x^1 * y^2 - 2 * x^2 * y^2

Remainder:

Do you want to continue with another calculation(y/n)n

STRAIGHT DIVISION

The Dividend is: 2 +4 * x^1 +6 * x^2 +4 * y^1 +8 * x^1 * y^1 +1 * x^2 * y^1

The Divisor is: 2 +1 * x^1 +1 * x^2 +2 * y^1 +3 * x^1 * y^1

Quotient: + 1 + 3/2 * x^1 + 7/4 * x^2 + 1 * y^1 + 1/2 * x^1 + y^1 - 17/4 * x^2 * y^1

Remainder: -13/4 * x^3 - 7/4 * x^4 - 3/2 * x^3 * y^1 - 2 * y^2 + 17/4 * x^4 * y^1 - 4 * x^1 * y^2 + 7 * x^2 * y^2 + 51/4 * x^3 * y^2

Do you want to continue with another calculation(y/n)n
STRAIGHT DIVISION

The Dividend is: $5 + 2 \cdot x^1 + 4 \cdot x^2 + 5 \cdot x^3 + 3 \cdot y^1 + 7 \cdot x^1 + 5 \cdot y^1 + 6 \cdot x^2 + 8 \cdot x^1 + y^2 + 8 \cdot y^3$

The Divisor is: $5 + 7 \cdot x^1 + 4 \cdot y^1$

Quotient: $+ 1 - 1 \cdot x^1 + 11/5 \cdot x^2 - 52/25 \cdot x^3 - 1/5 \cdot y^1$

$+ 62/25 \cdot x^1 \cdot y^1 + 19/25 \cdot y^2 - 381/125 \cdot x^1 \cdot y^2 + 74/125 \cdot y^3$

Remainder: $+ 384/25 \cdot x^1 - 654/25 \cdot x^2 \cdot y^1 + 208/25 \cdot x^3 + y^1 + 30867/125 \cdot x^2 \cdot y^2 + 1006/125 \cdot x^1 \cdot y^3 - 296/125 \cdot y^4$

Do you want to continue with another calculation(y/n)?

STRAIGHT DIVISION

The Dividend is: $8 + 18 \cdot x^1 + 9 \cdot x^2 + 8 \cdot y^1 + 19 \cdot x^1 \cdot y^1 + 12 \cdot x^2 \cdot y^1 + 2 \cdot y^2 + 5 \cdot x^1 \cdot y^2 + 3 \cdot x^2 \cdot y^2$

The Divisor is: $4 + 3 \cdot x^1 + 2 \cdot y^1 + 3 \cdot x^1 \cdot y^1$

Quotient: $+ 2 + 3 \cdot x^1 + 1 \cdot y^1 + 1 \cdot x^1 \cdot y^1$

Remainder:

Do you want to continue with another calculation(y/n)?

STRAIGHT DIVISION

The Dividend is: $3 - 4 \cdot x^1 - 4 \cdot x^2 - 5 \cdot y^1 - 1 \cdot x^1 \cdot y^1 + 6 \cdot x^2 \cdot y^1 - 12 \cdot y^2 + 13 \cdot x^1 \cdot y^2 + 4 \cdot x^2 \cdot y^2$

The Divisor is: $1 - 2 \cdot x^1 - 3 \cdot y^1 + 4 \cdot x^1 \cdot y^1$

Quotient: $+ 3 + 2 \cdot x^1 + 4 \cdot y^1 + 1 \cdot x^1 \cdot y^1$

Remainder:

Do you want to continue with another calculation(y/n)?
STRAIGHT DIVISION

The Dividend is: $3 + 4 \cdot x^1 - 4 \cdot x^2 - 5 \cdot y^1 - 1 \cdot x^1 \cdot y^1 + 6 \cdot x^2 \cdot y^1 - 12 \cdot y^2 + 13 \cdot x^1 \cdot y^2 + 4 \cdot x^2 \cdot y^2$

The Divisor is: $1 + 2 \cdot x^1 - 3 \cdot y^1 + 4 \cdot x^1 \cdot y^1$

Quotient: $+ 3 + 2 \cdot x^1 + 4 \cdot y^1 + 1 \cdot x^1 \cdot y^1$

Remainder:

Do you want to continue with another calculation (y/n)?

STRAIGHT DIVISION

The Dividend is: $8 + 22 \cdot x^1 + 12 \cdot x^2 + 10 \cdot y^1 - 7 \cdot x^1 \cdot y^1 + 4 \cdot x^2 \cdot y^1 + 6 \cdot x^3 \cdot y^1 + 1 \cdot y^2 + 5 \cdot x^1 \cdot y^2 - 18 \cdot x^2 \cdot y^2 + 4 \cdot x^3 \cdot y^2 - 1 \cdot y^3 + 4 \cdot x^1 \cdot y^3 - 4 \cdot x^2 \cdot y^3$

The Divisor is: $4 + 3 \cdot x^1 - 1 \cdot y^1 + 2 \cdot x^1 \cdot y^1$

Quotient: $+ 2 + 4 \cdot x^1 + 3 \cdot y^1 - 4 \cdot x^1 \cdot y^1 + 2 \cdot x^2 \cdot y^1 + 1 \cdot y^2 - 2 \cdot x^1 \cdot y^2$

Remainder:

Do you want to continue with another calculation (y/n)?

STRAIGHT DIVISION

The Dividend is: $8 + 22 \cdot x^1 + 12 \cdot x^2 + 10 \cdot y^1 - 7 \cdot x^1 \cdot y^1 + 4 \cdot x^2 \cdot y^1 + 6 \cdot x^3 \cdot y^1 + 1 \cdot y^2 + 5 \cdot x^1 \cdot y^2 - 18 \cdot x^2 \cdot y^2 + 4 \cdot x^3 \cdot y^2 - 1 \cdot y^3 + 4 \cdot x^1 \cdot y^3 - 4 \cdot x^2 \cdot y^3$

The Divisor is: $4 + 3 \cdot x^1 - 1 \cdot y^1 + 2 \cdot x^1 \cdot y^1$

Quotient: $+ 2 + 4 \cdot x^1 + 3 \cdot y^1 - 4 \cdot x^1 \cdot y^1 + 2 \cdot x^2 \cdot y^1 + 1 \cdot y^2 - 2 \cdot x^1 \cdot y^2$

Remainder:

Do you want to continue with another calculation (y/n)?
STRAIGHT DIVISION

The Dividend is: \(3 + 4 \cdot x^1 + 1 \cdot x^2 + 2 \cdot x^3 + 2 \cdot x^4 + 4 \cdot y^1 + 17 \cdot x^1 \cdot y^1 + 12 \cdot x^2 \cdot y^1 + 2 \cdot x^3 \cdot y^1 + 10 \cdot x^4 \cdot y^1 + 4 \cdot y^2 + 7 \cdot x^1 \cdot y^2 + 20 \cdot x^2 \cdot y^2 + 9 \cdot x^3 \cdot y^2 + 5 \cdot x^4 \cdot y^2 + 4 \cdot y^3 - 5 \cdot x^1 \cdot y^3 + 3 \cdot x^2 \cdot y^3 + 3 \cdot x^3 \cdot y^3 + 1 \cdot y^4 - 1 \cdot x^1 \cdot y^4 - 3 \cdot x^2 \cdot y^4 - 4 \cdot x^3 \cdot y^4 - 2 \cdot x^4 \cdot y^4\)

The Divisor is: \(3 - 2 \cdot x^1 + 2 \cdot x^2 + 4 \cdot y^1 + 2 \cdot x^2 + y^1 + 1 \cdot y^2 + 1 \cdot x^1 \cdot y^2 + 1 \cdot x^2 \cdot y^2\)

Quotient: \(+ 1 + 2 \cdot x^1 + 1 \cdot x^2 + 3 \cdot x^1 \cdot y^1 + 4 \cdot x^2 \cdot y^1 + 1 \cdot y^1 + 2 \cdot y^2 - 2 \cdot x^1 \cdot y^2 - 2 \cdot x^2 \cdot y^2\)

Remainder:

Do you want to continue with another calculation (y/n)?

(REMAINDER DIVISION)

STRAIGHT DIVISION

The Dividend is: \(5 + 2 \cdot x^1 + 4 \cdot x^2 + 5 \cdot x^3 + 3 \cdot y^1 + 7 \cdot x^1 + 8 \cdot x^2 + y^1 + 5 \cdot y^2 + 8 \cdot x^1 \cdot y^2 + 6 \cdot y^3\)

The Divisor is: \(5 + 7 \cdot x^1 + 4 \cdot y^1\)

Quotient: \(+ 1 - 1 \cdot x^1 + 11/5 \cdot x^2 - 52/25 \cdot x^3 - 1/5 \cdot y^1 \cdot 62/25 \cdot x^1 \cdot y^1 - 454/125 \cdot x^2 \cdot y^1 + 29/25 \cdot y^2 - 251/125 \cdot x^1 \cdot y^2 + 34/125 \cdot y^3\)

Remainder: \(+ 364/25 \cdot x^4 + 4218/125 \cdot x^3 + 3573/125 \cdot x^2 \cdot y^4 + 766/125 \cdot x^1 \cdot y^3 - 138/125 \cdot y^4\)

Do you want to continue with another calculation (y/n)?
STRAIGHT DIVISION (COMMON LCM 125)

The Dividend is: \(1820 \times x^4 + 4218 \times x^3 \times y^1 + 3573 \times x^2 \times y^2 + 768 \times x^1 \times y^3 - 138 \times y^4\)

The Divisor is: \(5 + 7 \times x^1 + 4 \times y^1\)

Quotient: \(+ 364 \times x^4 + 4218/5 \times x^3 \times y^1 + 3573/5 \times x^2 \times y^2 + 768/5 \times x^1 \times y^3 - 138/5 \times y^4\)

Remainder: \(-2548 \times x^5 - 36806/5 \times x^4 \times y^1 - 41883/5 \times x^3 \times y^2 - 19654/5 \times x^2 \times y^3 - 2112/5 \times x^1 \times y^4 + 544/5 \times y^5\)

Do you want to continue with another calculation(y/n)?

STRAIGHT DIVISION (COMMON LCM 625)

The Dividend is: \(-12740 \times x^5 - 36806 \times x^4 \times y^1 - 41883 \times x^3 \times y^2 - 19654 \times x^2 \times y^3 - 2112 \times x^1 \times y^4 + 544 \times y^5\)

The Divisor is: \(5 + 7 \times x^1 + 4 \times y^1\)

Quotient: \(-2548 \times x^5 - 36806/5 \times x^4 \times y^1 - 41883/5 \times x^3 \times y^2 - 19654/5 \times x^2 \times y^3 - 2112/5 \times x^1 \times y^4 + 544/5 \times y^5\)

Remainder: \(+ 17836 \times x^6 + 308002/5 \times x^5 \times y^1 + 88081 \times x^4 \times y^2 + 61022 \times x^3 \times y^3 + 18880 \times x^2 \times y^4 + 928 \times x^1 \times y^5 - 2176/5 \times y^6\)

Do you want to continue with another calculation(y/n)?
STRAIGHT DIVISION

The Dividend is: $89180 \cdot x^6 + 308602 \cdot x^5 \cdot y^1 + 440405 \cdot x^4 \cdot y^2 + 305110 \cdot x^3 \cdot y^3 + 93400 \cdot x^2 \cdot y^4 + 4640 \cdot x^1 \cdot y^5 - 2176 \cdot y^6$

The Divisor is: $5 + 7 \cdot x^1 + 4 \cdot y^1$

Quotient: $+ 17836 \cdot x^6 + 308602/5 \cdot x^5 \cdot y^1 + 88081 \cdot x^4 \cdot y^2 + 81022 \cdot x^3 \cdot y^3 + 18680 \cdot x^2 \cdot y^4 + 928 \cdot x^1 \cdot y^5 - 2178/5 \cdot y^6$

Remainder: $-124852 \cdot x^7 - 2516934/5 \cdot x^6 \cdot y^1 - 4317243/5 \cdot x^5 \cdot y^2 - 779478 \cdot x^4 \cdot y^3 - 374848 \cdot x^3 \cdot y^4 - 81216 \cdot x^2 \cdot y^5 - 3328/5 \cdot x^1 \cdot y^6 + 8704/5 \cdot y^7$

Do you want to continue with another calculation (y/n)?
STRAIGHT DIVISION 3 VARIABLES

STRAIGHT DIVISION

The Dividend is: 5 + 2 * x^1 + 3 * y^1 + 4 * z^1 + 2 * x^1 * y^1 + 3
* x^1 * z^1 + 4 * y^1 * z^1 + 5 * x^2 + 6 * y^2 + 7 * z^2 + 2 * x^2 * y^1 + 3 * x^2 * z^1
* y^1 + 4 * x^1 * y^2 + 9 * y^2 + z^1 + 5 * x^1 + z^2 + 4 * y^1 + z^2 + 6 * x^3 + 8 * y^3
3 + 3 * z^3 + 5 * x^2 * y^2 + 3 * y^2 * z^2 + 4 * x^2 * z^2

The Divisor is: 5 + 7 * x^1 + 4 * y^1 + 2 * z^1

Quotient: + 1 - 1 * x^1 - 1/5 * y^1 + 2/5 * z^1 + 37/25 * x^1
* y^1 + 11/25 * x^1 * z^1 + 14/25 * y^1 * z^1 + 125/25 * x^2 + 34/25 * y^2 +
31/25 * z^2 - 449/125 * x^2 * y^1 - 122/125 * x^1 * z^1 - 286/125 * x^1 * y^2
+ 101/125 * y^2 * z^1 - 114/125 * x^1 * z^2 - 52/125 * y^1 * z^2 - 54/25 * x^3
+ 64/125 * y^3 + 13/125 * z^3 + 4423/625 * x^2 * y^2 + 381/625 * y^2 * z^2 +
1542/625 * x^2 * z^2

Remainder: -216/25 * x^1 * y^1 * z^1 + 4223/125 * x^3 * y^1 +
1394/125 * x^3 * z^1 + 1386/125 * x^2 * y^1 * z^1 - 27/25 * x^1 * y^2 * z^1 +
696/125 * x^1 * y^3 - 532/125 * y^3 * z^1 + 164/25 * x^1 * y^1 * z^2 + 378/25
* x^4 + 137/125 * x^1 * z^3 + 52/125 * y^1 * z^3 - 258/125 * y^4 - 30961/625
* x^3 * y^2 - 2667/625 * x^1 * y^2 * z^2 - 17692/625 * x^2 * y^3 - 26/125 * z^4
- 10794/625 * x^3 * z^2 - 1524/625 * y^3 * z^2 - 8846/625 * x^2 * y^2 * z^1 - 61
68/625 * x^2 * y^1 * z^2 - 782/625 * y^2 * z^3 - 3084/625 * x^2 * z^3

Do you want to continue with another calculation (y/n)? N