Vedic Mathematics

Lecture Notes – 1

Multiplication

By
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INDEX

VEDIC MATHEMATICS
OR SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

SIXTEEN SUTRAS AND THEIR COROLLARIES

Sūtras	*Sub-Sūtras or Corollaries*
1. एकाधिकेन पूर्वेण *Ekādhikena Pūrveṇa* (also a corollary)	1. आनुरूप्येण *Ānurūpyeṇa*
2. निखिलं नवतश्चरमं दशतः *Nikhilaṃ Navataścaramaṃ Daśataḥ*	2. शिष्यते शेषसंज्ञः *Śiṣyate Śeṣasaṃjñaḥ*
3. ऊर्ध्वतिर्यग्भ्याम् *Ūrdhva-tiryagbhyām*	3. आद्यमाद्येनान्त्यमन्त्येन *Ādyamādyenāntya-mantyena*
4. परावर्त्य योजयेत् *Parāvartya Yojayet*	4. केवलैः सप्तकं गुण्यात् *Kevalaiḥ Saptakaṃ Guṇyāt*
5. शून्यं साम्यसमुच्चये *Śūnyaṃ Sāmyasamuccaye*	5. वेष्टनम् *Veṣṭanam*
6. (आनुरूप्ये) शून्यमन्यत् *(Ānurūpye) Śūnyamanyat*	6. यावदूनं तावदूनम् *Yāvadūnaṃ Tāvadūnam*
7. संकलनव्यवकलनाभ्याम् *Saṅkalana-vyavakalanābhyām* (also a corollary)	7. यावदूनं तावदूनीकृत्य वर्गं च योजयेत् *Yāvadūnaṃ Tāvadūnīkṛtya Vargañca Yojayet*
8. पूरणापूरणाभ्याम् *Pūraṇāpūraṇābhyām*	8. अन्त्ययोर्दशकेऽपि *Antyayordaśake'pi*
9. चलनकलनाभ्याम् *Calana-Kalanābhyām*	9. अन्त्ययोरेव *Antyayoreva*
10. यावदूनम् *Yāvadūnam*	10. समुच्चयगुणितः *Samuccayaguṇitaḥ*
11. व्यष्टिसमष्टिः *Vyaṣṭisamaṣṭiḥ*	11. लोपनस्थापनाभ्याम् *Lopanasthāpanābhyām*
12. शेषाण्यङ्केन चरमेण *Śeṣāṇyaṅkena Carameṇa*	12. विलोकनम् *Vilokanam*
13. सोपान्त्यद्वयमन्त्यम् *Sopāntyadvayamantyam*	13. गुणितसमुच्चयः समुच्चयगुणितः *Guṇitasamuccayaḥ* *Samuccayaguṇitaḥ*
14. एकन्यूनेन पूर्वेण *Ekanyūnena Pūrveṇa*	
15. गुणितसमुच्चयः *Guṇitasamuccayaḥ*	
16. गुणकसमुच्चयः *Guṇakasamuccayaḥ*	

This list has been complied from stray references in the text
(Editor of the original book on Vedic Mathematics)

MULTIPLICATION WORKING DETAILS WITH EXAMPLES

7. CHAPTER I

APPLICATION OF URDHVA TIRYAGBHYAM SUTRAM (VERTICAL AND CROSSWISE) (V.M.)

Modus operandi: (Column-wise Multiplication)

i) Right to Left Multiplication:

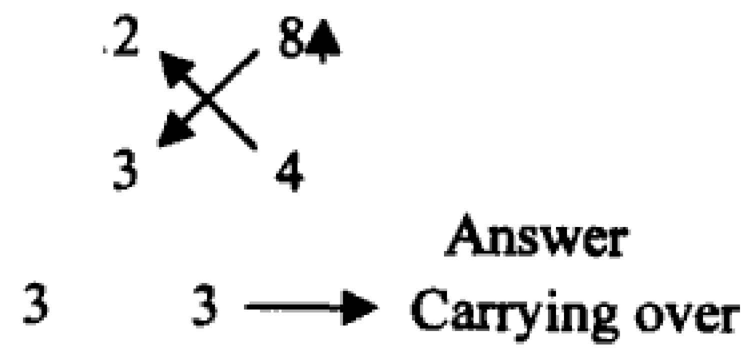
a) Two-digit Numbers Multiplication:

Example: 28 × 34

Current Method

28
34
112
84
952

Vedic Method



Ans.: 952

Fig. 1

V.M:

Considering two-digit number multiplication by a two-digit number, the first step is to multiply the first column (from right end) vertically. If the result is more than one digit, keep the first digit[*] under the answer and the other to the left of it a little below which is to be carried out. This is continued throughout.

The second step is the completion of the multiplication of the first column crosswise with the second column digits as shown in the figure 1. The two results of the cross multiplication are added up and finally added to the number carried over in the first step. The final result is similarly placed as in the first step. With this multiplication, the first column is exhausted.

The third step is to consider the second column and to perform vertical multiplication carrying over the addition from the previous step.

The multiplication starts with Urdhva of 1st column and ends with Urdhva of the last column.

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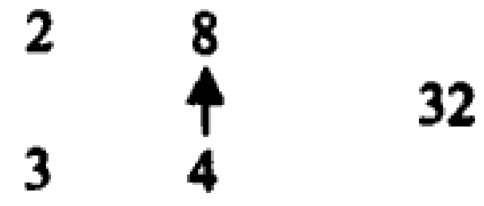
[*] The First digit corresponds to units place, second to tens place and so on.

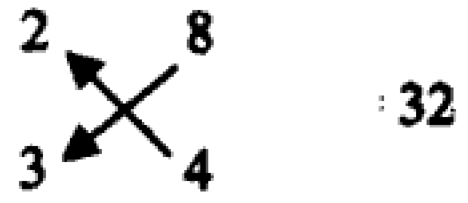
With this, the final answer is put in one line. This method is generally called **One - Line Method.**

Step Diagrams:

Step 1:

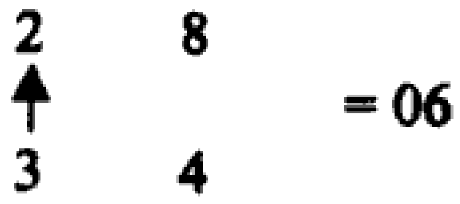


Step 2:



1 and 2 steps constitute the 1st column multiplication.

Step 3:

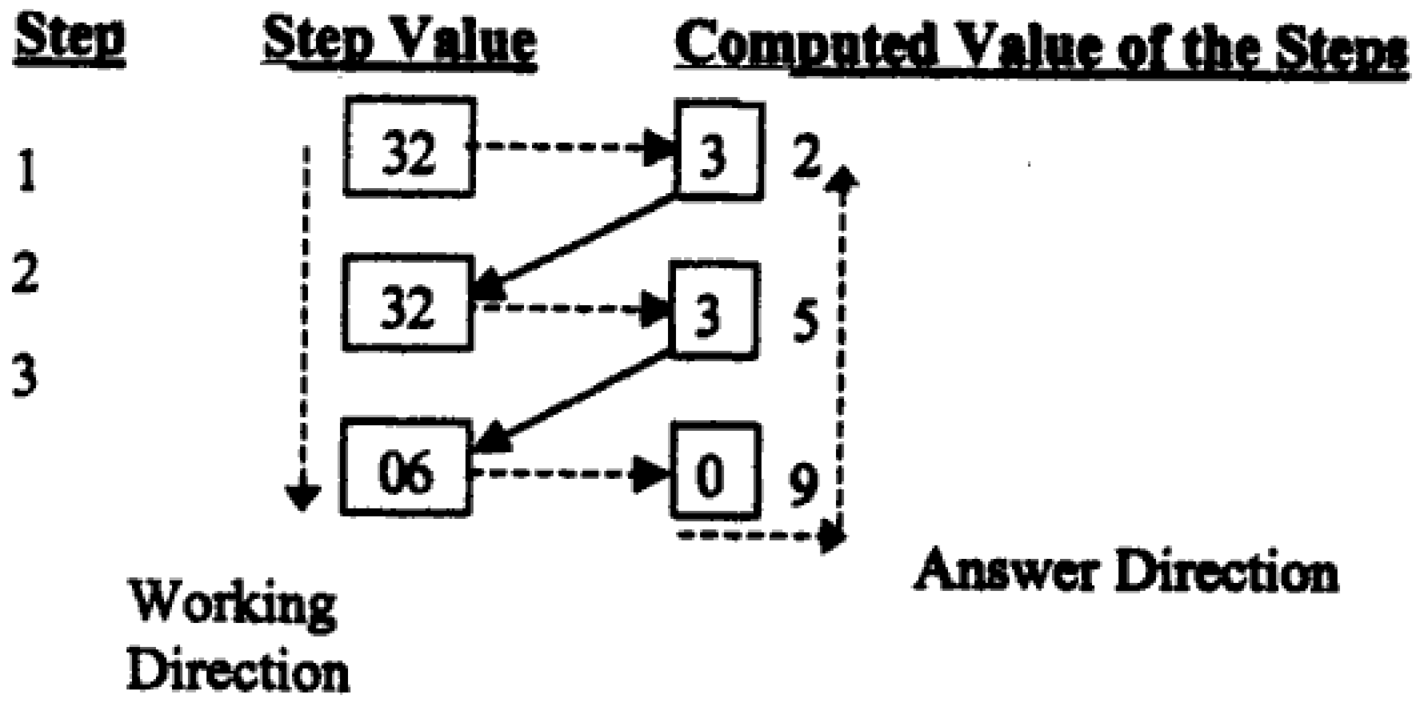


The last column multiplication (Urdhva only)

Multiplication with step diagrams (V.M.):

The answer of the multiplication can also be obtained by simply putting down the values of the various steps individually as shown in table 1, from which one can compute the answer as indicated. Here, the first step value and the corresponding computed value are same. The arrow mark in the results of the steps indicates the addition of corresponding numbers in blocks belonging to steps and computed values as shown in table 1 to get computed values of the successive steps. In the computed values, the blocks contain the carrying digits, which are to be carried to the succeeding step. Addition is self-explanatory as it is concerned with digits in the same status. The dashed lines lead the step values to the corresponding computed values.

Table 1



Ans.: = 952

The answer from the computed values can be read from the last digits 952.

b) Three-digit Numbers Multiplication:

Example: 852 × 395

Current Method

$$\begin{array}{r} 852 \\ \underline{395} \\ 4260 \\ 7668 \\ \underline{2556} \\ \underline{336540} \end{array}$$

Vedic Method

$$\begin{array}{r} 8\ 5\ 2 \\ 3\ 9\ 5 \\ \boxed{33\ 6\ 5\ 4\ 0} \\ 9\ 9\ 4\ 1 \end{array}$$

Ans.: 336540

V.M:

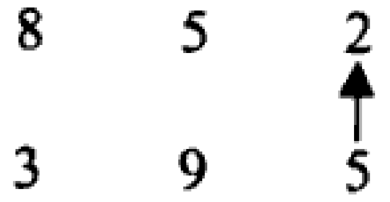
A three digit number multiplication is as follows:

In this multiplication, after the first column multiplication is exhausted, the remaining is worked out as en-block starting with second column and finally the last column is treated vertically.

The step diagrams and results are read from the computed values of the steps are given in figure 2.

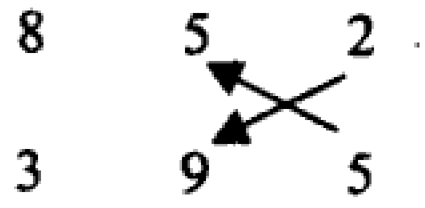
Step diagrams:

Step 1:



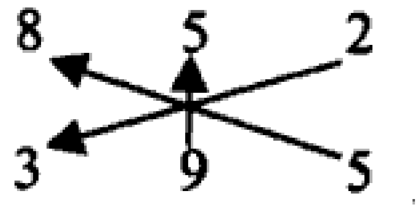
= 10

Step 2:



= 43

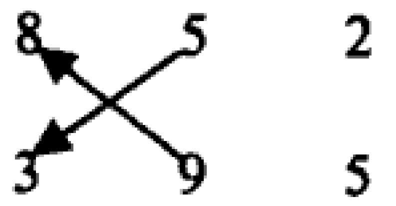
Step 3:



= 91

Steps 1 to 3 constitute the 1st column multiplication.

Step 4:



= 87

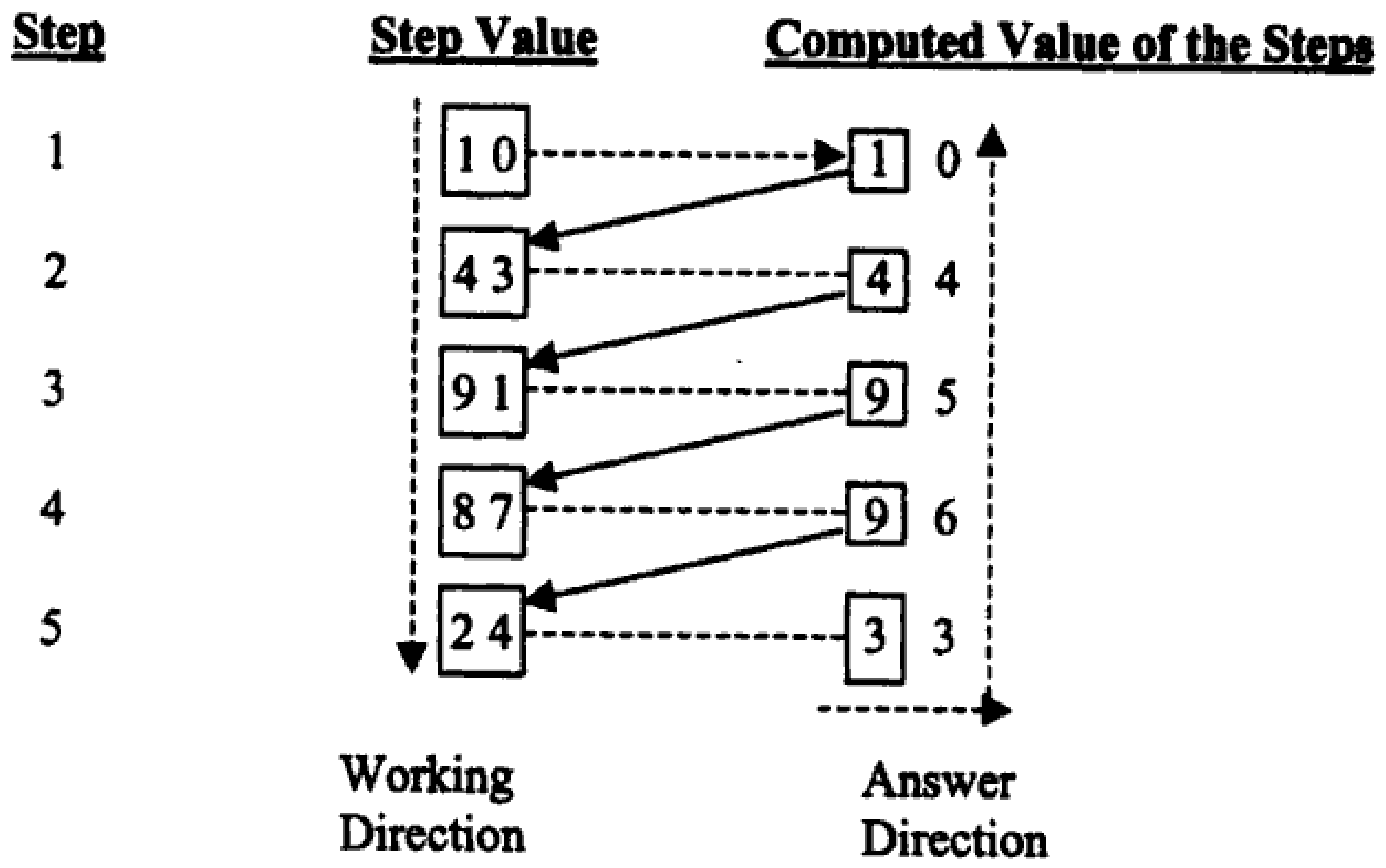
2nd column multiplication (en-block)

Step 5:

$$\begin{array}{ccc} 8 & 5 & 2 \\ \uparrow & & \\ 3 & 9 & 5 \end{array} \quad = 24$$

3rd column multiplication (Urdhva only)

Steps:



Ans.: 336540

Fig.2

Figure 3 and 4 explain the details of the five and eight digit number multiplication respectively.

c) Five-digit Numbers Multiplication:

Example: **23657 × 41893**

Current Method

23657
41893
70971
212913
189256
23657
94628
991062701

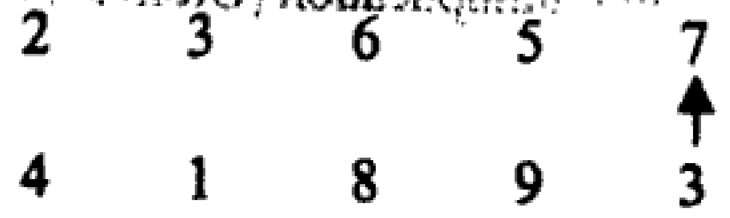
Vedic Method

2 3 6 5 7
4 1 8 9 3
$\boxed{9\ 9\ 1\ 0\ 6\ 2\ 7\ 0\ 1}$
1 5 8 12 12 12 8 2

Ans.: = 991062701

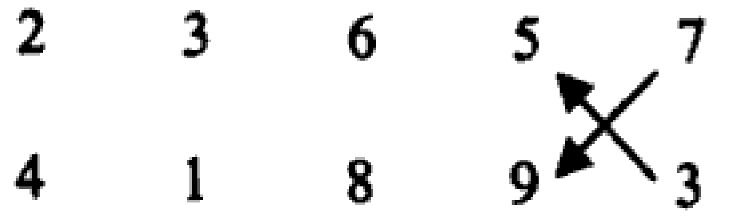
Step Diagrams:

Step 1:



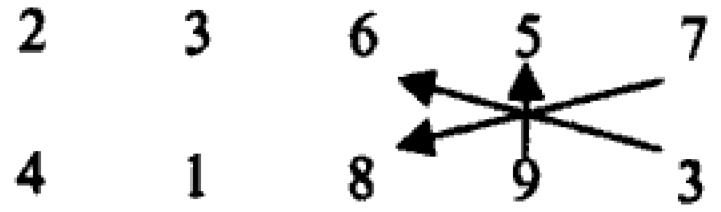
= 21

Step 2:



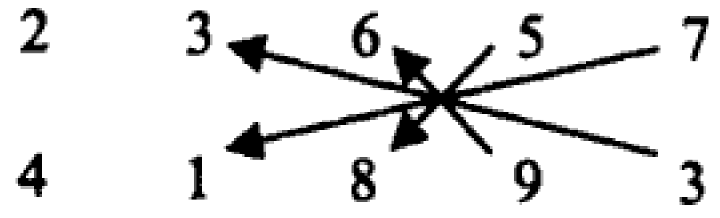
= 78

Step 3:



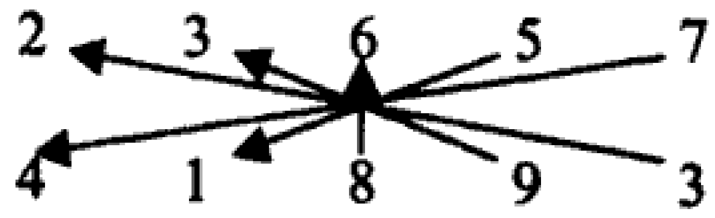
= 119

Step 4:



= 110

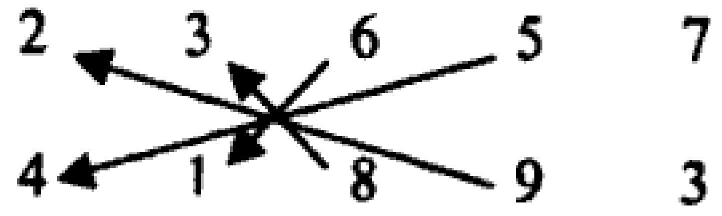
Step 5:



= 114

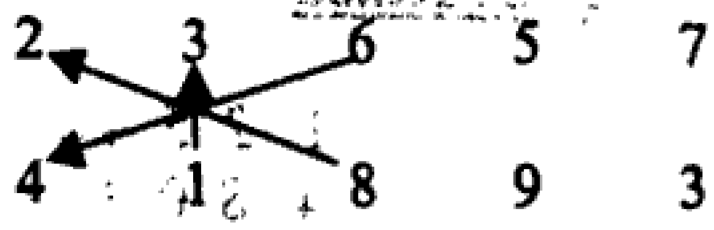
Steps 1 to 5 constitute the completion of 1st column multiplication.

Step 6:



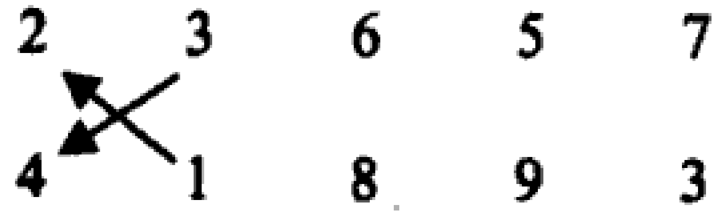
= 68 2nd column multiplication (en-block)

Step 7:



= 43 3rd column multiplication (en-block)

Step 8:



= 14 4th column multiplication (en-block)

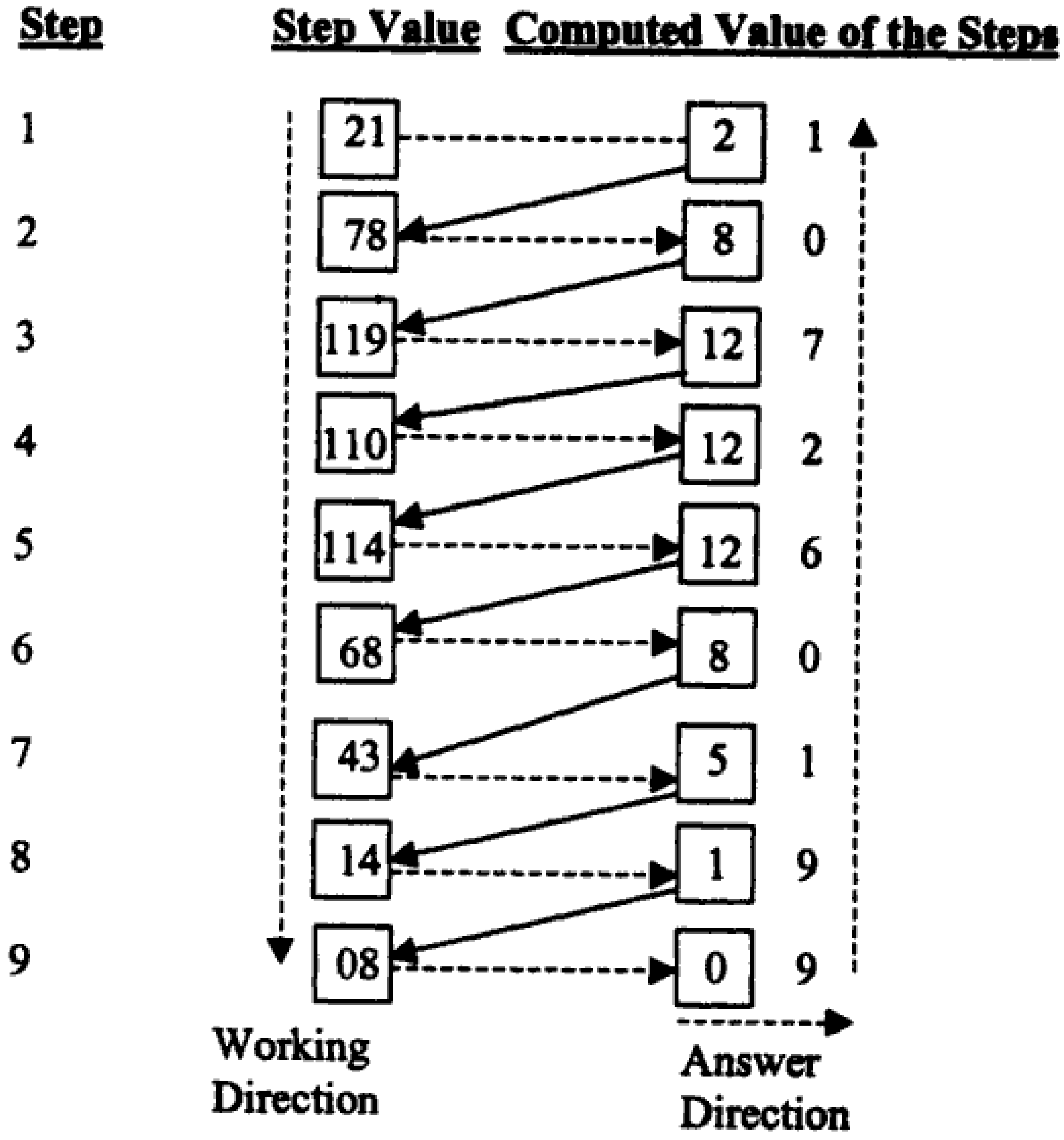
Step 9:

2 3 6 5 7

↑ = 08 5th column multiplication (Urdhva only)

4 1 8 9 3

Steps:



Ans.: 991062701

Fig.3

d) Eight-digit Numbers Multiplication:

Example: **12546873 × 48032162**

Current Method

```
        12546873
        48032162
       25093746
      75281238
     12546873
     5093746
   37640619
  00000000
 100374984
 50187492
 602653436529426
```

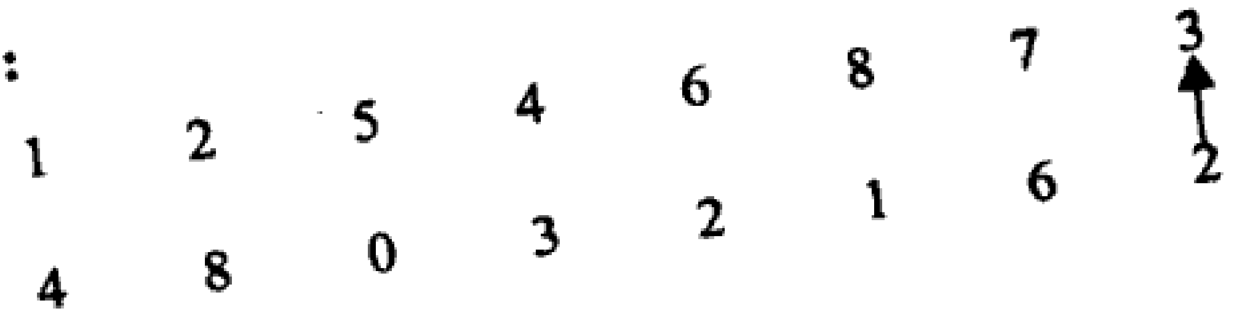
Vedic Method

```
        1 2 5 46 8 7 3
        4 8 0 3 2 1 6 2
  6 0 2 6 5 3 4 3 6 5 2 9 4 2 6
  2 4 6 7 11 13 12 10 8 8 7 6 3
```

Ans.: 6 0 2 6 5 3 4 3 6 5 2 9 4 2 6

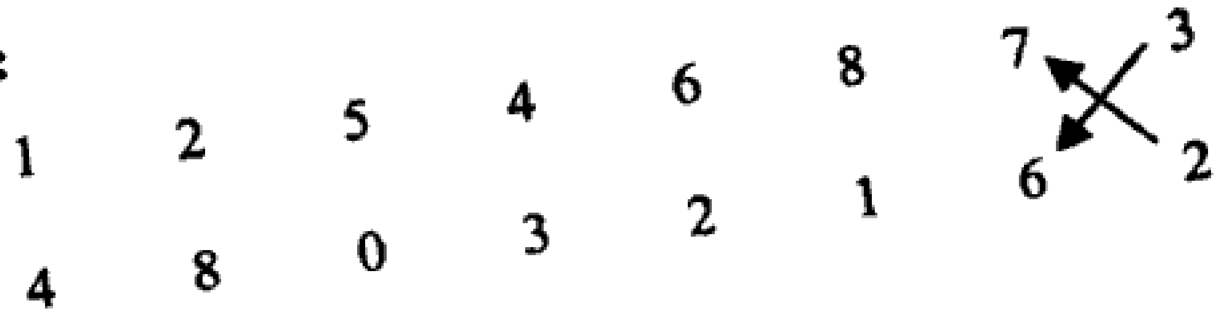
Step Diagrams:

Step 1:



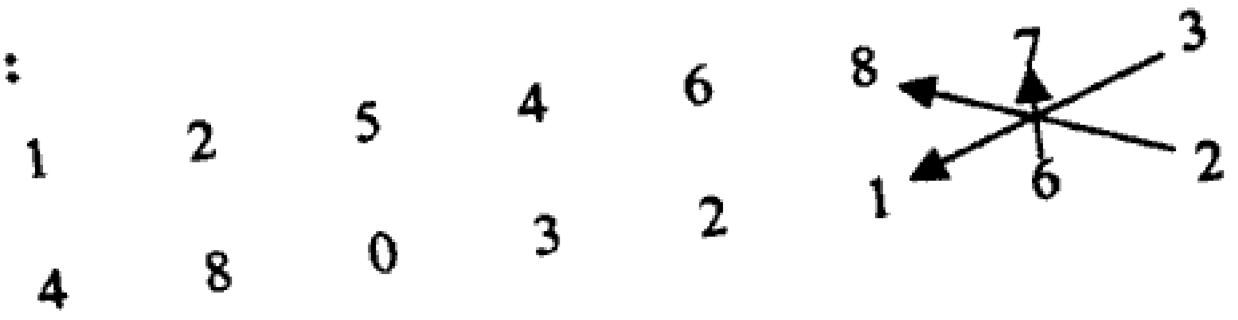
= 06

Step 2:



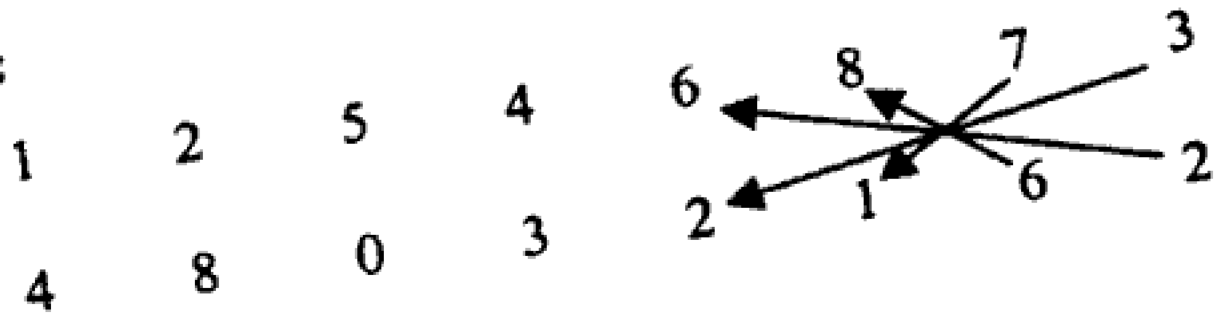
= 32

Step 3:



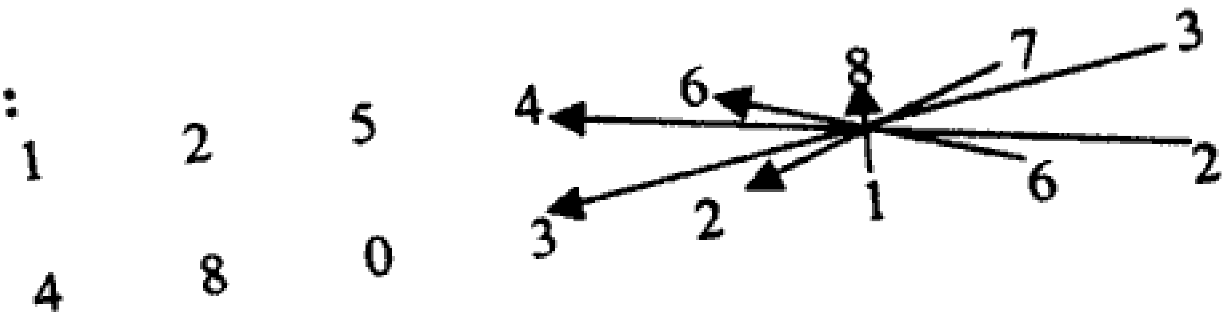
= 61

Step 4:



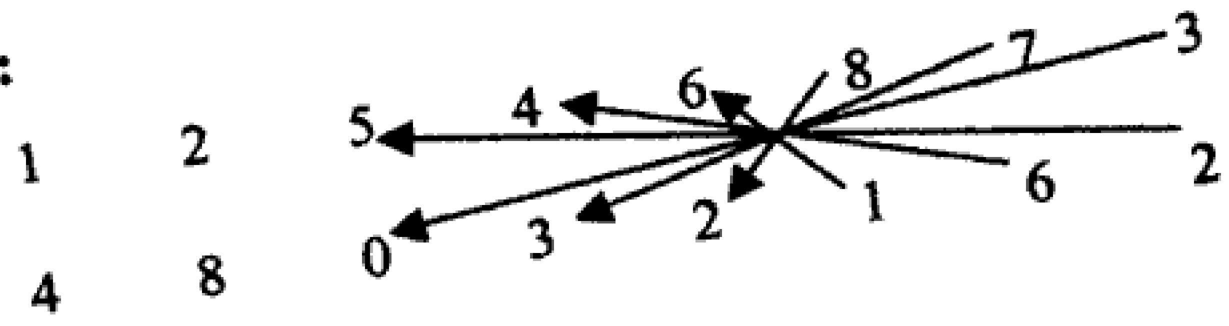
= 73

Step 5:



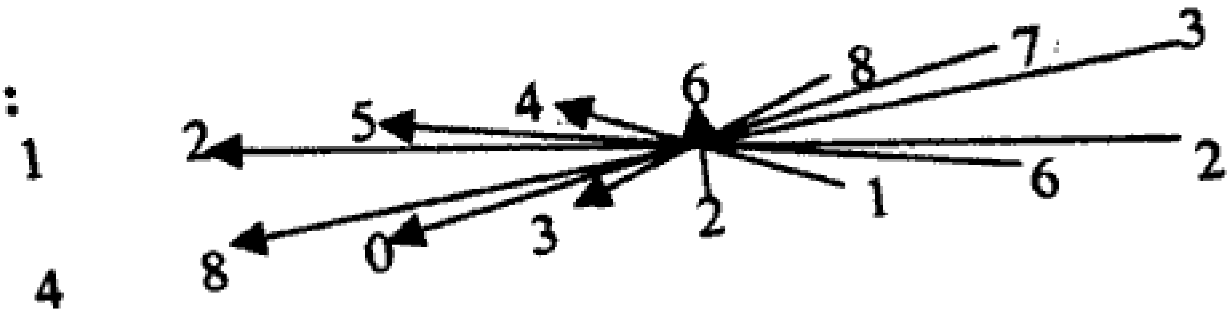
= 75

Step 6:



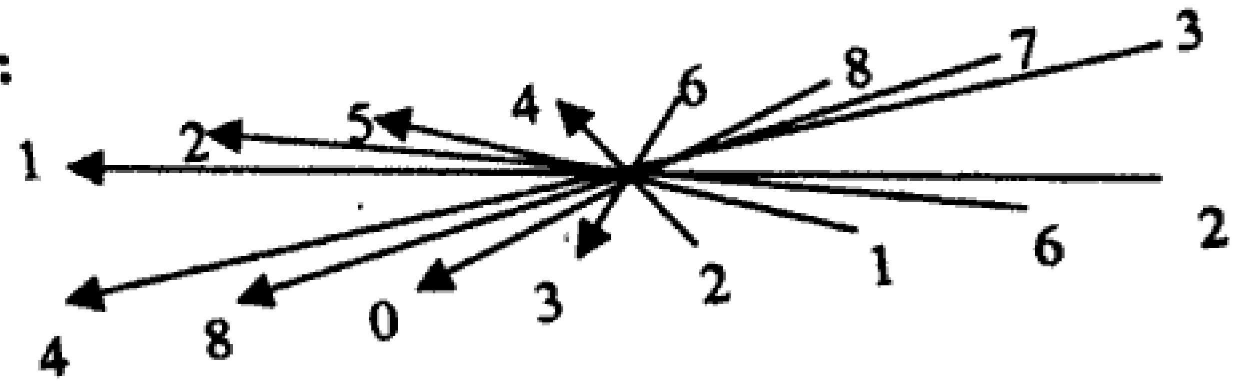
= 77

Step 7:



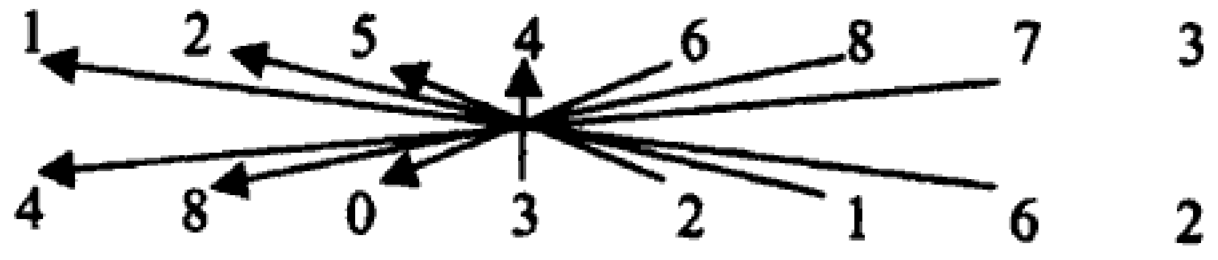
= 98

Step 8:



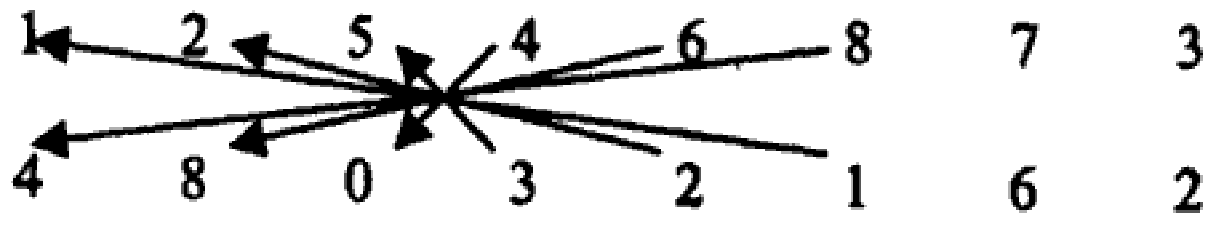
= 113

Step 9:



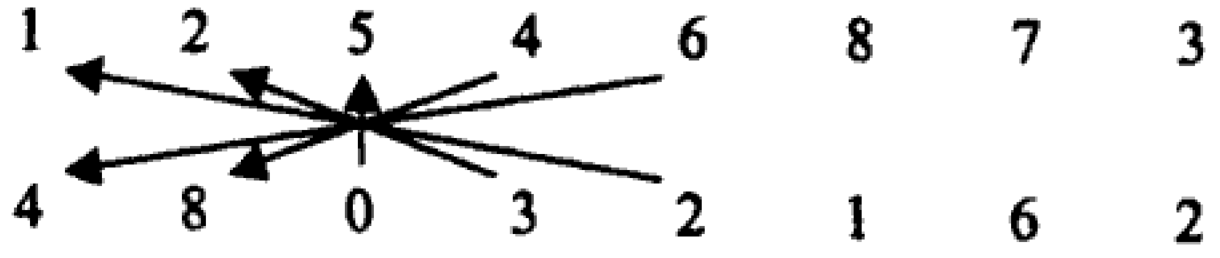
= 122

Step 10:



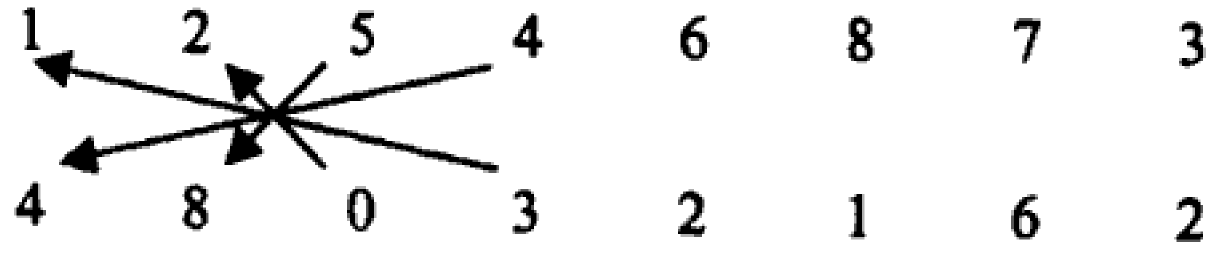
= 100

Step 11:



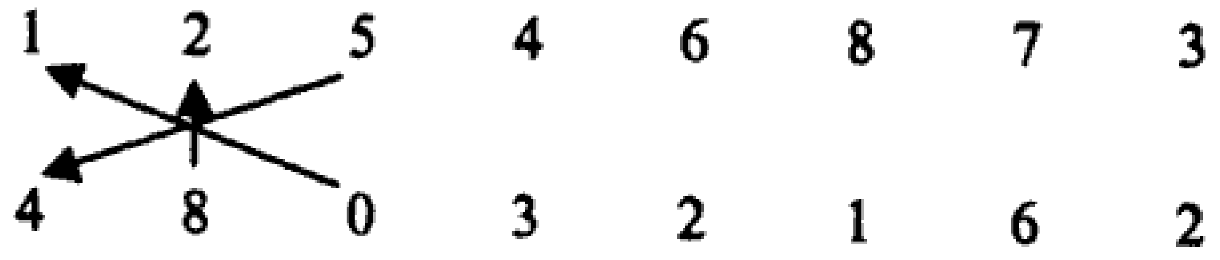
= 64

Step 12:



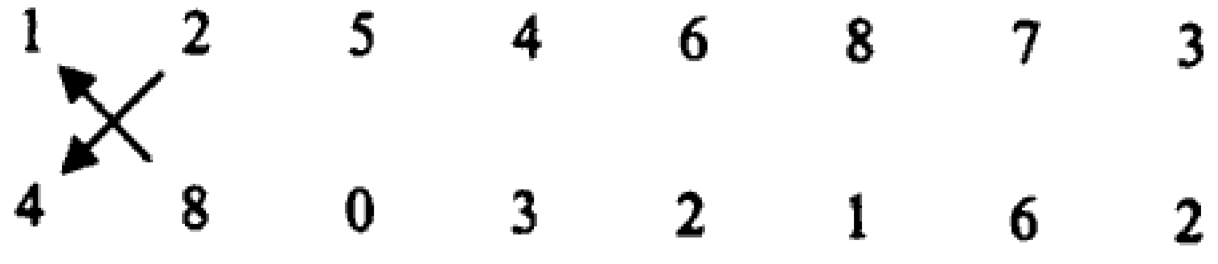
= 59

Step 13:



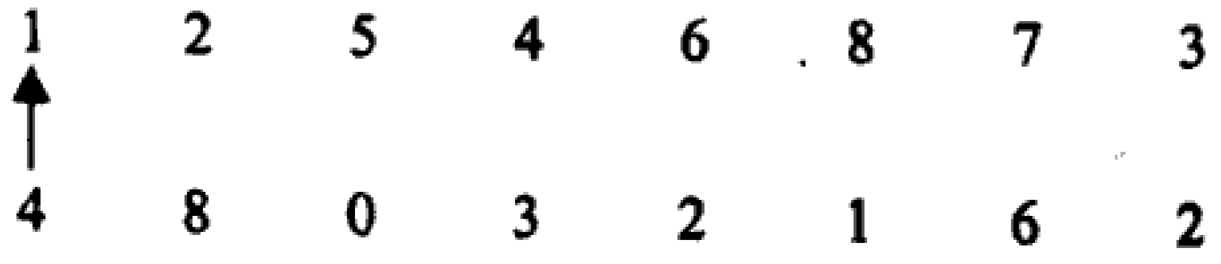
= 36

Step 14:



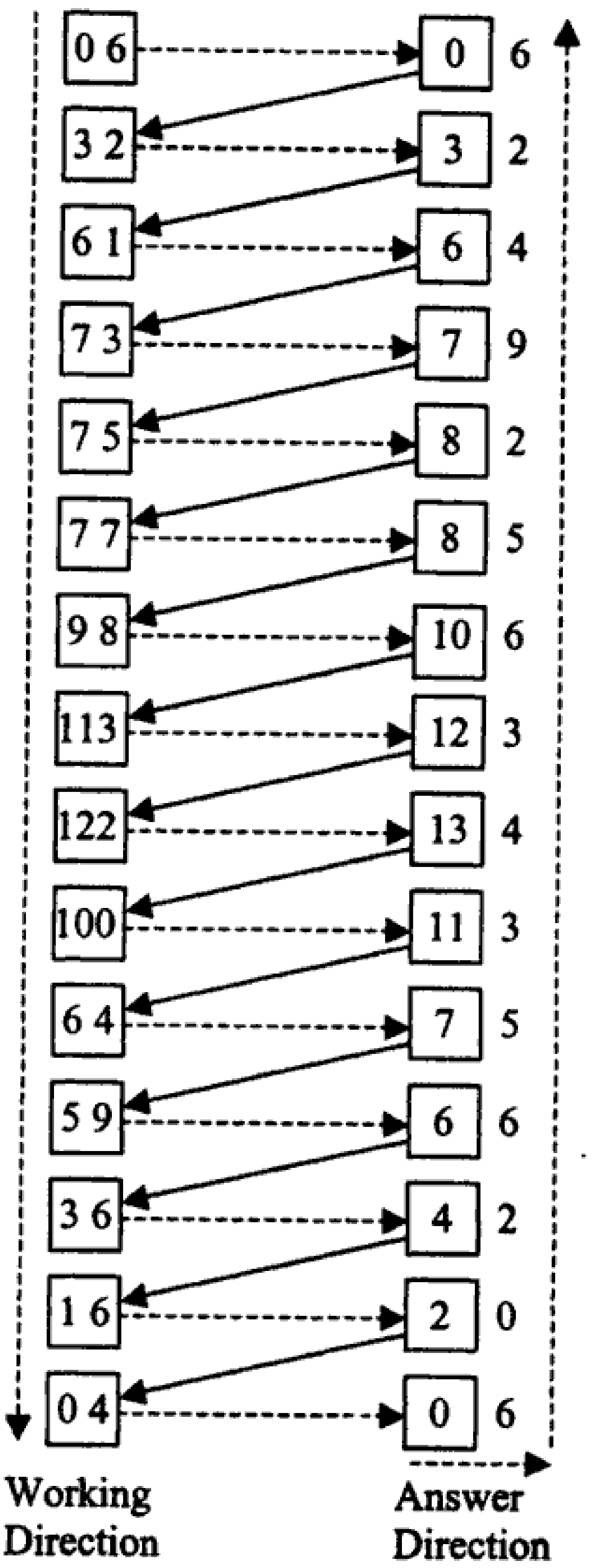
= 16

Step 15:



= 04

Steps:



Step	Step Value	Computed Value of the Steps	
1	0 6	0	6
2	3 2	3	2
3	6 1	6	4
4	7 3	7	9
5	7 5	8	2
6	7 7	8	5
7	9 8	10	6
8	113	12	3
9	122	13	4
10	100	11	3
11	6 4	7	5
12	5 9	6	6
13	3 6	4	2
14	1 6	2	0
15	0 4	0	6

Ans.: 602653436529426

Fig.4

In general, the method is applicable to multiplication of any number of digits by any number of digits.

In the stepwise method, one considers the results of various steps being worked out separately, which has perfect symmetry in working and much easier to remember in placing the results. The ease with which the Vedic Method can be worked out has to be experienced by each individual worker.

Proof:

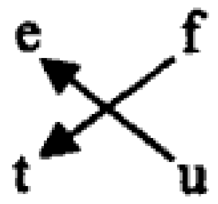
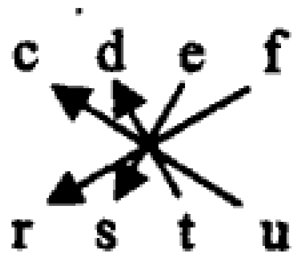
Algebrical proof for the multiplication using Urdhva Tiryagbhyam Sutram is as follows:

Suppose the polynomial form of given two numbers are $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ and $px^5 + qx^4 + rx^3 + sx^2 + tx + u$ respectively where $x = 10$ is the base.

$(ax^5 + bx^4 + cx^3 + dx^2 + ex + f)$ (Multiplicand)
$(px^5 + qx^4 + rx^3 + sx^2 + tx + u)$ (Multiplier)

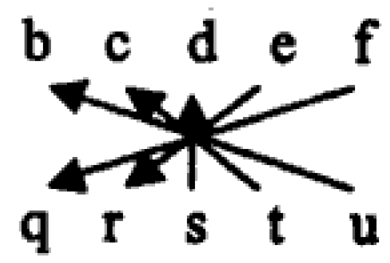
Result is:

$pax^{10} + (qa + pb)x^9 + (ra + pc + qb)x^8 + (sa + pd + rb + qc)x^7 + (ta + pe + sb + qd + rc)x^6 + (ua + pf + tb + qe + sc + rd)x^5 + (ub + qf + tc + re + sd)x^4 + (uc + rf + td + se)x^3 + (ud + sf + te)x^2 + (ue + tf)x + uf$

Current Method	**Vedic Method**
Multiplication is multiplying the terms of the multiplicand by each term of the multiplier and then collecting the coefficients of the powers of x.	Multiplication is by applying Urdhva Tiryagbhyam Sutram
1. Constant Term uf	≡  = uf
2. Coefficient of x ue + tf	≡  = ue + tf
3. Coefficient of x^2 ud + sf + te	≡  = ud + sf + te
4. Coefficient of x^3 uc + rf + td + se	≡  = uc + rf + td + se

5. Coefficient of x^4

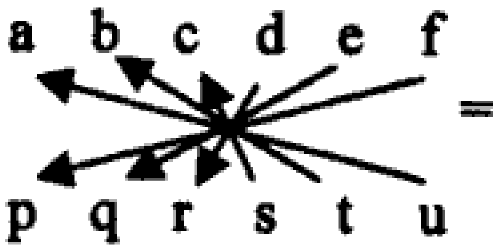
$ub + qf + tc + re + sd \equiv$



$= ub + qf + tc + re + sd$

6. Coefficient of x^5

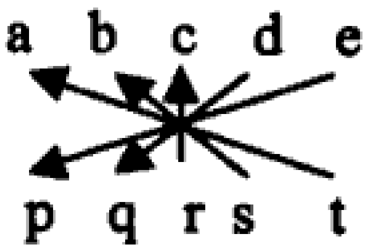
$ua + pf + tb + qe + sc + rd \equiv$



$= ua + pf + tb + qe + sc + rd$

7. Coefficient of x^6

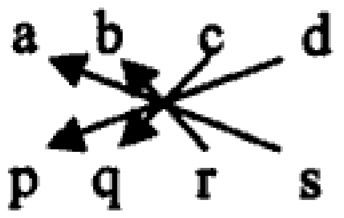
$ta + pe + sb + qd + rc \equiv$



$= ta + pe + sb + qd + rc$

8. Coefficient of x^7

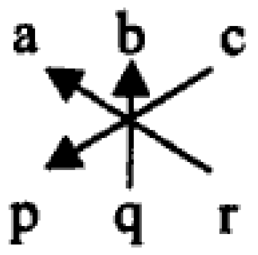
$sa + pd + rb + qc \equiv$



$= sa + pd + rb + qc$

9. Coefficient of x^8

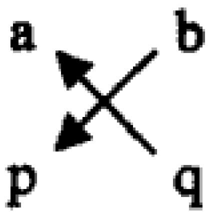
$ra + pc + qb \equiv$



$= ra + pc + qb$

10. Coefficient of x^9

$qa + pb \equiv$



$= qa + pb$

11. Coefficient of x^{10}

$pa \equiv$

$= pa$

Both results are same confirming the Urdhva Tiryagbhyam mode of Multiplication.

ii) Left to Right Multiplication (V.M.):

It is surprisingly clear that the Vedic method (VM) is applicable with the same ease when the multiplication is carried out from left to right. This type of left to right multiplication is not explained or not in vogue in the current system (for number). This is explained using a two digit number multiplication, which is extendable to any number of digits. There are two methods, which are denoted as VM.1 and VM.2.

a) Two-digit Numbers Multiplication:

Example: **28 × 34**

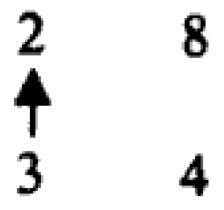
Current Method

Vedic Method 1
(V.M.1)

2 8
3 4
6 2 2 → First Line
3 3
9 5 2| Ans.: 952

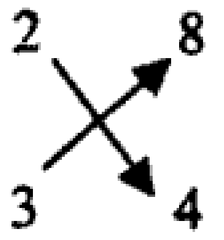
Step diagrams:

Step 1:



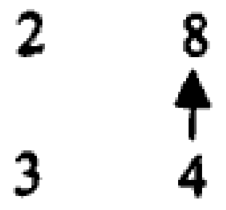
= 06

Step 2:



= 32

Step 3:



= 32

V.M.1:

This method works in two lines. In the first step, the first column in the left is considered for vertical multiplication (Urdhva) and the result (6) is put below, as it is in the first line.

In the second step, the first column (in the left) is multiplied with the second column crosswise. The results are added together, which gives 32. The first digit of this result, i.e., 2, is placed along the first line and remaining digit (3) are placed below the result of the first column multiplication. The last step is to multiply the last column vertically and the placement of this result is similarly followed. Now an addition is required for final result.

Current Method

Vedic Method 2 (One Line Method)

2 8
3 4
$\boxed{0\ \ 9\ \ 52}$ → Answer
6 2 → Carrying for Addition after multiplying with 10
Ans.: 952

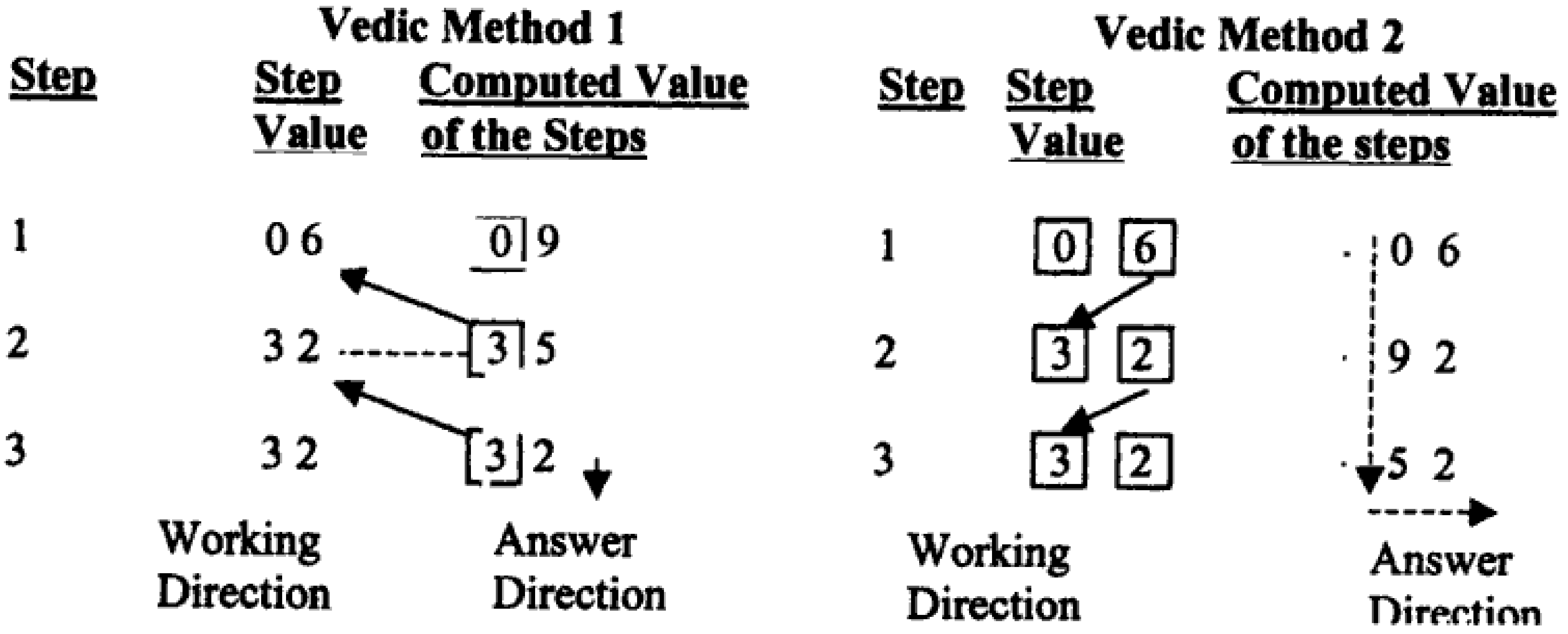
V.M.2:

There is also another way of getting the result in Vedic Method as shown in Vedic Method 2.

In this method step values are same as that obtained in V.M.1, but the computed values are obtained by different procedure. The first step value and its corresponding computed value are the same, i.e., 6. The first digit (6) of the first step is used for carrying the addition, where as the other digits are placed in the answer line and hence, 6 is placed in the carrying digit allotment. Coming to the calculation of the computed value of the second step, which is 32, the first digit remains as it is, whereas the other digit(s) (3) are added to the first digit of the first step (6), in tune with the status of the digits giving the computed value as 92. The first digit of this computed value, i.e., 2, is used for carrying the addition and the rest of the digits (9) are placed in the answer line. This type of placement is to be continued. The procedure is followed for the other steps until the last step, which ends with vertical multiplication. The result of this vertical multiplication is added as in the previous step, but is kept along the line of answer. This also gives one line method as in the case of Right to Left multiplication. This can be viewed also as

1st Step: $2 \times 3 = 06$
2nd Step: $6{\times}10 + 32 = 92$
3rd Step: $2{\times}10 + 32 = 52$
The result as read from the steps is shown in Fig 5.

Steps:



Answer from the Computed Values can be read from last digits
Ans.: 952

Answer from the Final Values can be read from first digits.

Fig.5

The three-digit, five-digit and eight-digit numbers multiplications are shown in figures 6, 7 and 8 respectively.

b) Three-digit Numbers Multiplication:

Example: **852 × 395**

Current Method

```
        8 5 2
        3 9 5

      4 2 6 0
    7 6 6 8
  2 5 5 6
  ___________
  3 3 6 5 4 0
```

Vedic Method 1

```
 8  5  2
 3  9  5
 ______________
24  7  1  3  0
 8  9  4  1
|33  6  5  4  0|
```

Vedic Method 2

```
 8  5  2
 3  9  5
|2 12 16 5 40|
  4  7  1 3
```

Ans.: 336540

V.M.2:

If we get in the answer, two digits as a unit as is clear in the above three digit numbers multiplication, ie.,

2 12 16 5 40 = (1+2) (1+2) 6540 = 336540
└┘ └┘

The answer needs to be read from right to left as 336540.

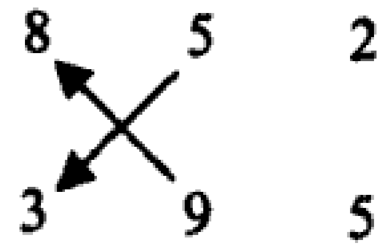
This is in keeping with the usual procedure adopted through out the Vedic Mathematical Computations based on status of the digits.

Step Diagrams:

Step 1:

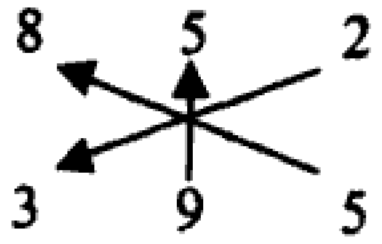
```
8     5     2
↑                 = 24
3     9     5
```

Step 2:



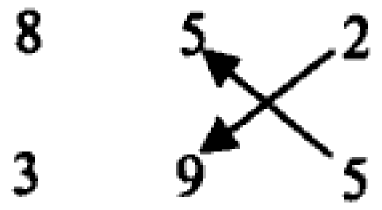
= 87

Step 3:



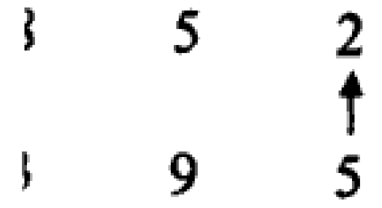
= 91

Step 4:



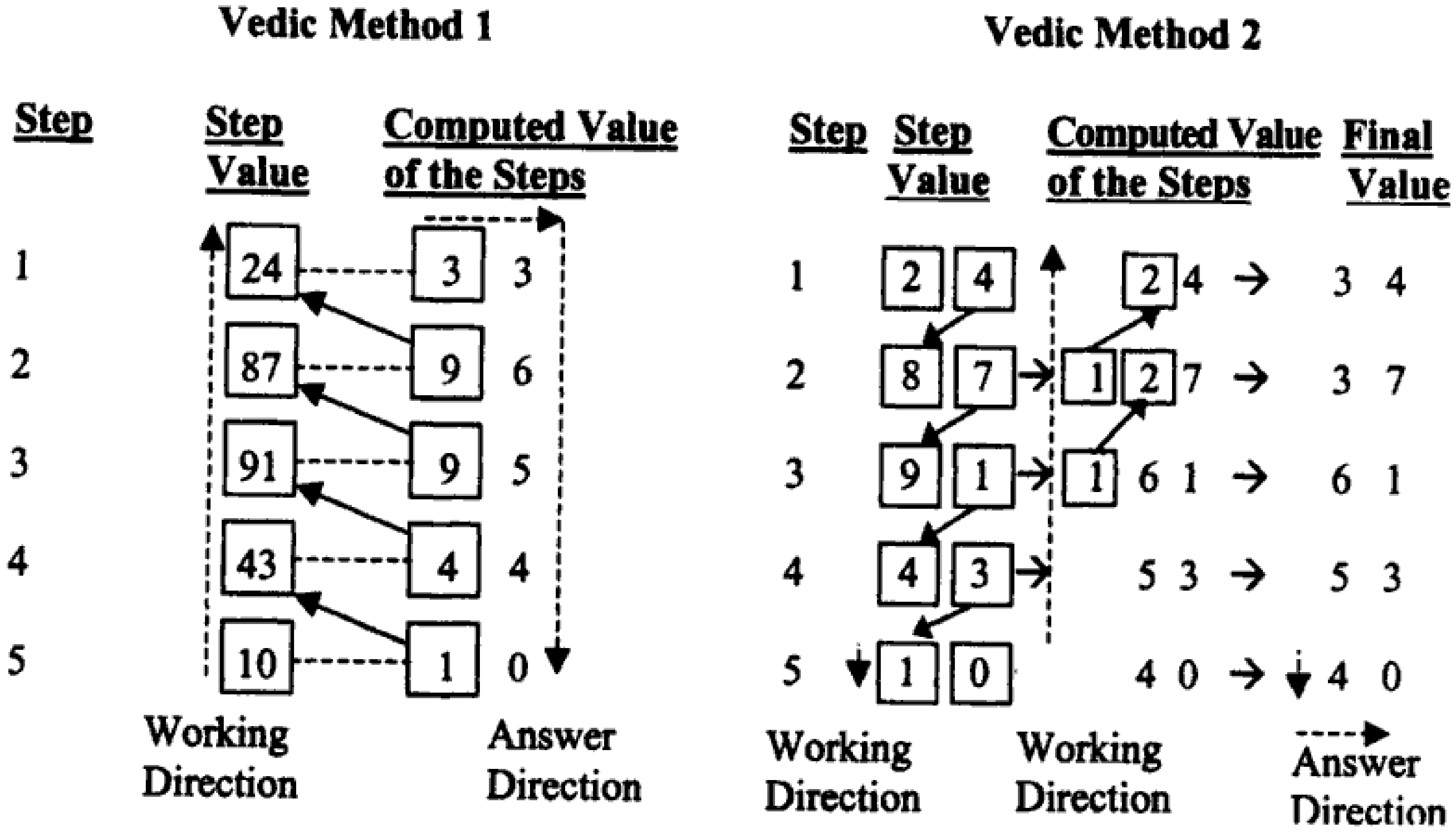
= 43

Step 5:



= 10

Steps:



Vedic Method 1

Step	Step Value	Computed Value of the Steps
1	24	3 3
2	87	9 6
3	91	9 5
4	43	4 4
5	10	1 0

Vedic Method 2

Step	Step Value	Computed Value of the Steps	Final Value
1	2 4	2 4 →	3 4
2	8 7	1 2 7 →	3 7
3	9 1	1 6 1 →	6 1
4	4 3	5 3 →	5 3
5	1 0	4 0 →	4 0

Ans.: 336540

Fig.6

c) Five-digit Numbers Multiplication:

Example: *23657 × 41893*

Current Method

					2	3	6	5	7
				x	4	1	8	9	3
					7	0	9	7	1
			2	1	2	9	1	3	
		1	8	9	2	5	6		
		2	3	6	5	7			
	9	4	6	2	8				
	9	9	1	0	6	2	7	0	1

Vedic Method 1

2	3	6	5	7				
4	1	8	9	3				
8	4	3	8	4	0	9	8	1
1	4	6	11	11	11	7	2	
9	9	1	0	6	2	7	0	1

Vedic Method 2

2 3 6 5 7
4 1 8 9 3
0 9 8 9 19 15 11 16 101
8 4 3 8 4 0 9 8

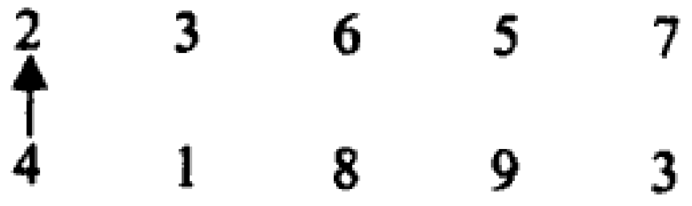
Ans.: 0 9 8 9 (19) (15) (11) (16) (101)

= 9 8 (10) (10) 6 2 7 0 1

= 9 9 1 0 6 2 7 0 1

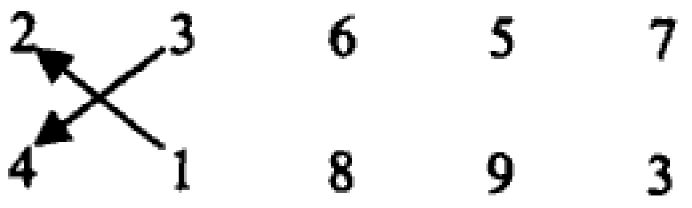
Step Diagrams:

Step 1:



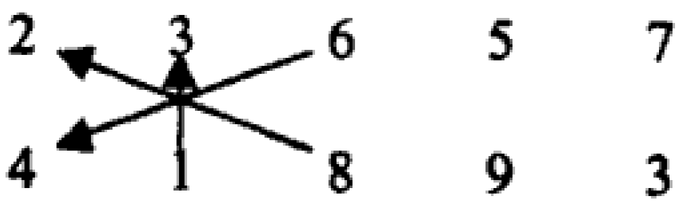
= 08

Step 2:



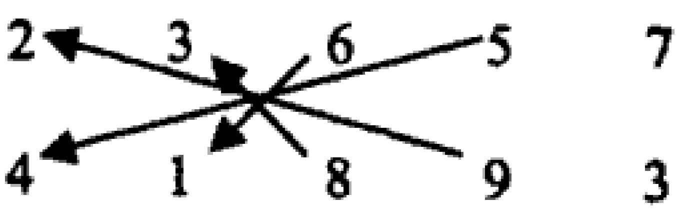
= 14

Step 3:



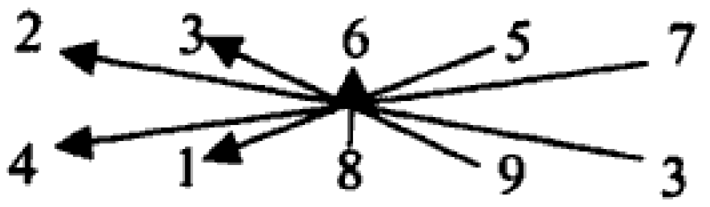
= 43

Step 4:



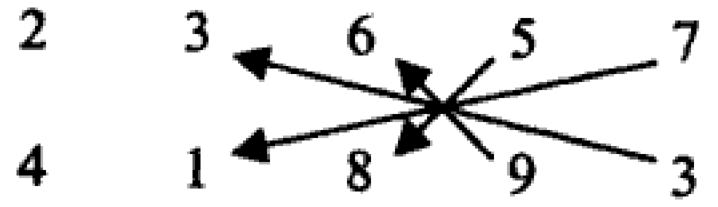
= 68

Step 5:



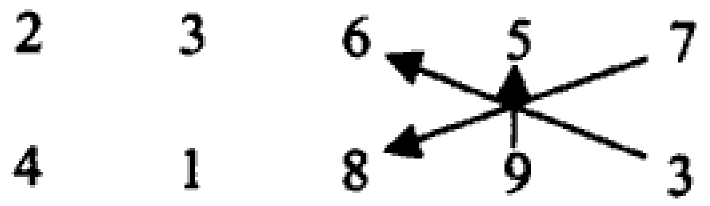
= 114

Step 6:



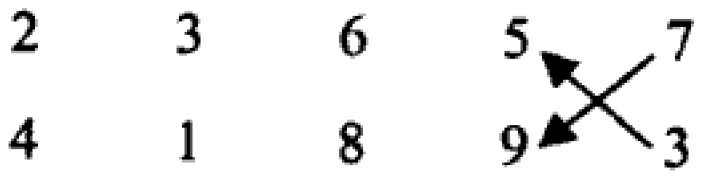
= 110

Step 7:



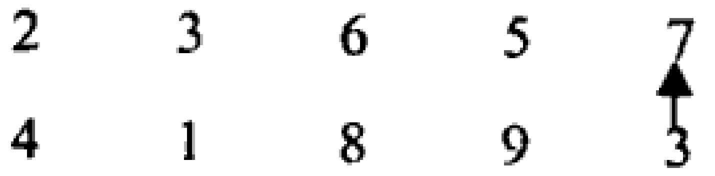
= 119

Step 8:



= 78

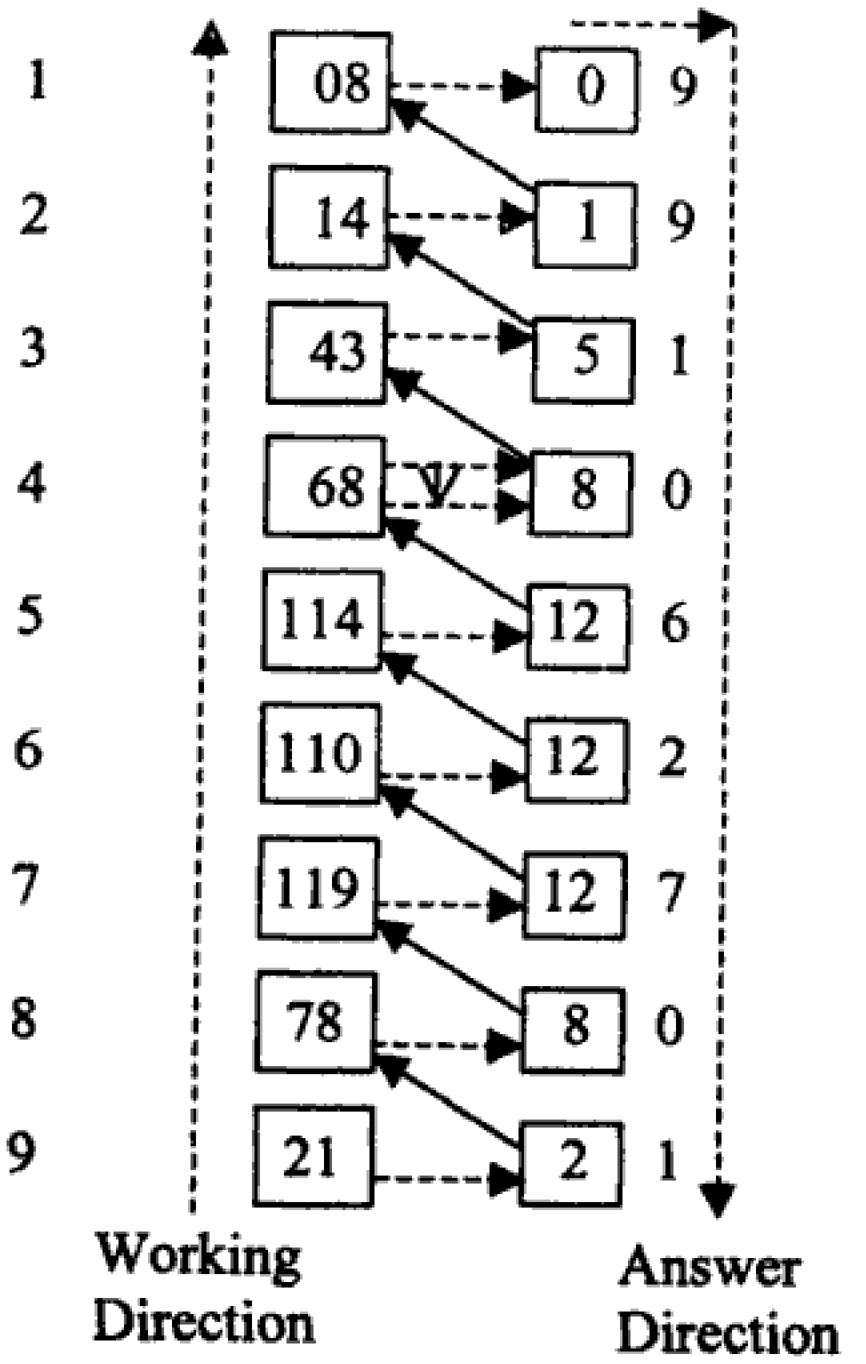
Step 9:



= 21

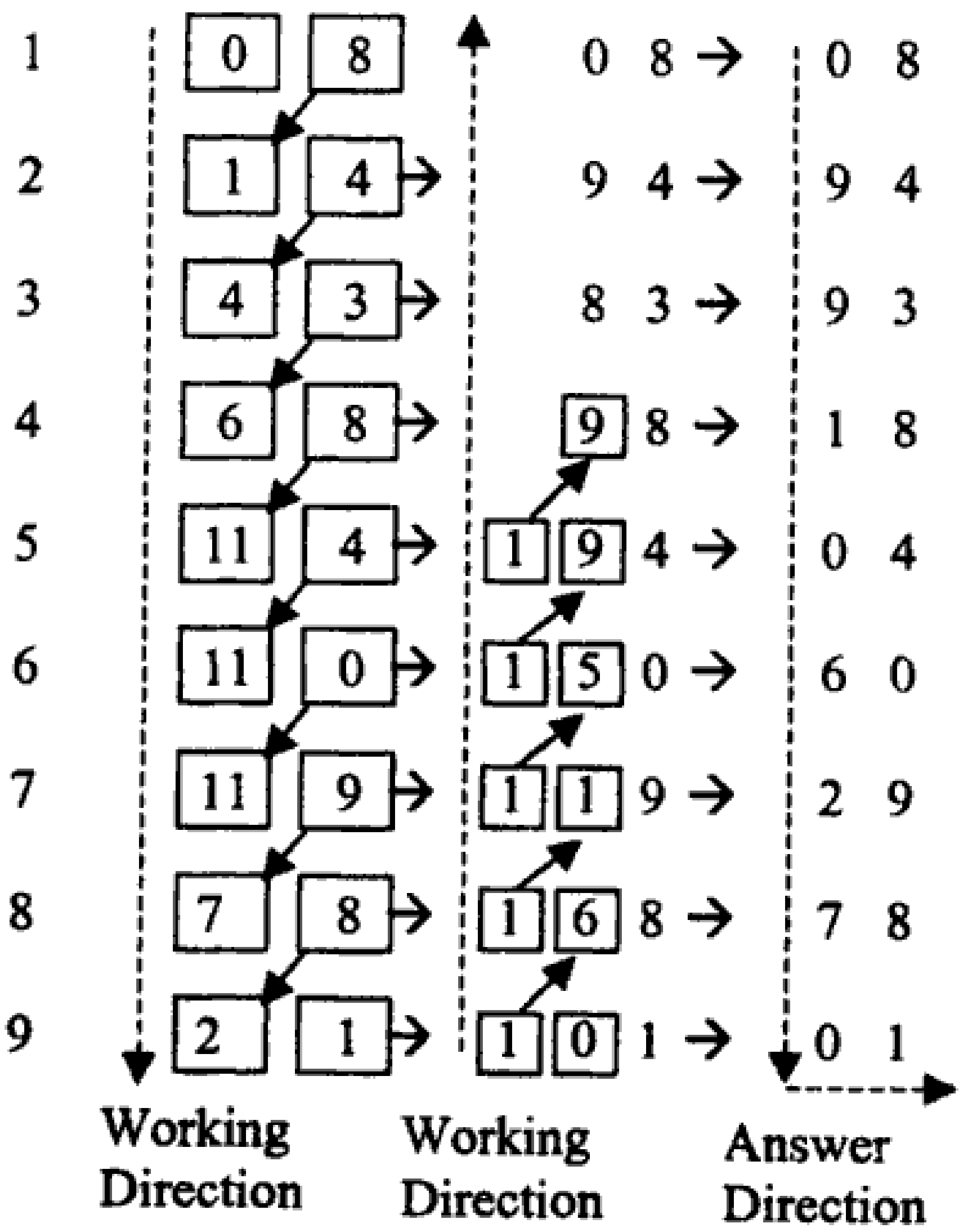
Steps:

Vedic Method 1



Step	Step Value	Computed Value of the Steps	
1	08	0	9
2	14	1	9
3	43	5	1
4	68	8	0
5	114	12	6
6	110	12	2
7	119	12	7
8	78	8	0
9	21	2	1

Vedic Method 2



Step	Step Value		Computed Value	Final Value
1	0	8	0 8 →	0 8
2	1	4	9 4 →	9 4
3	4	3	8 3 →	9 3
4	6	8	9 8 →	1 8
5	11	4	1 9 4 →	0 4
6	11	0	1 5 0 →	6 0
7	11	9	1 1 9 →	2 9
8	7	8	1 6 8 →	7 8
9	2	1	1 0 1 →	0 1

Fig: 7

Ans: 991062701

d) Eight-digit Numbers Multiplication:

Example: **12546873 × 48032162**

Current Method

Vedic Method 1

1 2 5 4 6 8 7 3
4 8 0 3 2 1 6 2
4 6 6 9 4 0 2 3 8 7 5 3 1 2 6
1 3 5 6 10 12 11 9 7 7 7 6 3 0
$\boxed{6\ 0\ 2\ 6\ 5\ 3\ 4\ 3\ 6\ 5\ 2\ 9\ 4\ 2\ 6}$

Current Method

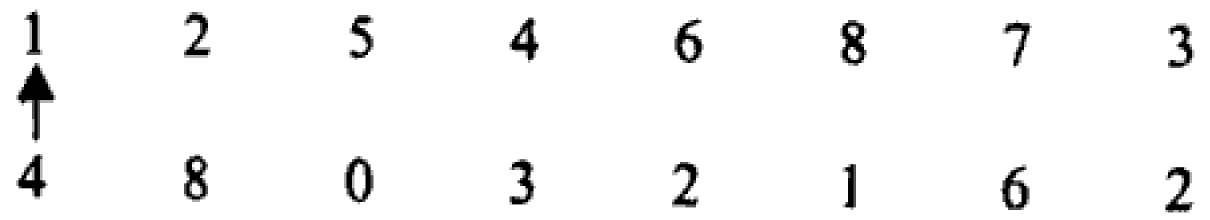
Vedic Method 2

1 2 5 4 6 8 7 3
4 8 0 3 2 1 6 2
$\boxed{0\ 5\ 9\ 11\ 15\ 14\ 12\ 13\ 12\ 15\ 14\ 12\ 9\ 4\ 26}$
4 6 6 9 4 0 2 3 8 7 5 3 1 2

Ans.: 602653436529426

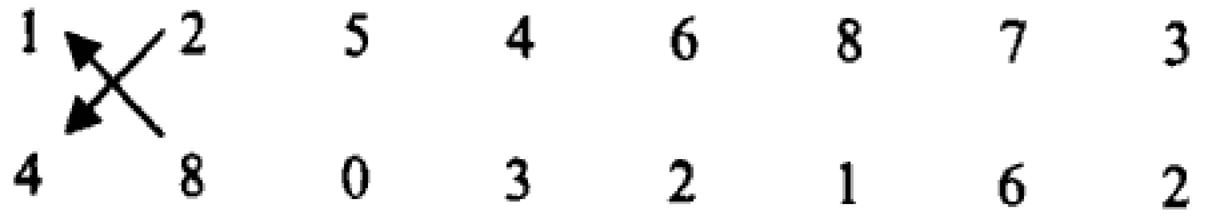
Step Diagrams:

Step 1:



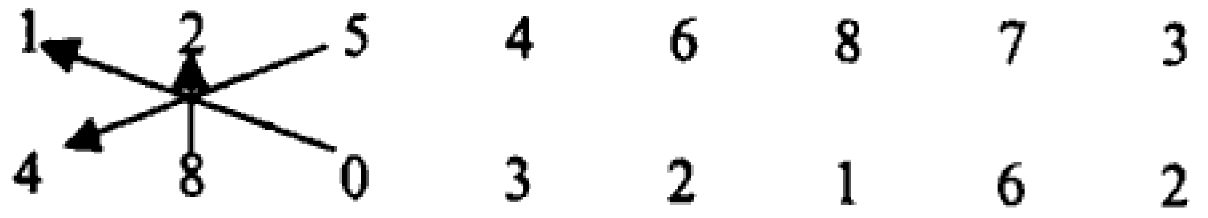
= 04

Step 2:



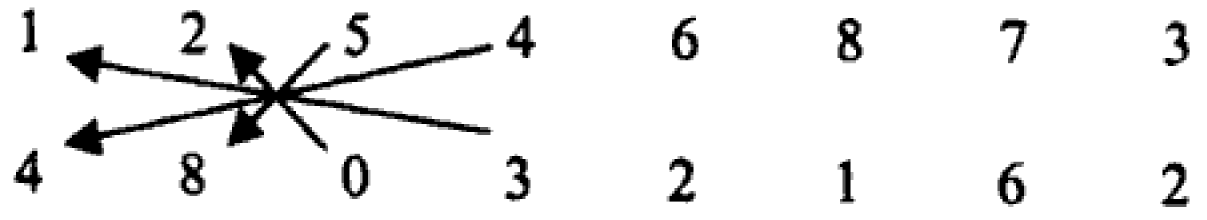
= 16

Step 3:



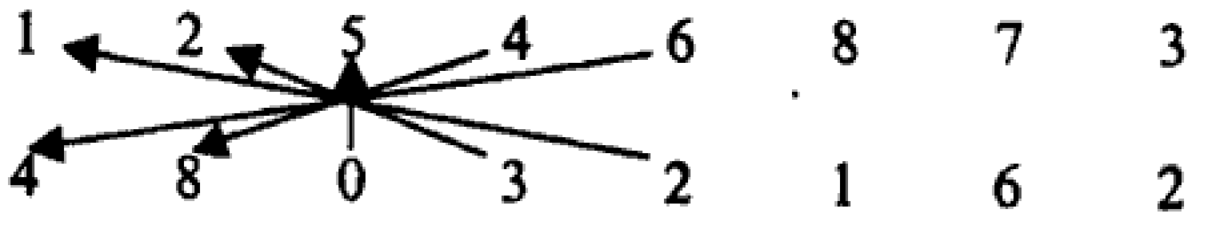
= 36

Step 4:



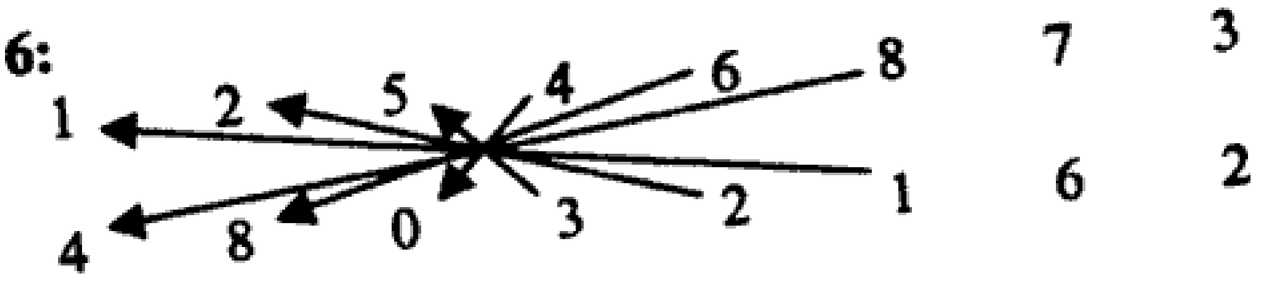
= 59

Step 5:



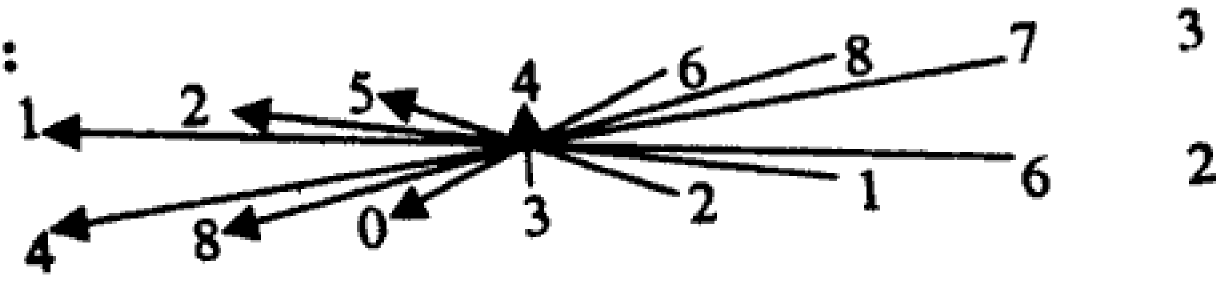
= 64

Step 6:



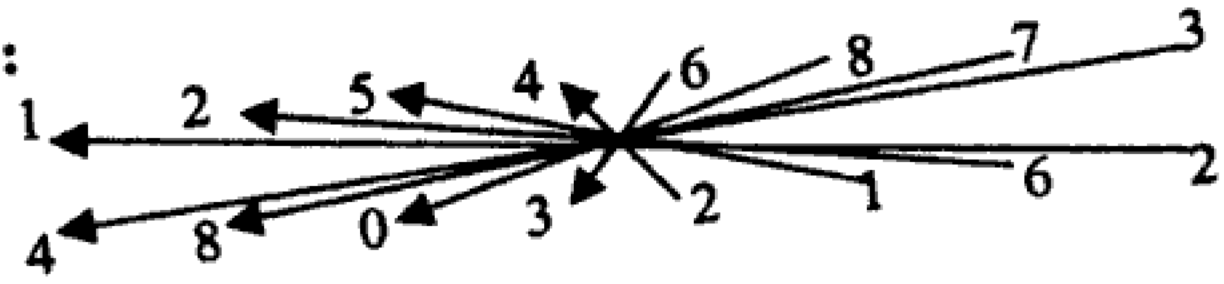
= 100

Step 7:



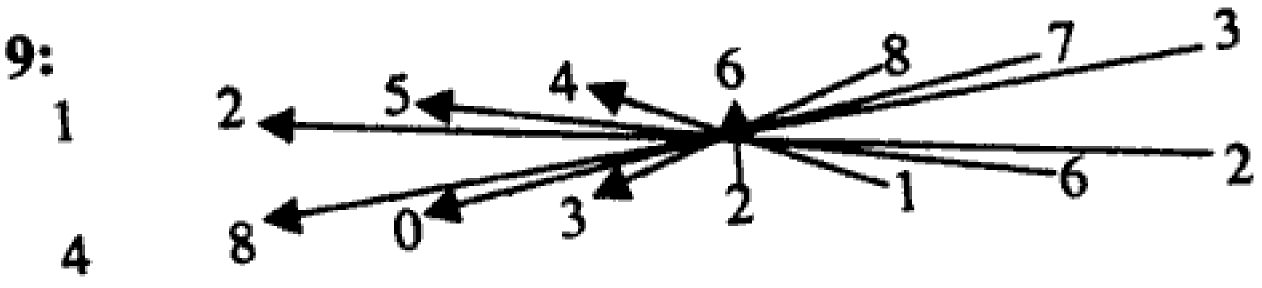
= 122

Step 8:



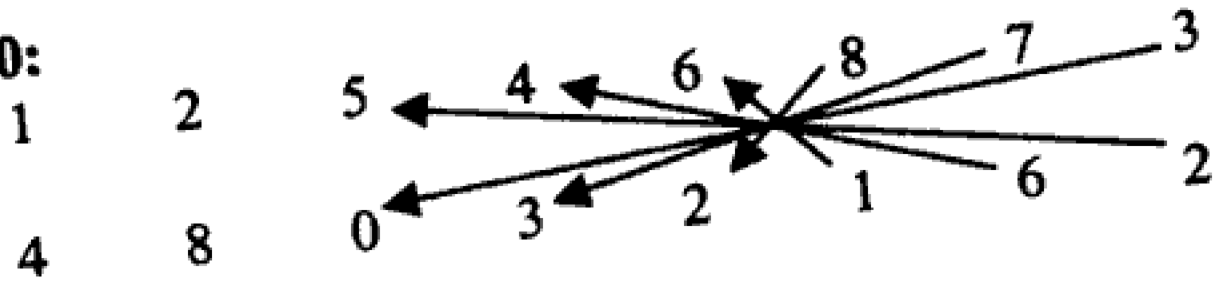
= 113

Step 9:



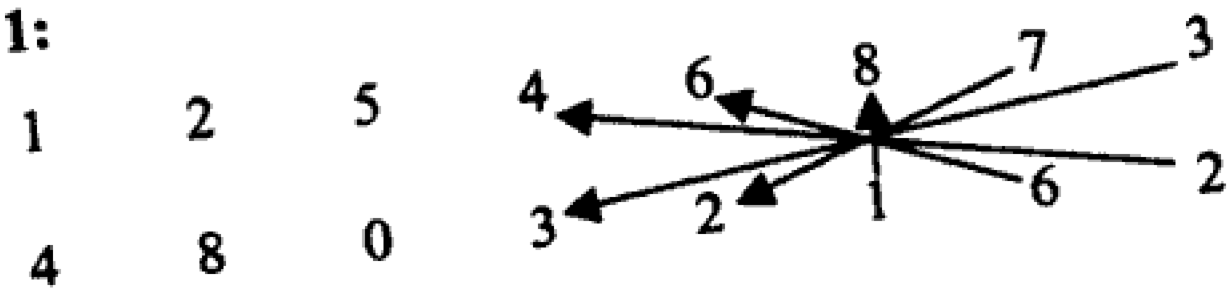
= 98

Step 10:



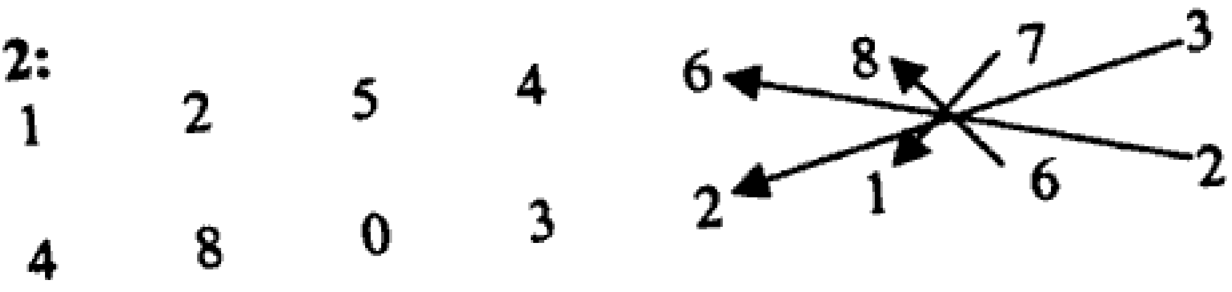
= 77

Step 11:



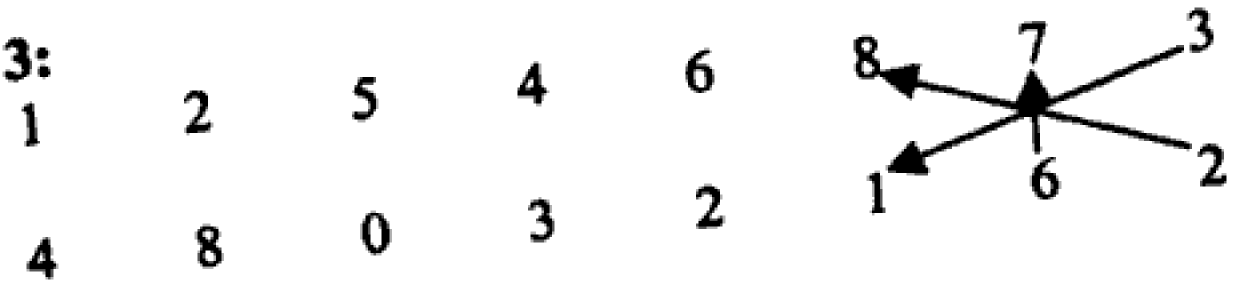
= 75

Step 12:



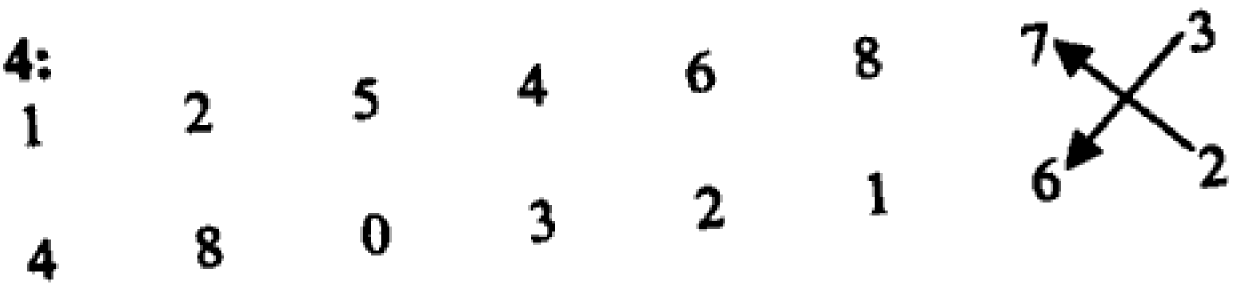
= 73

Step 13:



= 61

Step 14:



= 32

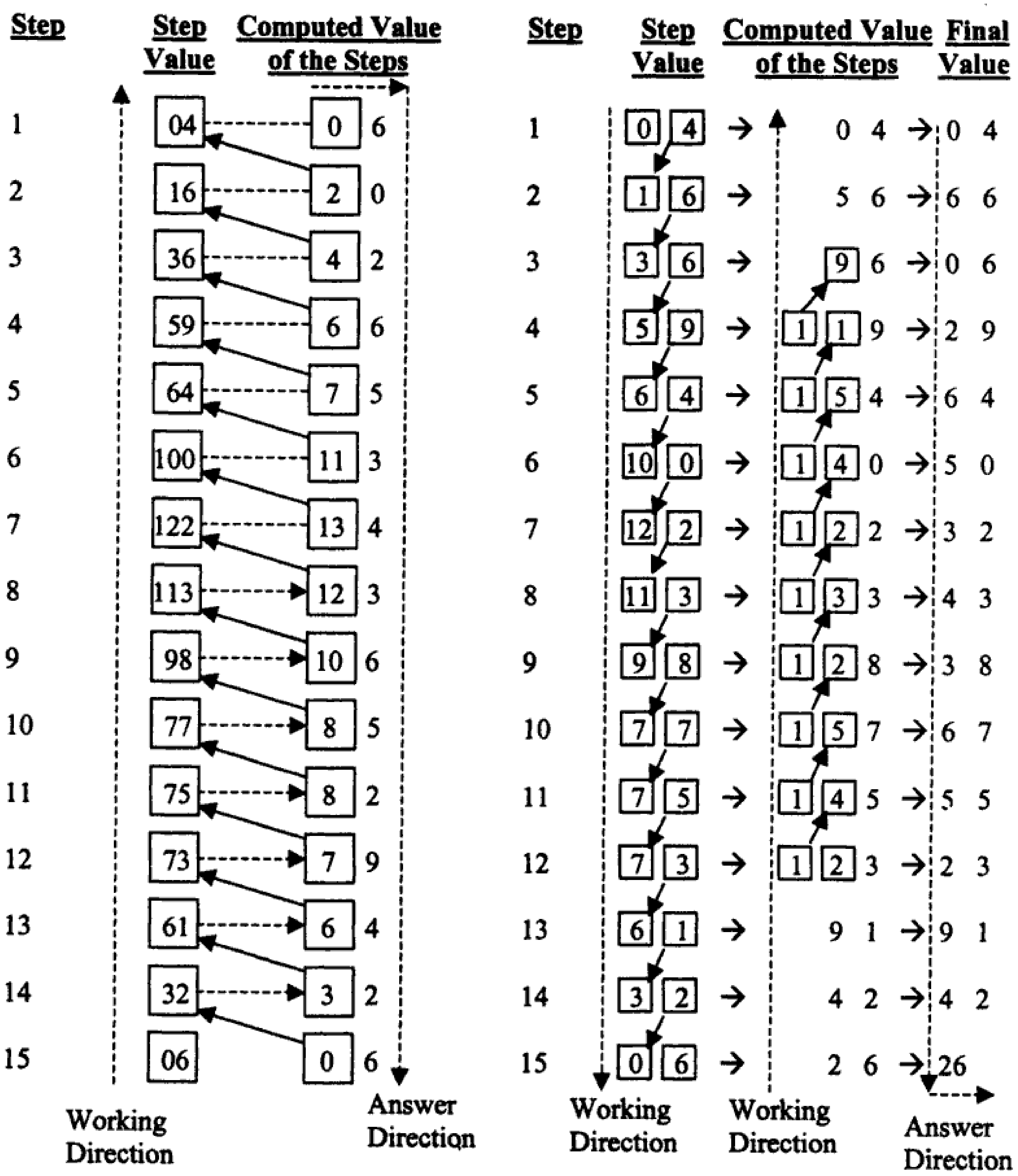
Step 15:

1 2 5 4 6 8 7 3

= 06

4 8 0 3 2 1 6 2

Steps:



Vedic Method 1

Step	Step Value	Computed Value of the Steps	
1	04	0	6
2	16	2	0
3	36	4	2
4	59	6	6
5	64	7	5
6	100	11	3
7	122	13	4
8	113	12	3
9	98	10	6
10	77	8	5
11	75	8	2
12	73	7	9
13	61	6	4
14	32	3	2
15	06	0	6

Vedic Method 2

Step	Step Value	Computed Value of the Steps	Final Value
1	0 4	0 4	0 4
2	1 6	5 6	6 6
3	3 6	9 6	0 6
4	5 9	1 1 9	2 9
5	6 4	1 5 4	6 4
6	10 0	1 4 0	5 0
7	12 2	1 2 2	3 2
8	11 3	1 3 3	4 3
9	9 8	1 2 8	3 8
10	7 7	1 5 7	6 7
11	7 5	1 4 5	5 5
12	7 3	1 2 3	2 3
13	6 1	9 1	9 1
14	3 2	4 2	4 2
15	0 6	2 6	26

Ans.: 602653436529426

Fig. 8

iii) Multiplying a long number by a shorter number (V.M.):

Regarding multiplication of a number by a shorter number in Vedic Method, zeros are supplemented to make the number of digits same and then multiply. Two examples are given below.

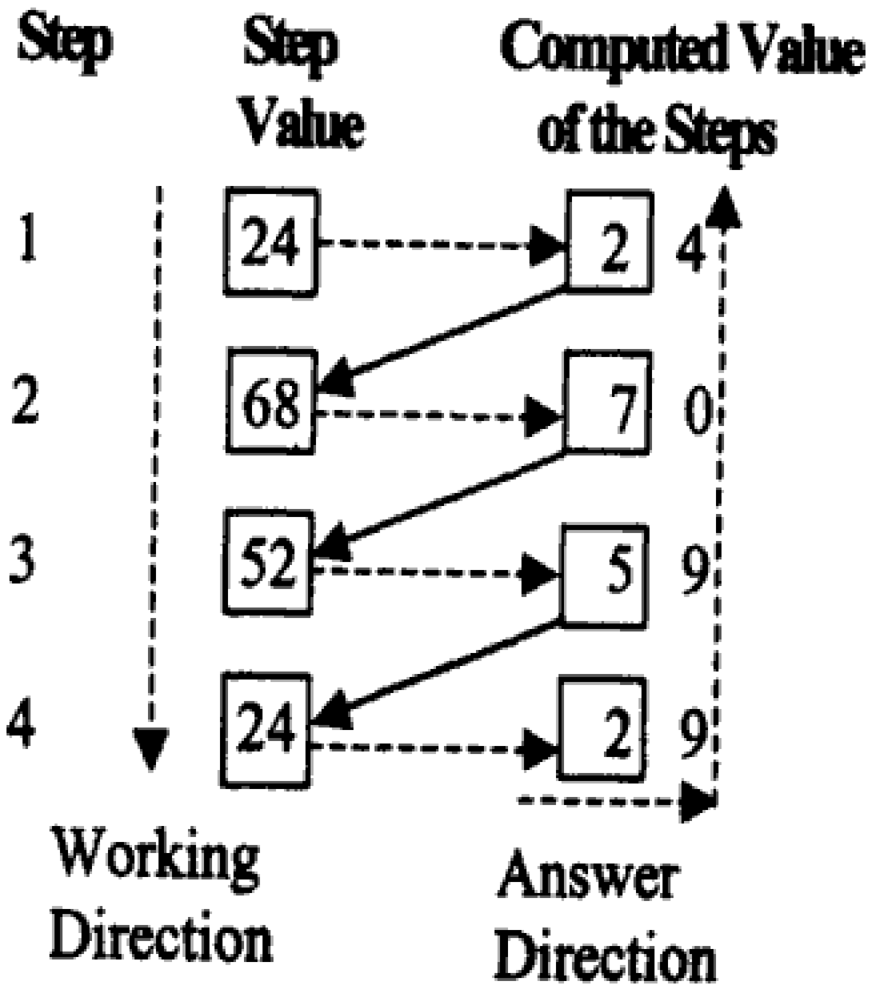
Example : 356 x 84

Current Method

$$\begin{array}{r} 356 \\ \times\ 84 \\ \hline 1424 \\ 2848 \\ \hline 29904 \end{array}$$

Steps:

Vedic Method (Right to Left)



Vedic Method (Right to Left)

$$\begin{array}{ccccc} & & 3 & 5 & 6 \\ & & 0 & 8 & 4 \\ \hline \boxed{2} & \boxed{9} & \boxed{9} & \boxed{0} & \boxed{4} \\ 2 & 5 & 7 & 2 & \end{array}$$

Ans.: 29904

Vedic Method 1 (Left to Right)

$$\begin{array}{l} 356 \\ \underline{084} \\ 04284 \\ 2562 \\ \boxed{29904} \end{array}$$

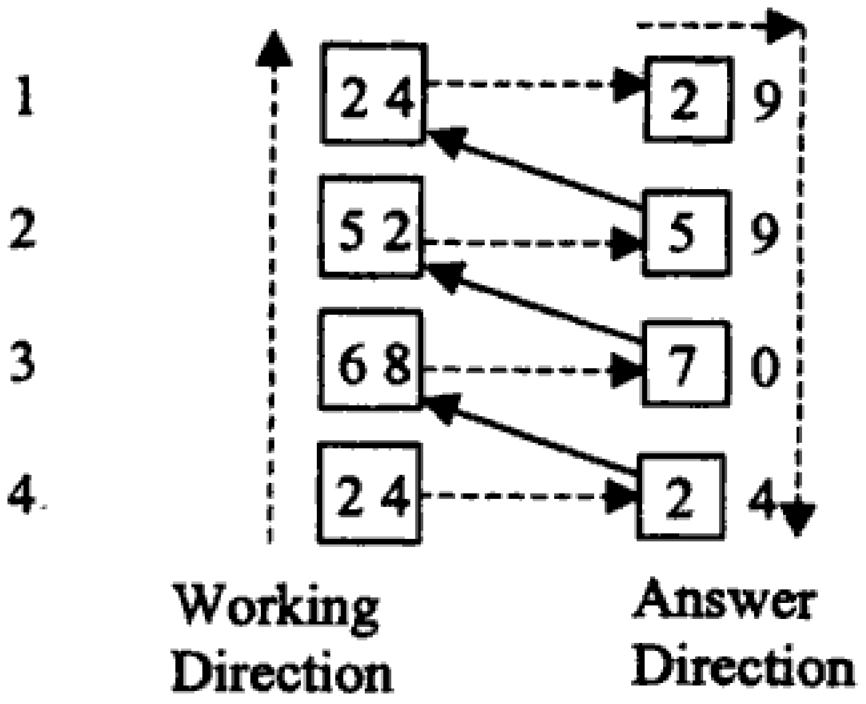
Vedic Method 2 (Left to Right)

$$\begin{array}{cccc} 3 & 5 & 6 & \\ 0 & 8 & 4 & \\ \hline 2 & 9 & 8 & 104 \\ \hline & 4 & 2 & 8 \end{array}$$

Ans.: 29904

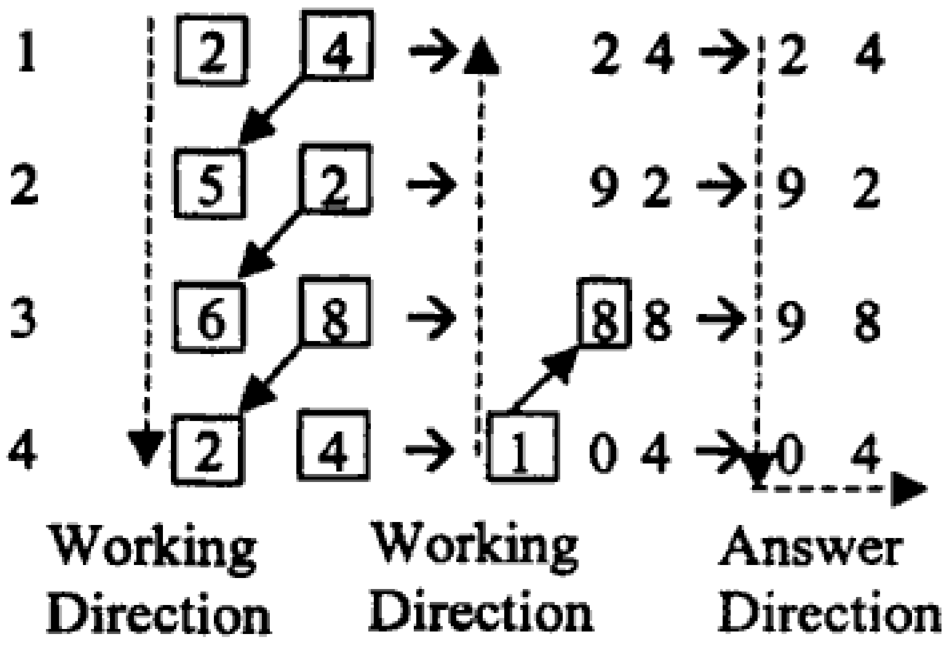
Vedic Method 1 (Left to Right)

Step **Step Value** **Computed Value of the Steps**



Vedic Method 2 (Left to Right)

Step **Step Value** **Computed Value of the Steps** **Final Value**



Ans.: 29904

Fig. 9

Example: 8215 × 32

Current Method

$$\begin{array}{r} 8215 \\ \underline{32} \\ 16430 \\ \underline{24645} \\ \underline{262880} \end{array}$$

Vedic Method (Right to Left)

$$\begin{array}{cccccc} & & 8 & 2 & 1 & 5 \\ & & 0 & 0 & 3 & 2 \\ \hline 2 & 6 & 2 & 8 & 8 & 0 \\ \hline & 2 & 2 & 0 & 1 & 1 \end{array}$$

Ans.: 262880

Vedic Method 1 (Left to Right)

$$\begin{array}{l} 8215 \\ \underline{0032} \\ 242770 \\ \quad 2011 \\ \boxed{262880} \end{array}$$

Vedic Method 2 (Left to Right)

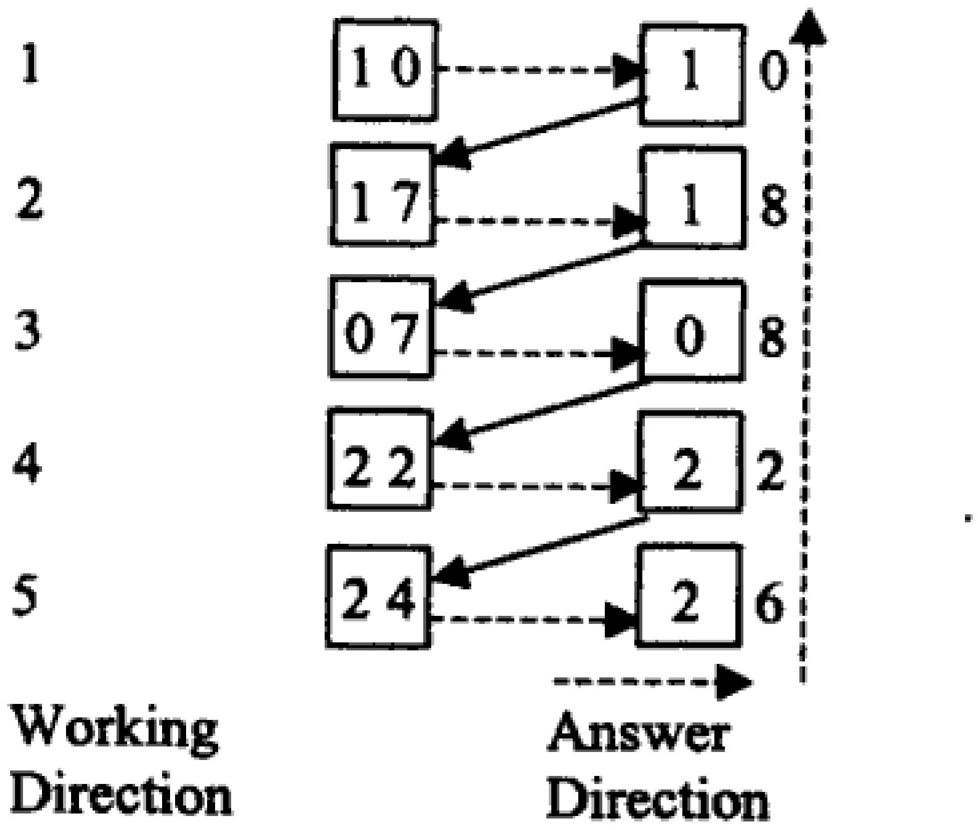
$$\begin{array}{l} 8\ 2\ 1\ 5 \\ 0\ 0\ 3\ 2 \\ \boxed{2\ 6\ 2\ 8\ 80} \\ \quad 4\ 2\ 7\ 7 \end{array}$$

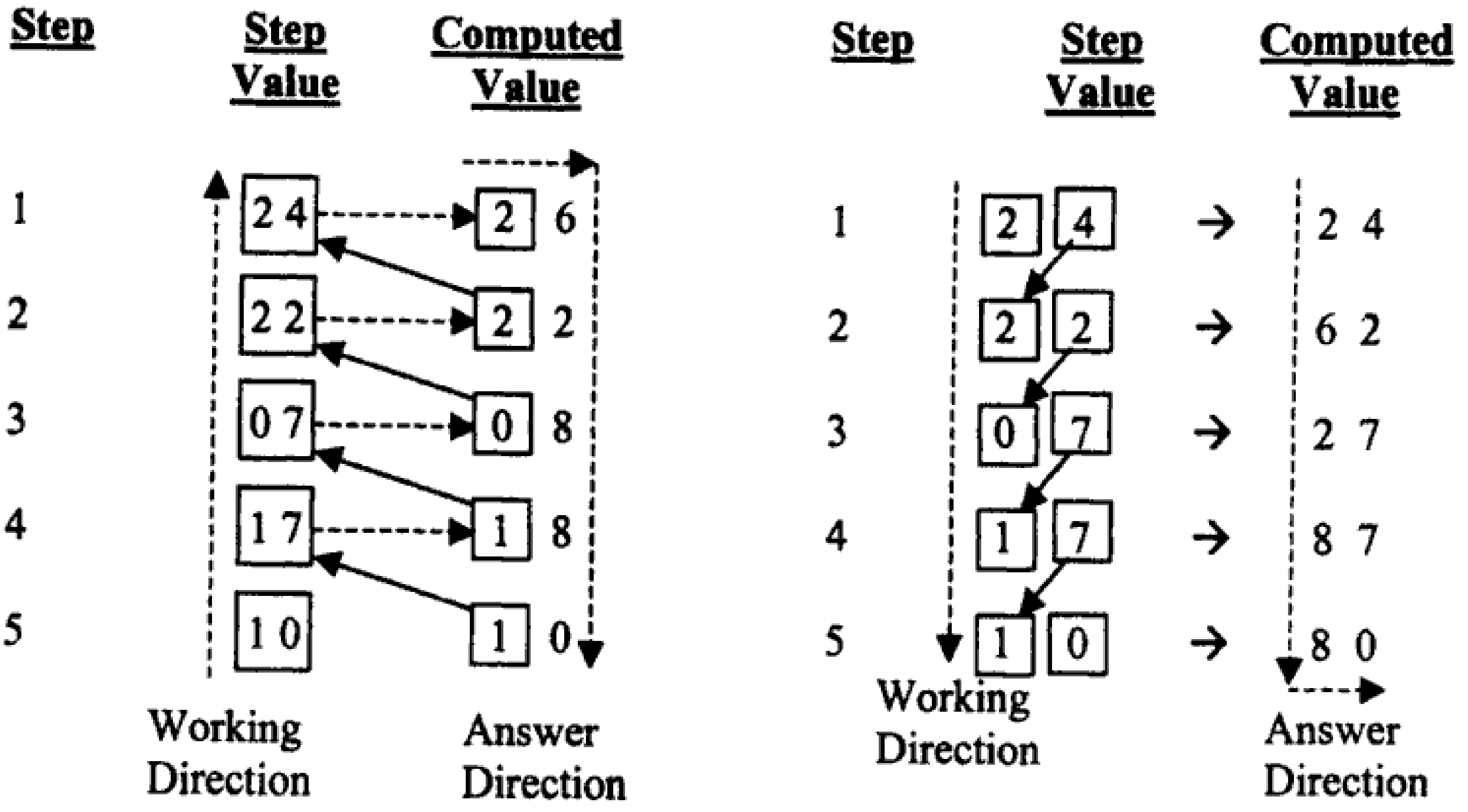
Ans: 262880

Steps :

Vedic Method (Right to Left)

Step **Step Value** **Computed Value of the steps**





Vedic Method 1 (Left to Right)

Vedic Method 2 (Left to Right)

Ans.: 262880

Fig.10

iv) Multiplying a long number by a shorter number: Moving Multiplier (V.M.):

It is also suggested that instead of considering zeros, one can use the principle of moving multiplier, i.e., the multiplier is being moved from right till the last digit of the multiplier comes under the last digit of the multiplicand, and this is surprisingly innovative that the same Vedic method of multiplication is carried out using the moving multiplier. Fig.11 shows moving multiplier taking different positions and the results obtained in different steps with the reading of the final result. This is clearly shown in the step diagrams.

Examples:

1) **12345678 × 32**

Current Method

$$\begin{array}{r} 1\;2\;3\;4\;5\;6\;7\;8 \\ 3\;2 \\ \hline 2\;4\;6\;9\;1\;5\;6 \\ 3\;7\;0\;3\;7\;0\;3\;4 \\ \hline 3\;9\;5\;0\;6\;1\;6\;9\;6 \end{array}$$

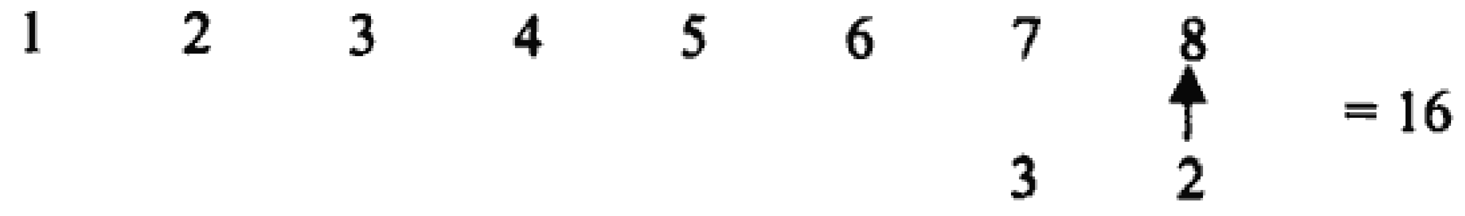
Vedic Method (Right to Left)

$$\begin{array}{r} 1\;2\;3\;4\;5\;6\;7\;8 \\ 3\;2 \\ \boxed{3\;9\;5\;0\;6\;1\;6\;9\;6} \\ 1\;2\;2\;3\;3\;3\;1 \end{array}$$

Ans.: 395061696

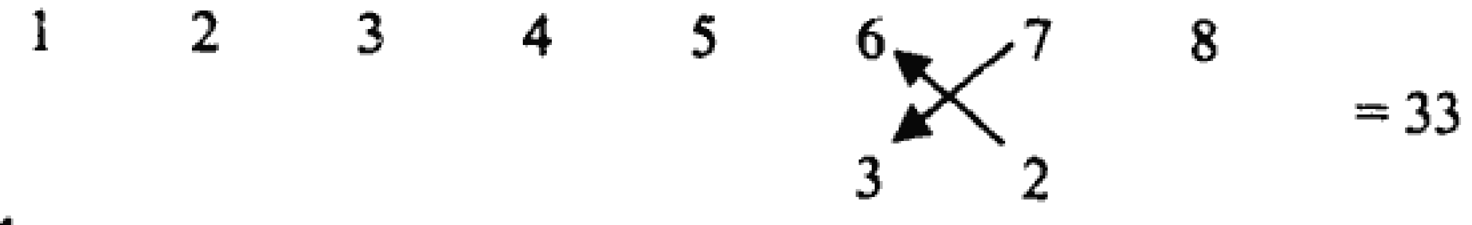
Step Diagrams (Right To Left):

Step 1:

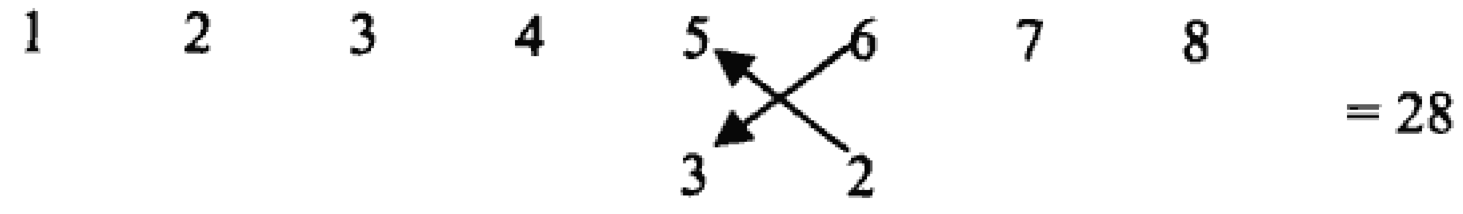


Step 2:

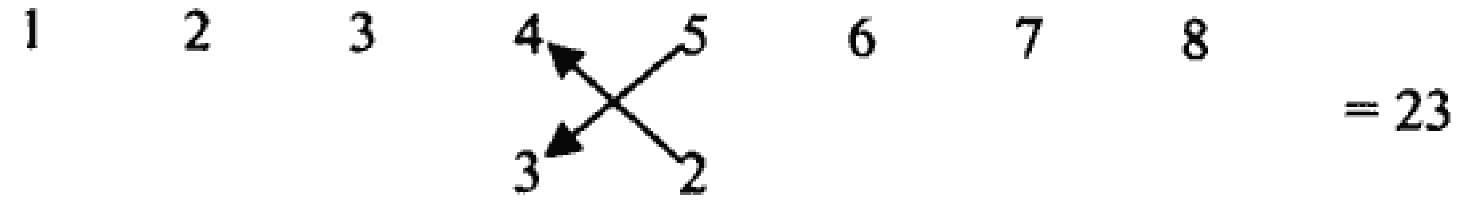
Step 3:



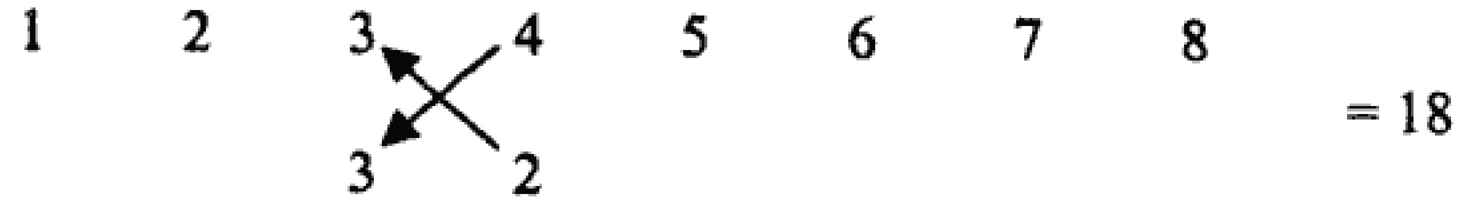
Step 4:



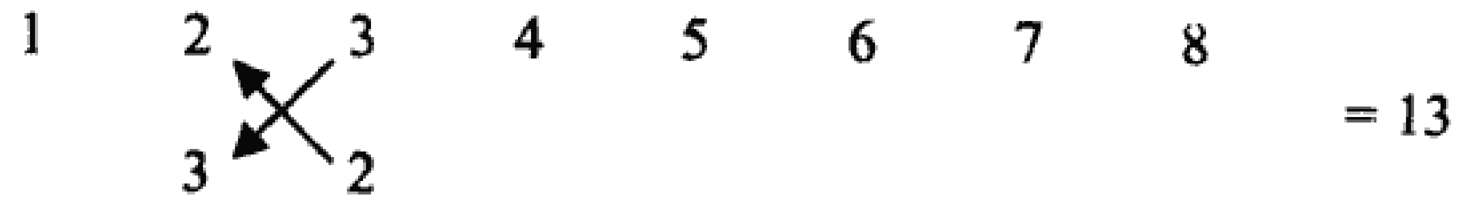
Step 5:



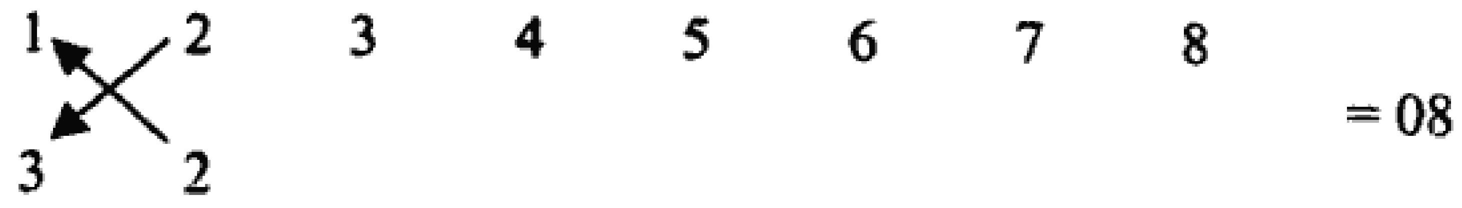
Step 6:



Step 7:



Step 8:



Step 9:

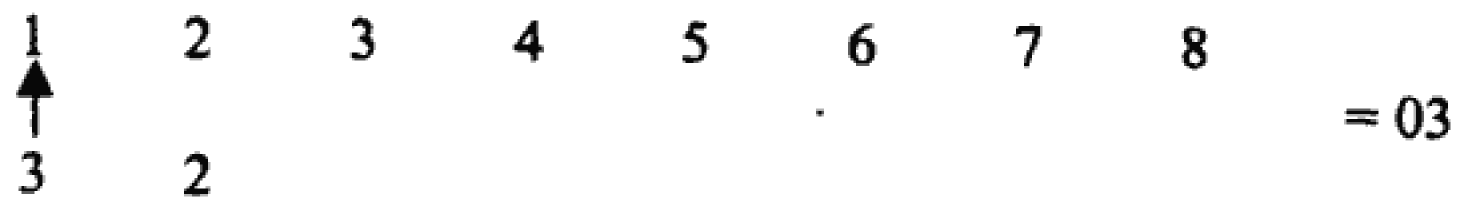


Fig.11

This moving multiplier method is also applicable to Left to Right Vedic Multiplication Fig.12.

Vedic Method 1 (Left to Right)

12345678
32

383838386
1122331

$\boxed{395061696}$

Vedic Method 2 (Left to Right)

1 2 3 4 5 6 7 8
3 2

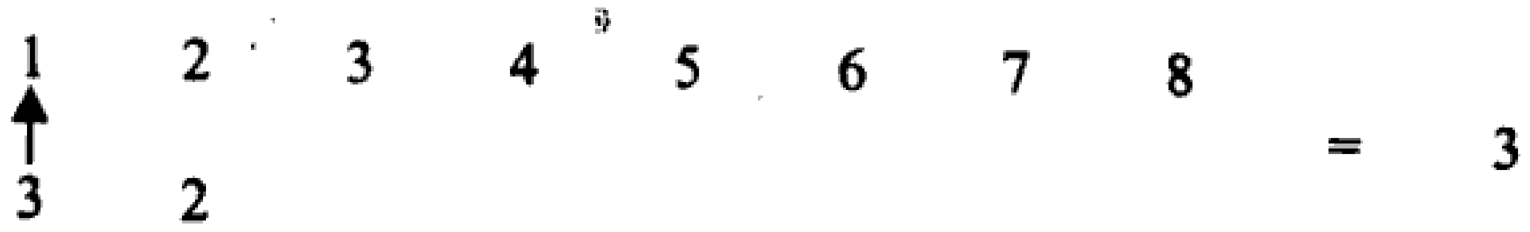
$\boxed{0\ \ 3\ \ 9\ \ 4\ \ 10\ 5\ \ 11\ 6\ \ 96}$

3 8 3 8 3 8 3 8

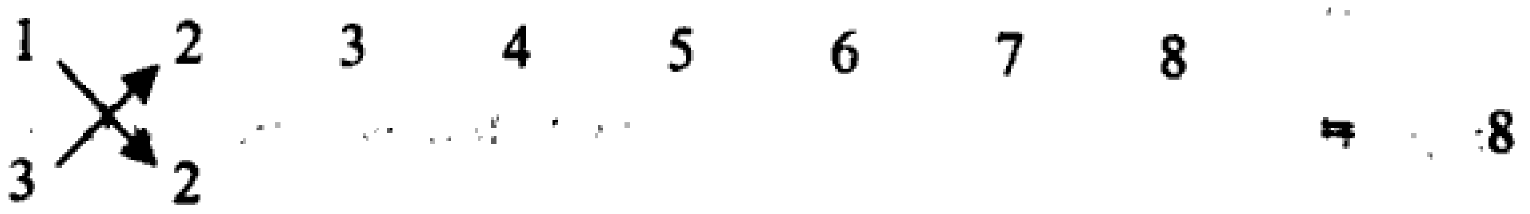
Ans.: 395061696

Step Diagrams (Left To Right):

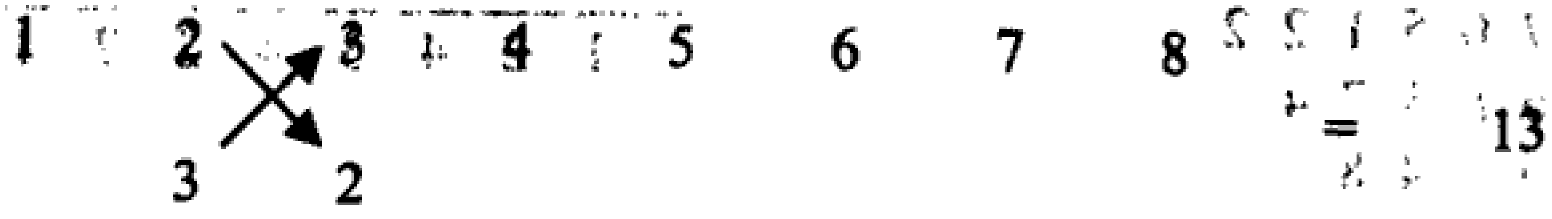
Step 1:



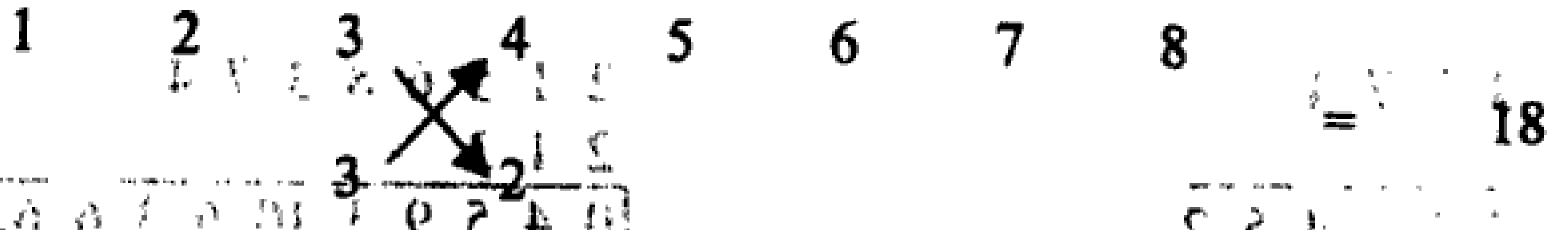
Step 2:



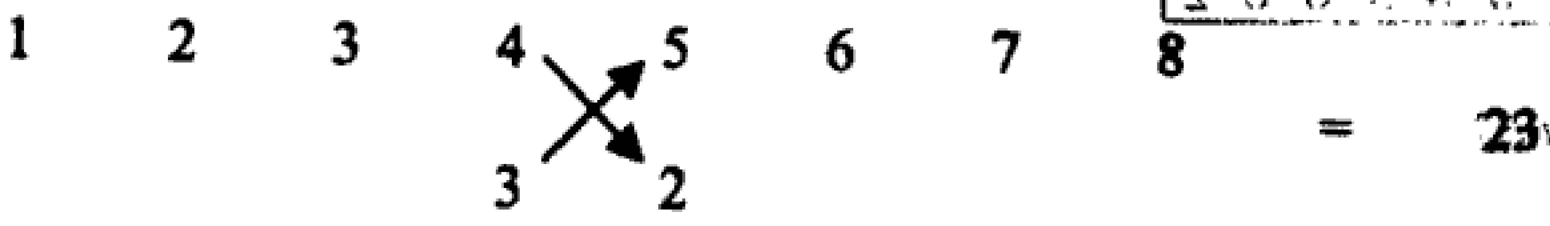
Step 3:



Step 4:



Step 5:



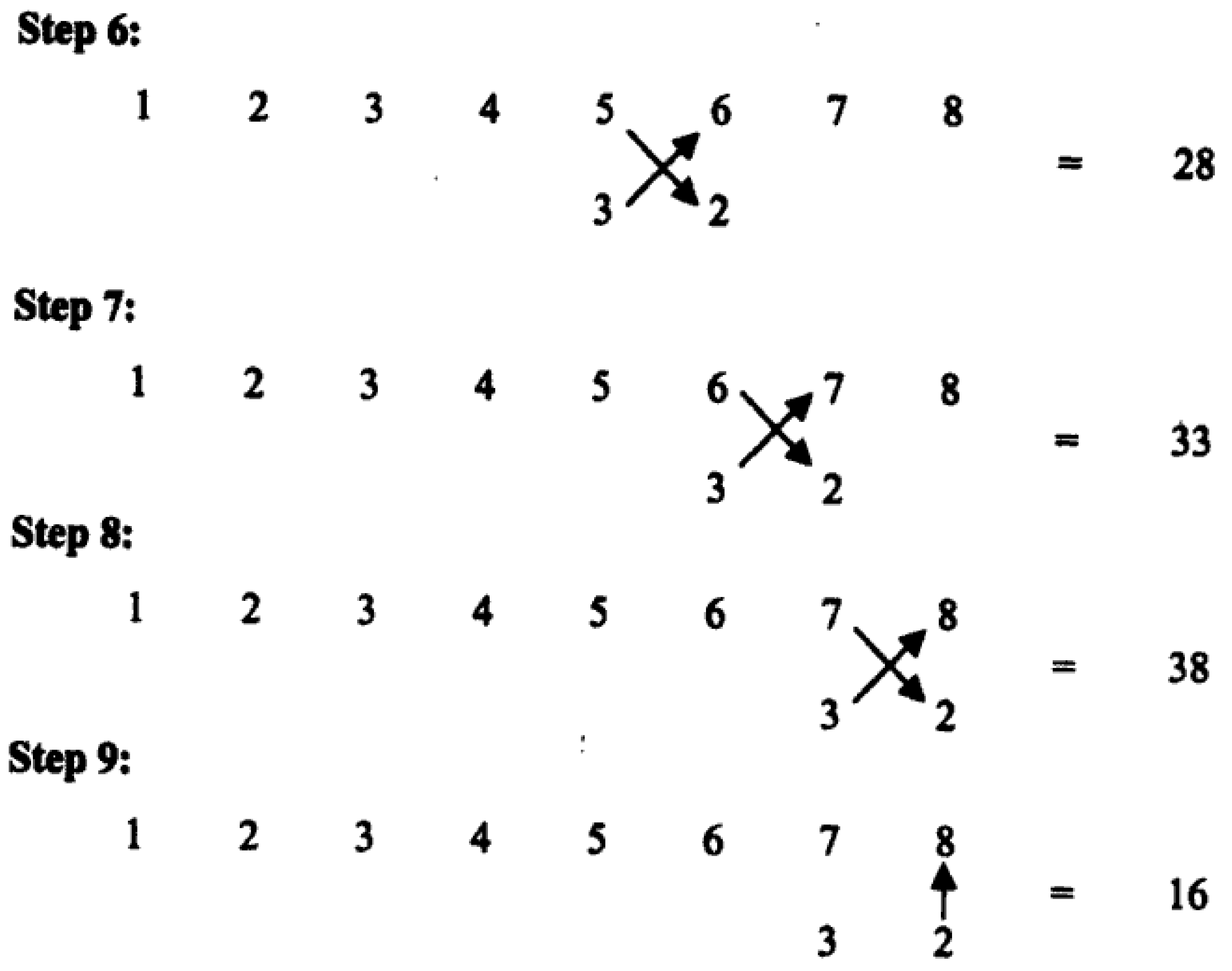


Fig.12

2) 21568374 × 213

Current Method

2 1 5 6 8 3 7 4
2 1 3
6 4 7 0 5 1 2 2
2 1 5 6 8 3 7 4
4 3 1 3 6 7 4 8
4 5 9 4 0 6 3 6 6 2

Vedic Method (Right to Left)

2 1 5 6 8 3 7 4
2 1 3
$\boxed{4\ 5\ 9\ 4\ 0\ 6\ 3\ 6\ 6\ 2}$
1 2 4 3 4 2 2 1

Vedic Method 1(Left to Right)

2 1 5 6 8 3 7 4
2 1 3
4 4 7 0 7 2 1 4 5 2
0 1 2 3 3 4 2 2 1
$\boxed{4\ 5\ 9\ 4\ 0\ 6\ 3\ 6\ 6\ 2}$

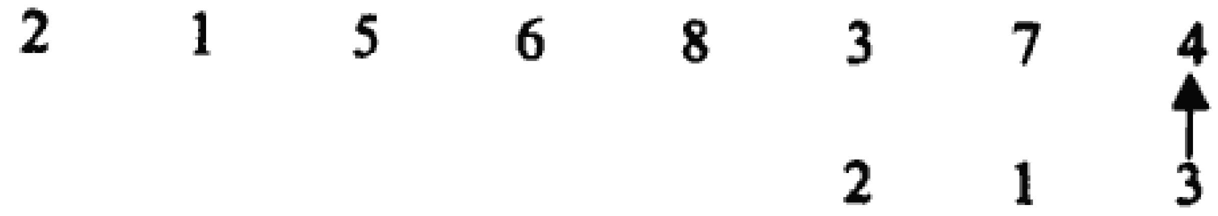
Vedic Method 2 (Left to Right)

2 1 5 6 8 3 7 4
2 1 3
$\boxed{0\ 4\ 5\ 9\ 3\ 10\ 6\ 3\ 6\ 62}$
4 4 7 0 7 2 1 4 5

Ans.: 4594063662

Step Diagrams (Right to Left):

Step 1:



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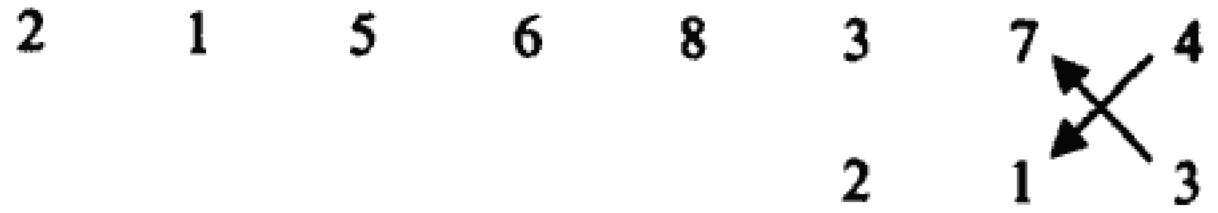
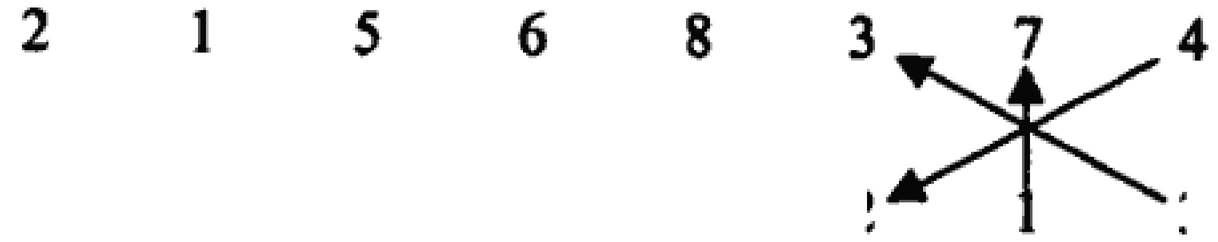
Step 2:



= 25

Step 3:



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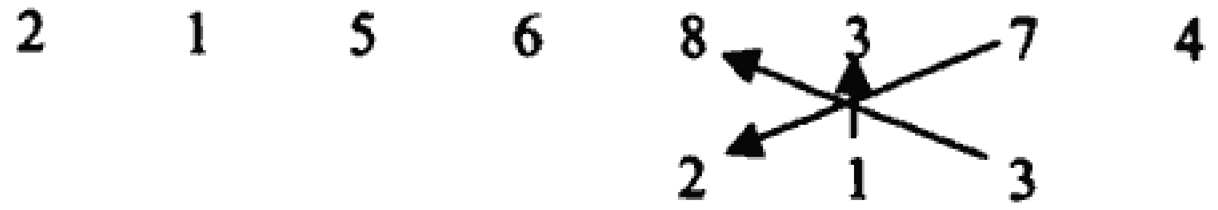
Step 4:



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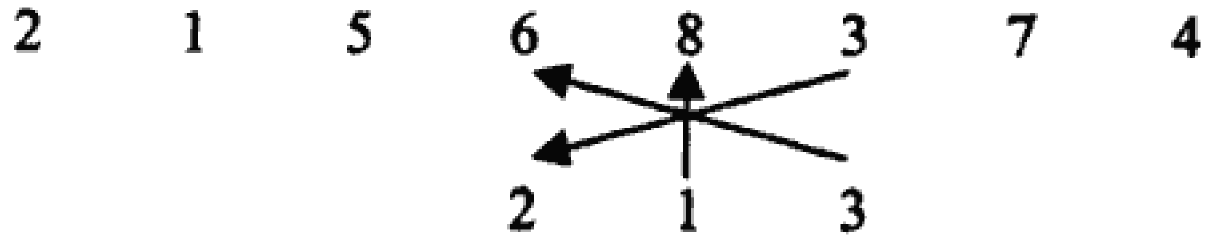
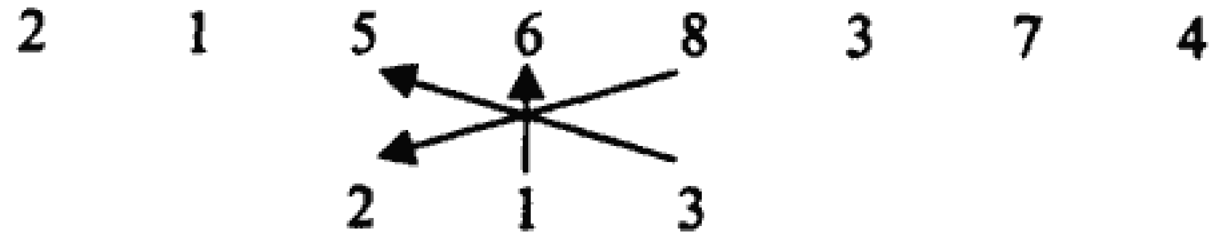
Step 5:



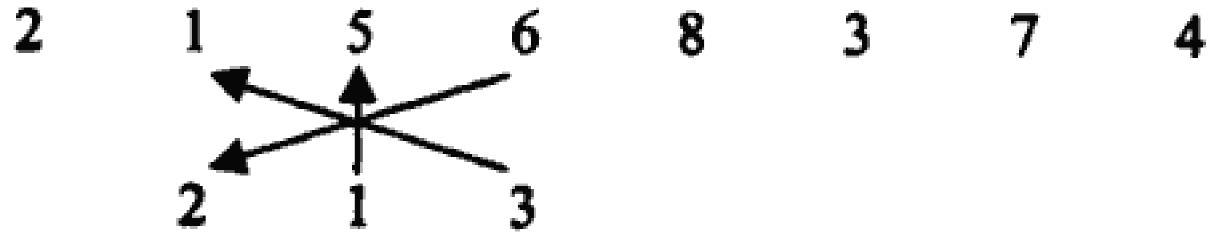
= 32

Step 6:



= 37

Step 7:



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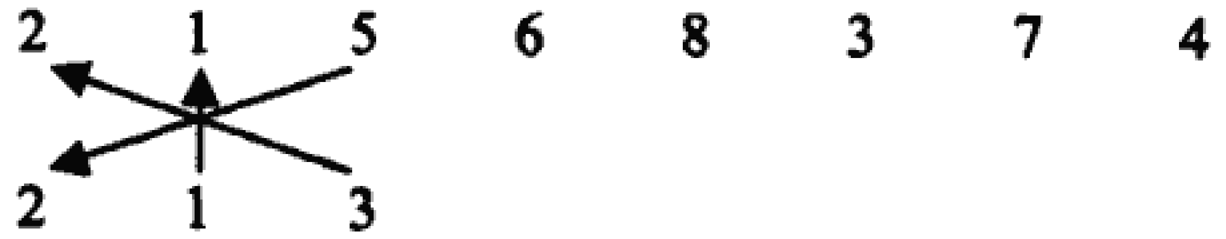
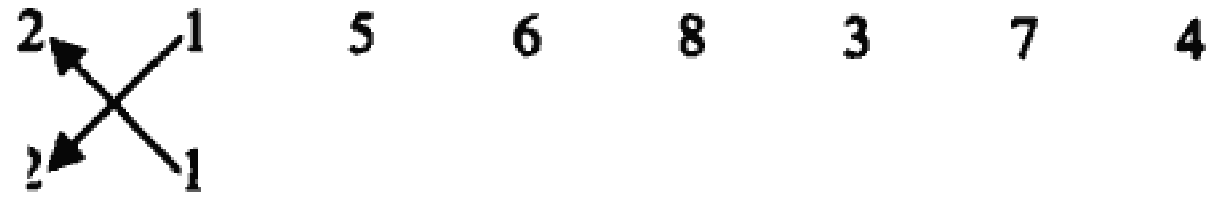
Step 8:



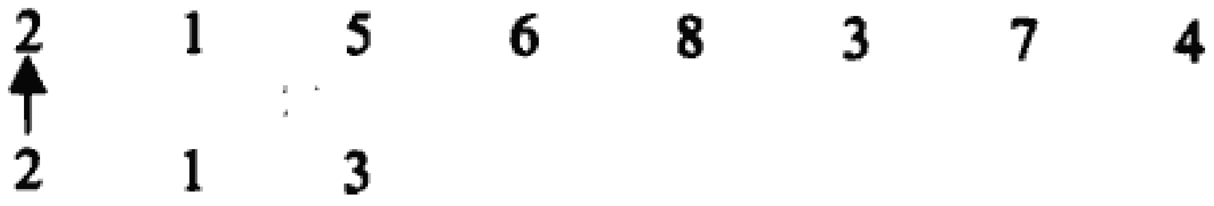
= 17

Step 9:



= 4

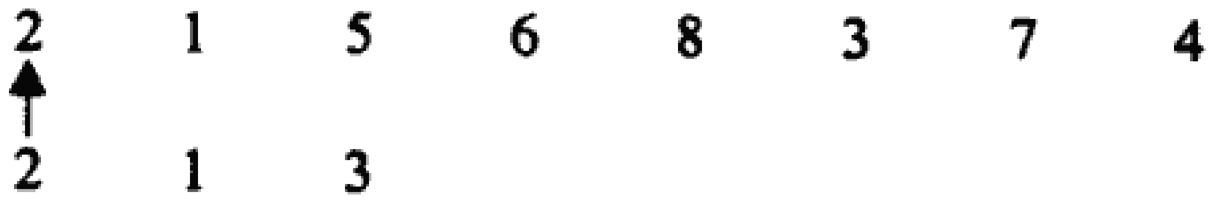
Step 10:



= 4

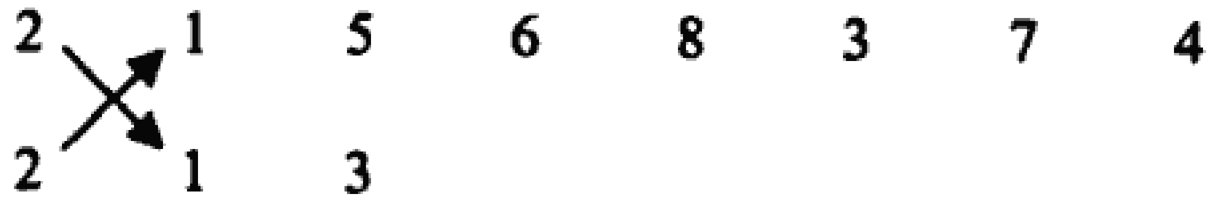
Step Diagrams (Left to Right):

Step 1:



= 4

Step 2:



= 4

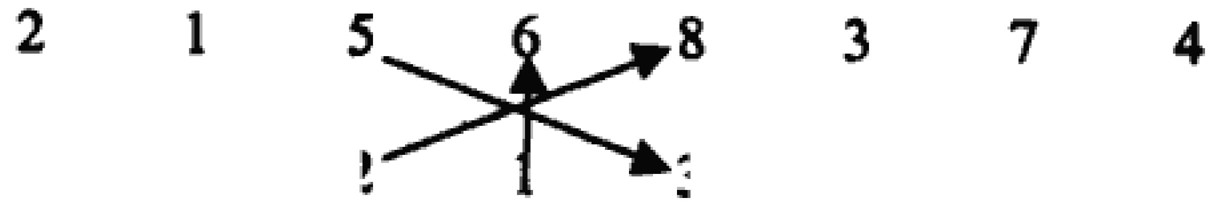
Step 3:

= 17

Step 4:

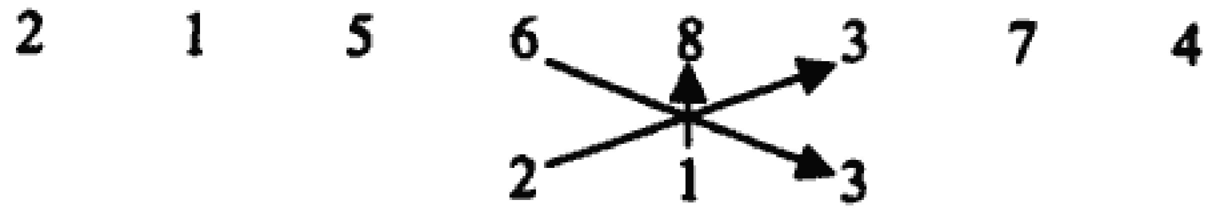
= 20

Step 5:



= 37

Step 6:



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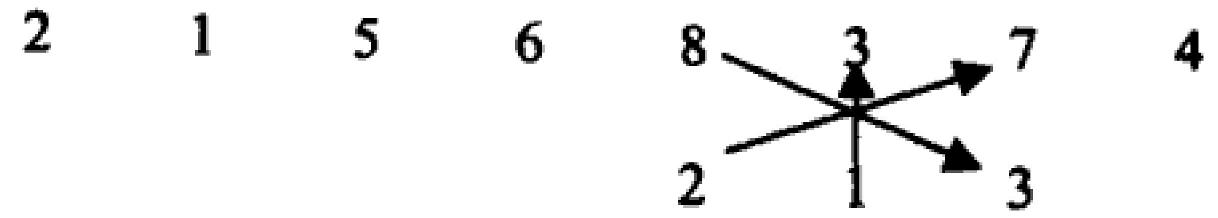
Step 7:



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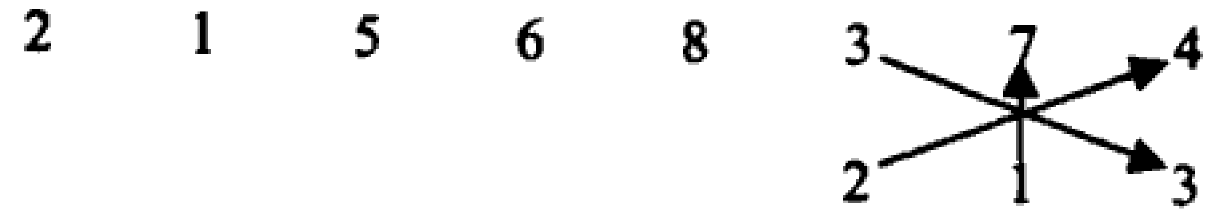
Step 8:



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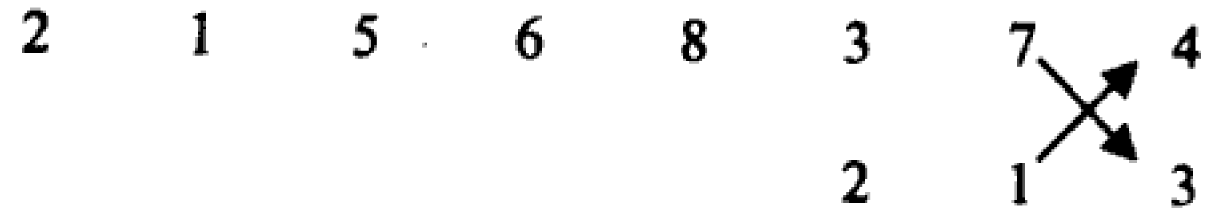
Step 9:



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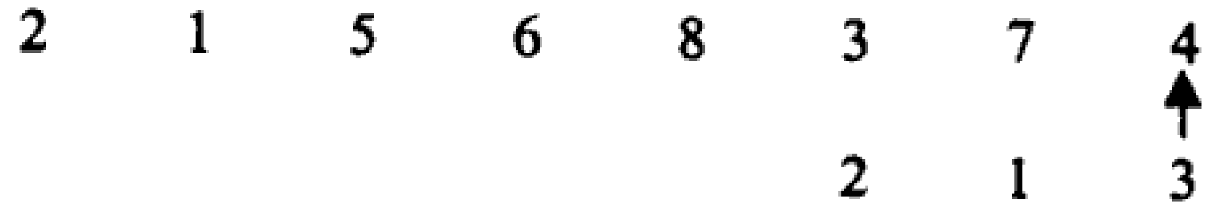
Step 10:



= 12

This method of moving multiplier is not existing at all in the current system. It can be applied to any number of digits in a multiplier. The moving multiplier can be moved from any position with limits intact and has to be covered completely. From the middle if the multiplier is being multiplied then it is as follows:

<u>Vedic Method 3</u> : (From middle)

(1) 1 2 3 4 5 6 7 8 x 3 2

1 2 3 4 5 6 7 8

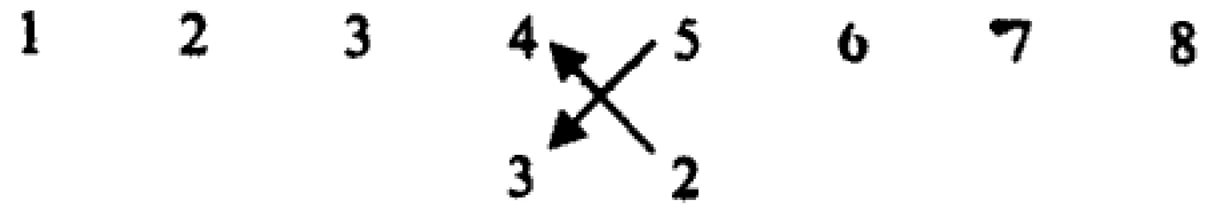
3 2

3	8	3	8	3	8	3	8	6
	0	1	1	2	2	3	3	1

Answer = 395061696

Step Diagrams (Middle)

Step 1:



= 23

Step 2: moving towards left of step 1

(2) 2 1 5 6 8 3 7 4 x 2 1 3

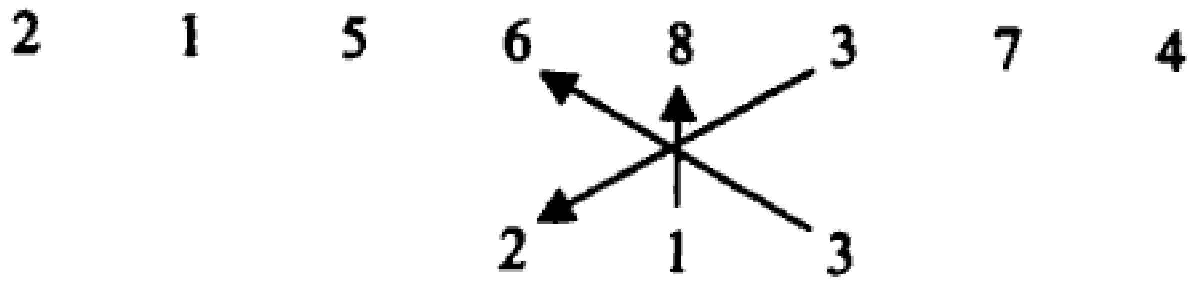
2 1 5 6 8 3 7 4

2 1 3

4	4	7	0	7	2	1	4	5	2
0	0	1	2	3	3	4	2	2	1

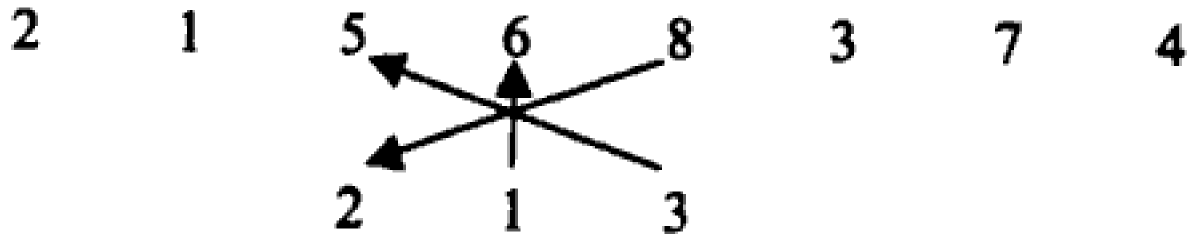
Step Diagrams :

Step 1:



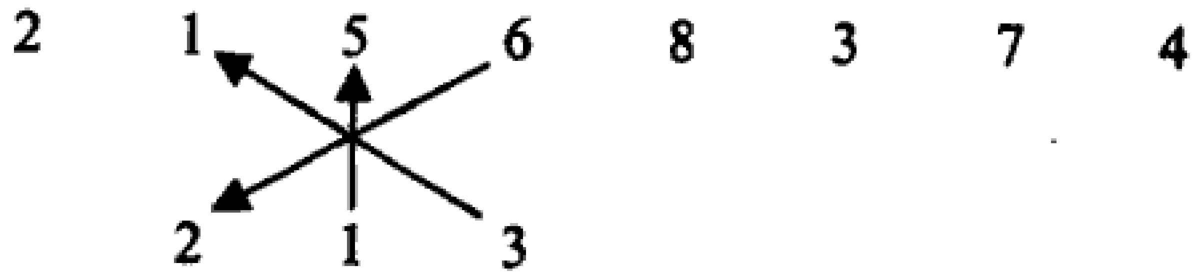
= 32

Step 2: Moving towards left of step 1



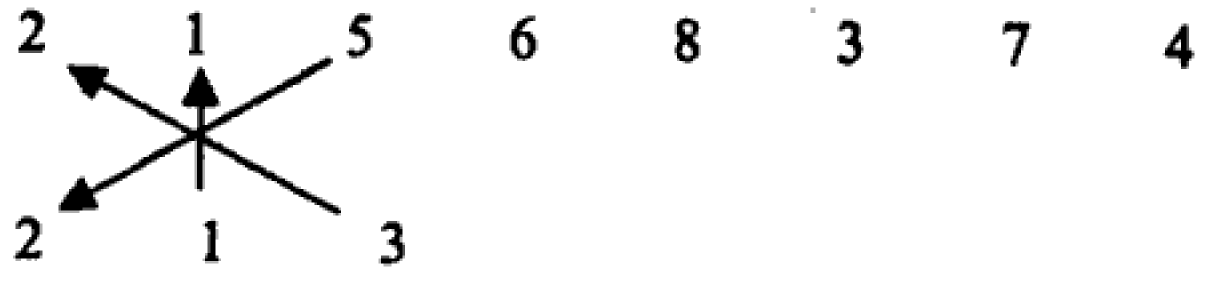
= 37

Step 3:



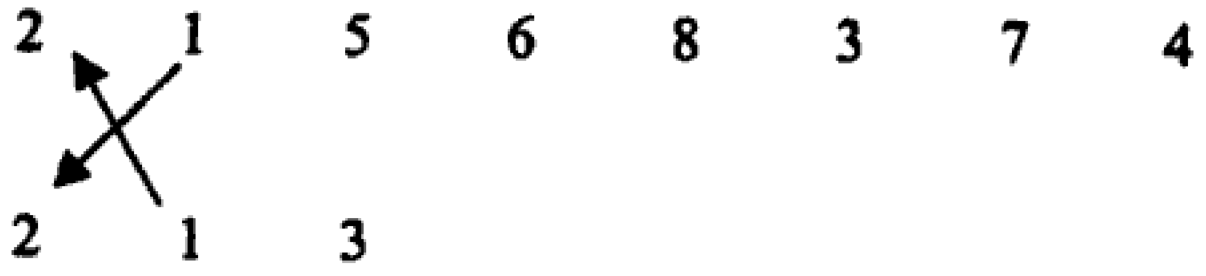
= 20

Step 4:



= 17

Step 5:

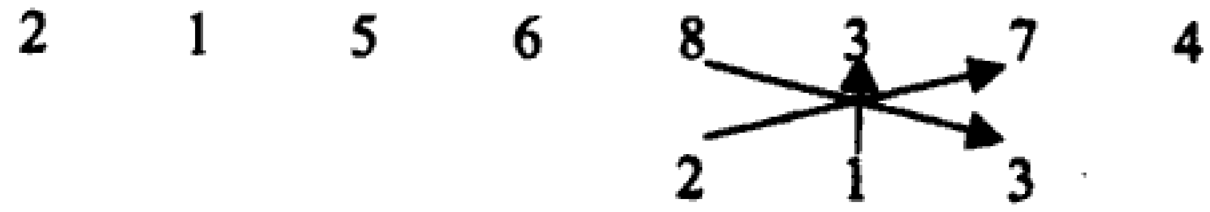


= 04

Step 6:

= 04

Step 7: Moving towards right of step 1



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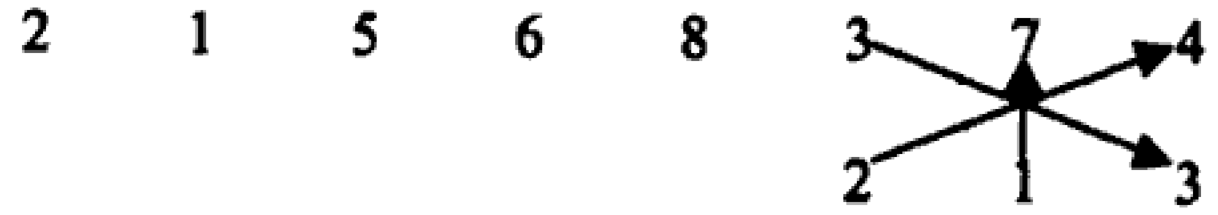
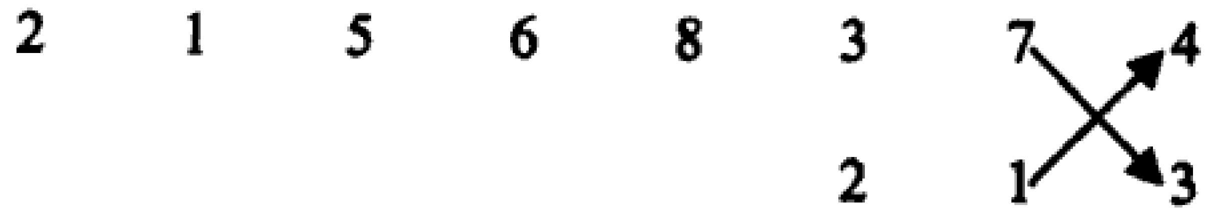
Step 8:



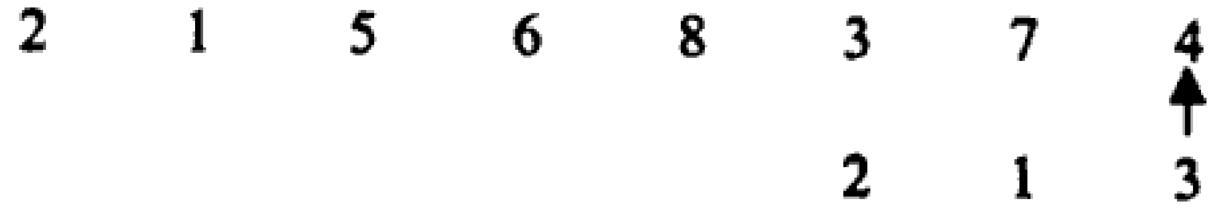
= 24

Step 9:



= 25

Step 10:



= 12

v) To find the digit in a particular position in the product of two numbers (V.M)

The stepwise method is significant in the sense that one can get the digit in a particular position in the product of two numbers.

For example, in the working of a 9 digit number by a 9 digit number, if we want to know 8th digit in the answer from right to left we have to work out simply values of 8 steps the last digit in the computed value of the 8th step is the required digit.

Example :

a)

3 4 5 6 2 1 7 5 8
1 2 4 0 8 2 0 3 5

$\boxed{4\ 2\ 8\ 8\ 5\ 4\ 5\ 1\ 0\ 7\ 2\ 9\ 1\ 7\ 5\ 3\ 0}$

1 2 3 6 7 7 10 11 9 14 9 9 4 5 5 4

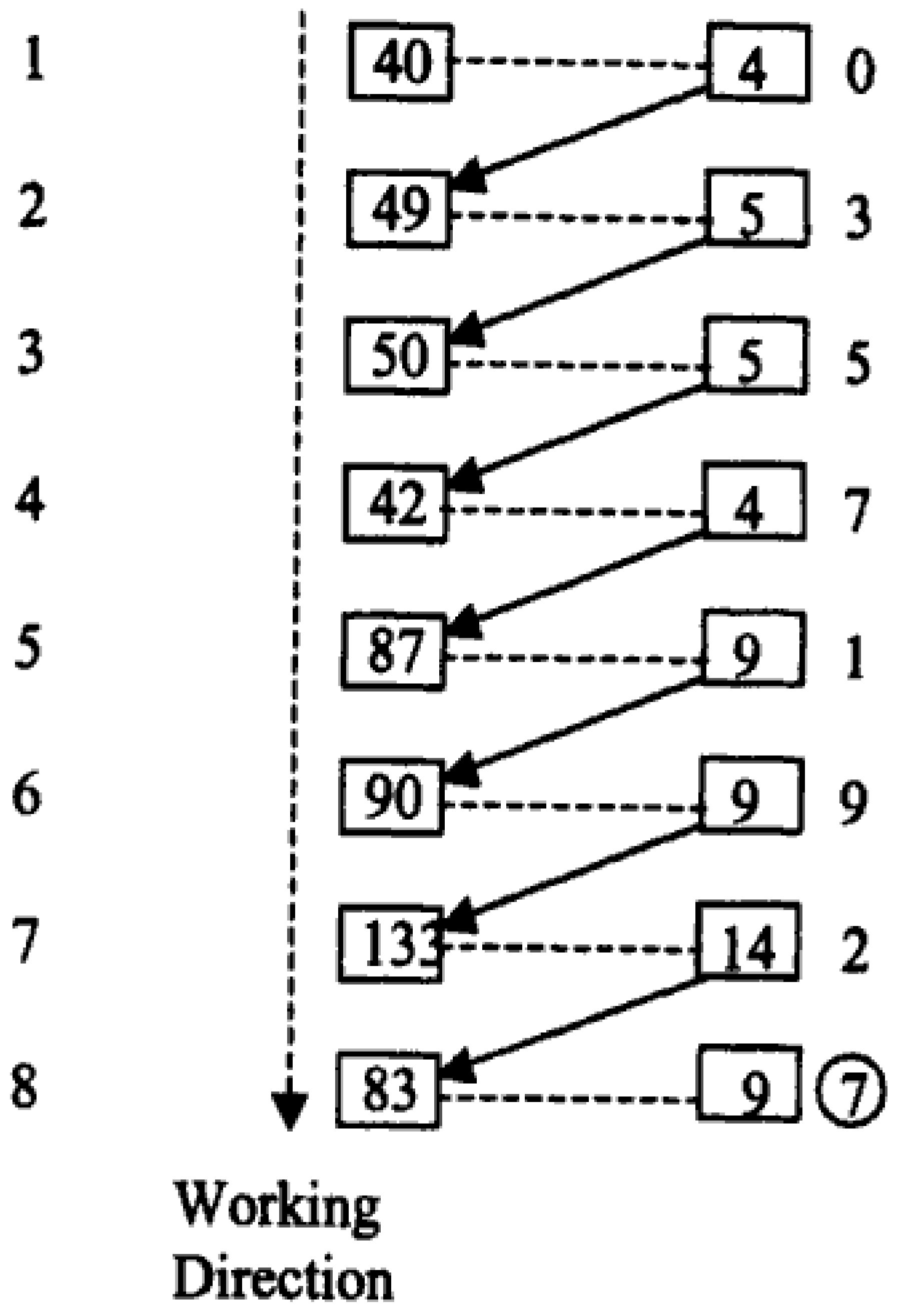
Answer : 4 2 8 8 5 4 5 1 0 7 2 9 1 7 5 3 0

Position of Digits 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Suppose we want to find digit in the 8th position, then from right of left, we have to go for 8 (Eight) steps8.

Steps:

Step **Step Value** **Computed Value of the Steps**



So '7' is the answer.

Fig.13

b) 1 2 5 4 6 8 7 3

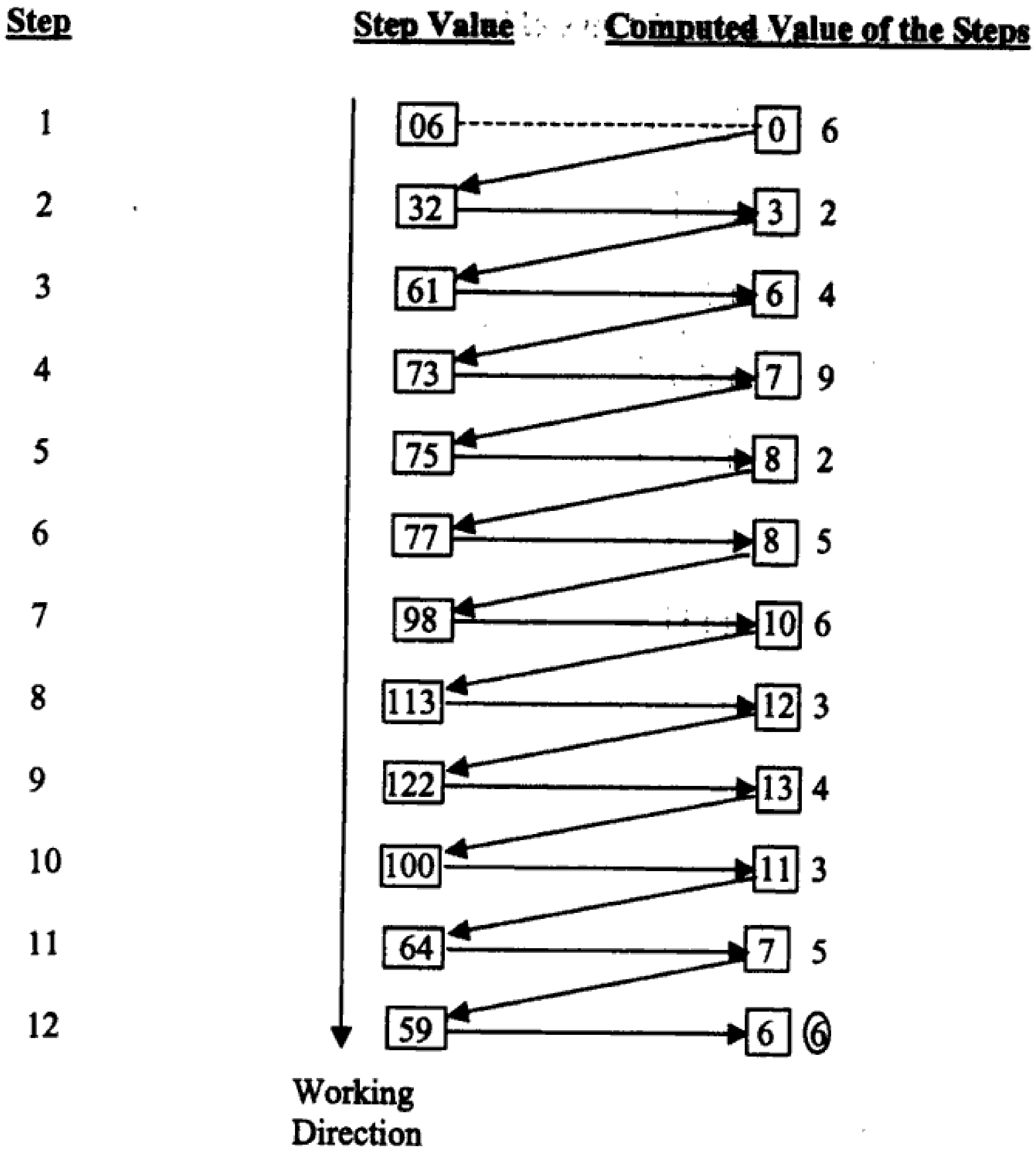
4 8 0 3 2 1 6 2

$\boxed{6\ 0\ 2\ 6\ 5\ 3\ 4\ 3\ 6\ 5\ 2\ 9\ 4\ 2\ 6}$

Answer:	6	0	2	6	5	3	4	3	6	5	2	9	4	2	6
Position: of Digits	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

To find a number in the 12th position, from Right to Left, we have to go for 12 Steps.

Steps:



So '6' is the answer.

Fig.14

vi) Combined Operations (V.M.):

Sum of Products:

Vedic Multiplication is useful and is found to be easy for combined operations such as Sum of Products or a Series of Products.

Let us consider a sum of three products of numbers of two digits. (34 × 52) + (84 × 36) + (25 × 71). In this case the Urdhva Tiryagbhyam Sutram can be applied simultaneously to all the products in the sum in a single step, whereas the current method multiplies each product separately and adds them to get the result.

Current Method

$$\begin{array}{r} 34\\ \underline{52}\\ 68\\ \underline{170}\\ \underline{1768} \end{array} \qquad \begin{array}{r} 84\\ \underline{36}\\ 504\\ \underline{525}\\ \underline{3024} \end{array} \qquad \begin{array}{r} 25\\ \underline{71}\\ 25\\ \underline{175}\\ \underline{1775} \end{array}$$

1768 + 3024 + 1775 = 6567

Vedic Method
(Right to Left)

$$\begin{array}{ccccc} 34 & & 84 & & 25\\ & + & & + & \\ \underline{52} & & \underline{36} & & \underline{71}\\ \hline \multicolumn{1}{|c}{65} & & 6 & & \multicolumn{1}{c|}{7}\\ \hline & 12 & & 3 & \end{array}$$

Ans. 6567

In the Vedic method the application of Urdhva Tiryagbhyam is as follows. The corresponding numbers of the first column in each product are multiplied by Urdhva and added as the first step.

$$\begin{bmatrix}5\\ \uparrow\\ 1\end{bmatrix}+\begin{bmatrix}4\\ \uparrow\\ 6\end{bmatrix}+\begin{bmatrix}4\\ \uparrow\\ 2\end{bmatrix}=(1\times 5)+(6\times 4)+(2\times 4)=37$$

The result is placed exactly as in the Vedic Multiplication (R→L).

In the second step, the corresponding Tiryak multiplication results of three products are added.

$$\begin{pmatrix}2 & & 5\\ & \times & \\ 7 & & 1\end{pmatrix}+\begin{pmatrix}8 & & 4\\ & \times & \\ 3 & & 6\end{pmatrix}+\begin{pmatrix}3 & & 4\\ & \times & \\ 5 & & 2\end{pmatrix}$$

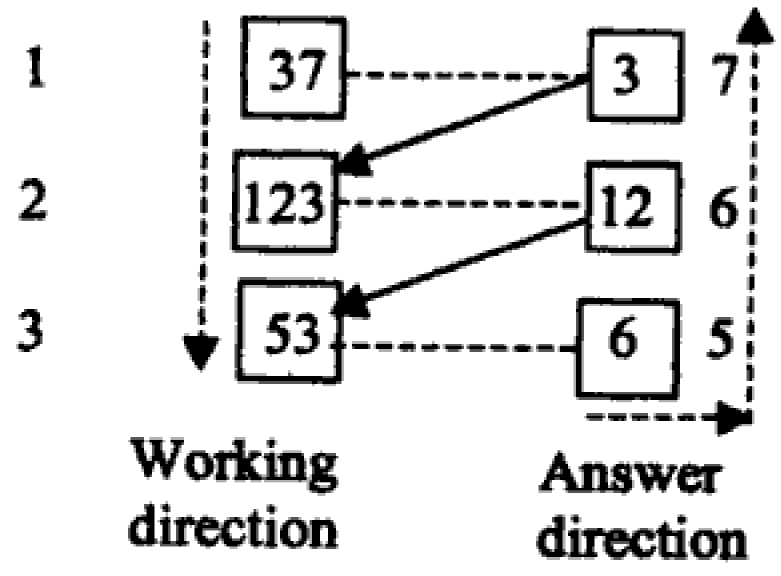
$$=(2\times 1+5\times 7)+(8\times 6+3\times 4)+(3\times 2+5\times 4)=123$$

The third step is again the addition of the corresponding Urdhava multiplication of the last columns.

$$\begin{bmatrix}3\\ \uparrow\\ 5\end{bmatrix}+\begin{bmatrix}8\\ \uparrow\\ 3\end{bmatrix}+\begin{bmatrix}2\\ \uparrow\\ 7\end{bmatrix}=(5\times 3)+(3\times 8)+(7\times 2)=53$$

The answer can be deduced from the steps.

<u>**Step**</u> <u>**Step Value**</u> <u>**Computed Value of the Steps**</u>



Ans: 6567

The same principle holds good to sum of the products of any number of digits. This is also a column-wise multiplication starting and ending with the Urdhva Multiplication. This combined operation can be carried out L→R, wise as well.

Example 2: (245 × 326) + (821 × 654) + (732 × 510)

Current Method

245	821	732
x 326	x 654	x 510
1470	3284	000
490	4105	732
735	4926	3660
79870	536934	373320

Ans:79870 + 536934 + 373320 = 990124

Vedic Method
(Right to Left)

245 821 732
+ +
326 654 510
$\boxed{99\ \ 0\ \ 1\ \ 2\ \ 4}$
10 10 5 3

Ans: 990124

Example 3: (3456 × 4217) + (9236 × 7851) + (4570 × 6234)

Current Method

3456	9236	4570
x 4217	x 7851	x 6234
24192	9236	18280
3456	46180	13710
6912	73888	9140
13824	64652	27420
14573952	72511836	28489380

Ans: 14573952 + 72511836 + 28489380 = 115575168

Vedic Method (Right to Left)

3456 9236 4570
+ +
4217 7851 6234
$\boxed{115\ \ 5\ \ 7\ \ 5\ \ 1\ \ 6\ \ 8}$
16 19 20 16 10 4

Ans.: 115575168

Example 4: (65231 × 42763) + (92107 × 35216) + (85310 × 64531)

Current Method

65231	92107	85310
x42763	x35216	x64531
195693	552642	85310
391386	92107	255930
456617	184214	426550
130462	460535	341240
260924	276321	511860
2789473253	3243640112	5505139610

Ans: 5505139610+3243640112+2789473253=11538252975

Vedic Method (Right to Left)

65231 92107 85310
+ +
42763 35216 64531
$\boxed{115\ \ 3\ \ 8\ \ 2\ \ 5\ \ 2\ \ 9\ \ 7\ \ 5}$
16 18 19 20 12 5 2 4

Ans.: 11538252975

vii) Multiplication of more than two numbers (V.M.): (Series Multiplications)

Application of Urdhva Tiryagbhyam Sutram for successive multiplication as a single unit:

Two-digit Numbers Multiplication:

Example let us consider three numbers multiplication: 12 × 23 × 34

In the current system, multiplication is carried out first by any two numbers and the result is used for the multiplication by the third number. For eg. 12 × 23 = 276, 276 × 34 = 9384.

Current Method:

12
×23
36
24
276
× 34
1104
828
.9384

The Associative Law is valid.

Vedic Method (Right to Left):

Consider a simultaneous multiplication of all the three numbers applying Vedic sutram Urdhva Tiryagbhyam .

Tens Units
1 2
2 3
3 4
$\boxed{9\ 3\ 8\ 4}$
3 4 2

Ans.: 9384

To elaborate a general procedure of column-wise multiplication of n numbers:

1) To multiply only digits belonging to the different numbers
2) To adopt the column-wise multiplication for all columns
3) Continuing the multiplication from one column to the rest of the columns systematically performing Urdhva and Tiryak Multiplication
4) To complete all permutations
5) To sort out the status of the result of the step

The working details are as follows: (Column-wise).

We can, in the first instance sort out units, tens, hundreds, etc.

Step 1: All the digits in the first column (Only Urdhva)

Status

For Units: 1 × 1 × 1 = 1 (All the three digits belonging to units place)

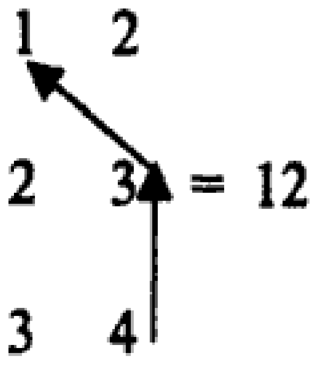
Tens Units
1 2
2 3 = 24
3 4

First Step considers the Urdhva Multiplication of all the digits in the units place as $2 \times 3 \times 4$. The result is noted as 24 units.

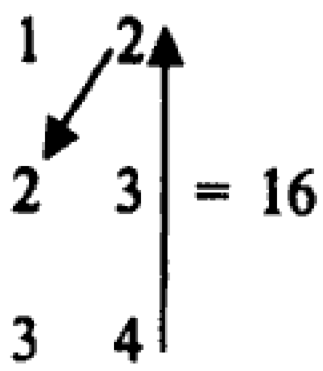
Step 2: Entering into 2nd column with one digit in the 2nd column and two digits in the 1st column (completing the permutations).

Status

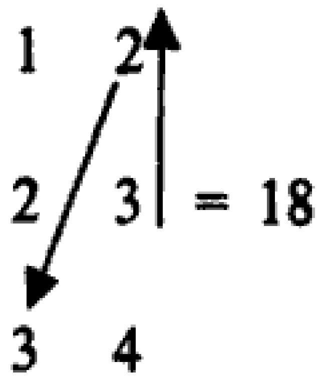
For Tens: $1 \times 1 \times 10 = 10$ (Two digits are taken from units place and one digit from tens place)



+



+



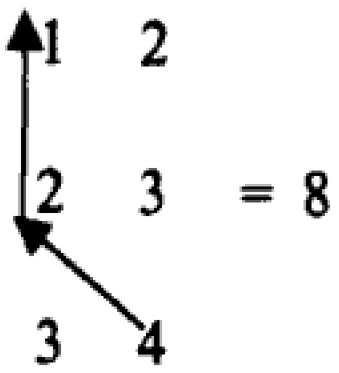
$12 + 16 + 18 = 46$

In the second step the consideration of obtaining the digits under tens place is aimed as shown in the diagrams, following Urdhva Tiryagbhyam. Thus the three possibilities are exhausted and the results are finally added up.

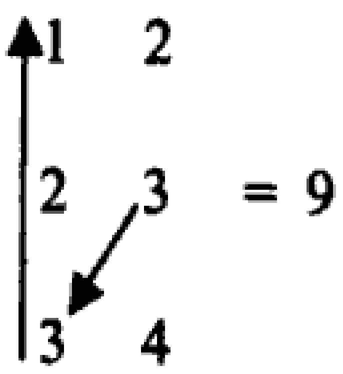
Step 3: Entering into 2nd column with two digits in the 2nd column and one digit in 1st column (completing the Permutations).

Status

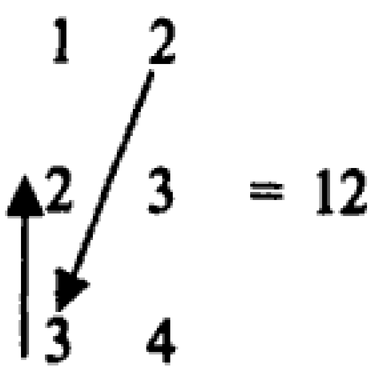
For Hundreds: $1 \times 10 \times 10 = 100$ (two digits from tens place and one digit from units place)



+



+



$8 + 9 + 12 = 29$

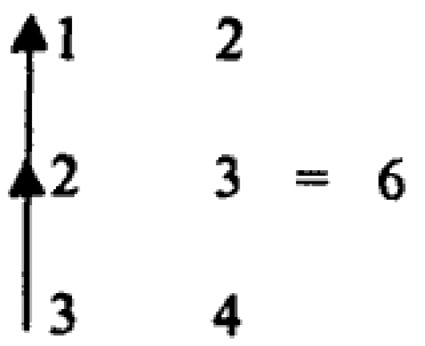
In this third step we aim at hundreds. Hundreds can be obtained by multiplying 2 digits under tens place and 1 digit under units place as shown in the above diagrams.

The multiplication of the digits in the units column is thus exhausted.

Step 4: All the digits of the 2nd column (Only Urdhva)

Status

For Thousands: $10 \times 10 \times 10 = 1000$ (All the digits are taken from tens place)



In this fourth step we work for thousands by applying Urdhva to all the three digits in the second column as given in the diagram, which is denoted by $1 \times 2 \times 3 = 6$.

The procedure takes care of multiplication of all the three numbers in a single step accordingly as the expected result in Units' place, Tens' place, Hundreds' place, etc. limiting to Urdhva Tiryagbhyam multiplication only. A perfect symmetry is very clear in the working of multiplication of three numbers as a pack. Final placement of result can be viewed as follows.

Units = 24
Tens = 46
Hundreds = 29
Thousands = 6 i.e., $24 + 460 + 2900 + 6000 = 9384$
This can be placed in two different ways:

Vedic Method (Right to Left):

1 2

2 3

3 4

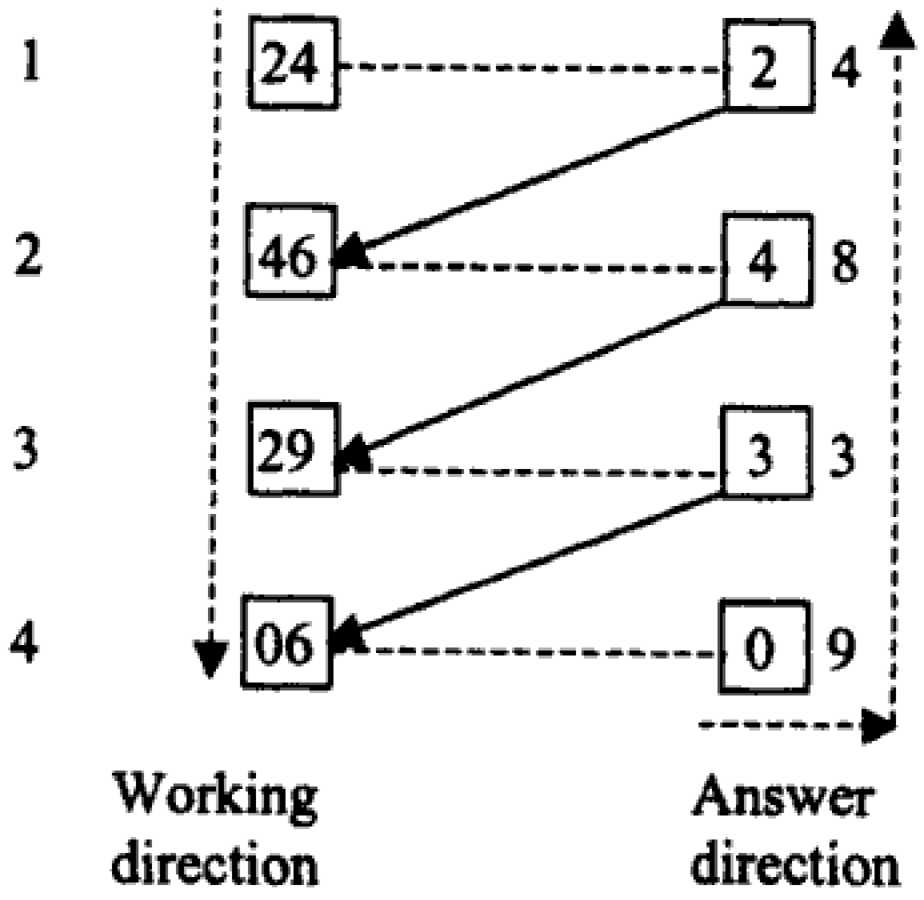
$\boxed{9\ \ 3\ \ 8\ \ 4}$

3 4 2

Ans.: 9384

Step Step Value Computed Value

of the Steps



This method is useful in finding out the exact value of the individual contributions of such successive multiplications starting from any position, i.e., units, tens, hundreds, thousands, ten-thousands, lakhs, etc. where as it is not possible to obtain such a value in current system unless the entire multiplication is carried out

Similarly, this method is extendable to multiplication of any number of digits by any number of numbers. The principle is same in working out in bits, the value in Units place, in tens place, in hundreds place, etc separately.

A few more illustrations are given below to show the symmetry in working and the ease with which one can separate out the working into bits.

Example 2: 12 × 23 × 34 × 45 × 56

Vedic Method (Right to Left)

Tens Units

1 2

2 3

3 4

4 5

5 6

$\boxed{236\quad 4\quad 7\quad 6\quad 8\quad 0}$

116 290 386 262 72

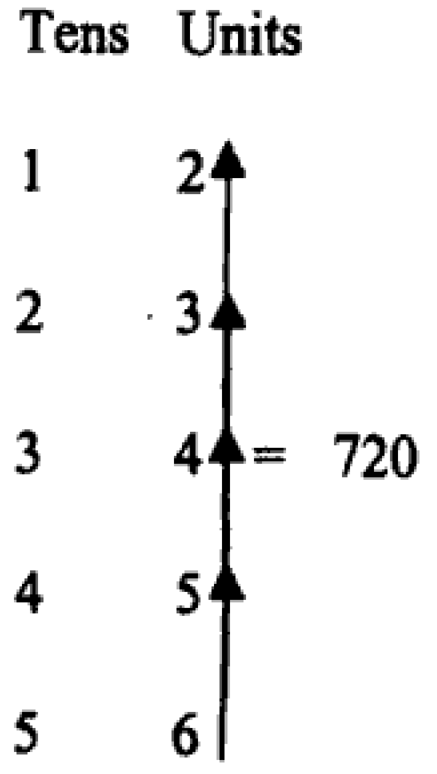
Ans.: 23647680

Steps are as follows: (Column-wise Multiplication)

Step 1: 1st column multiplication. All digits in the first column (Only Urdhva)

Status

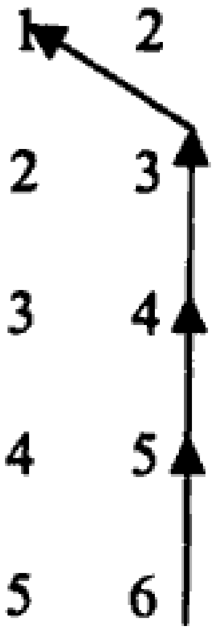
For Units: 1 × 1 × 1 × 1 × 1 = 1(all the digits are from the units place)



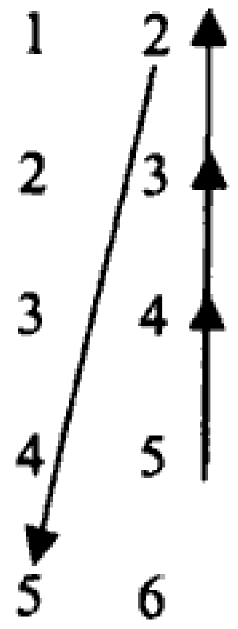
Step 2: Entering into 2nd column partially with 4 digits in 1st column and 1 digit in 2nd column and completing permutations.

Status

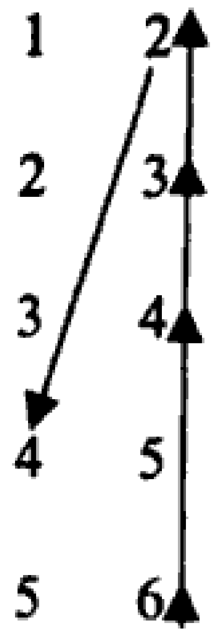
For Tens: $1 \times 1 \times 1 \times 1 \times 10 = 10$



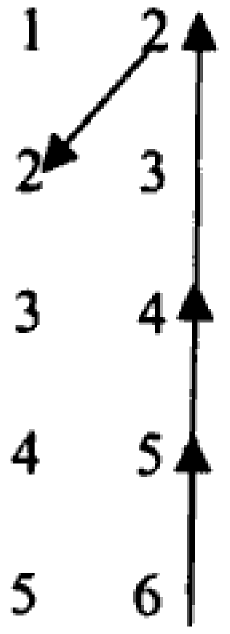
=360 +



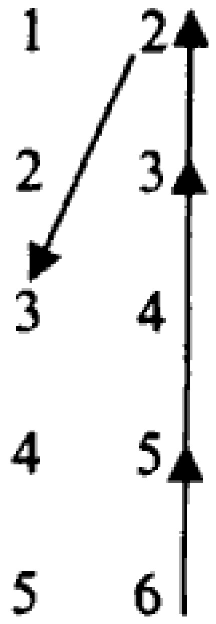
=600 +



=576 +



=480 +



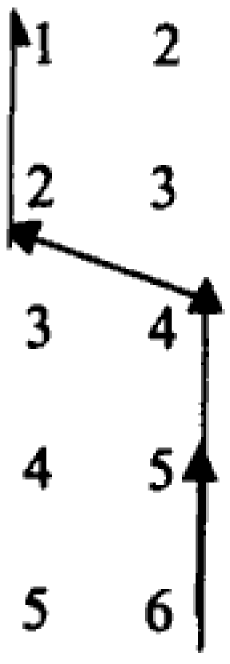
=540

360 + 600 + 576 + 480 + 540 = 2556

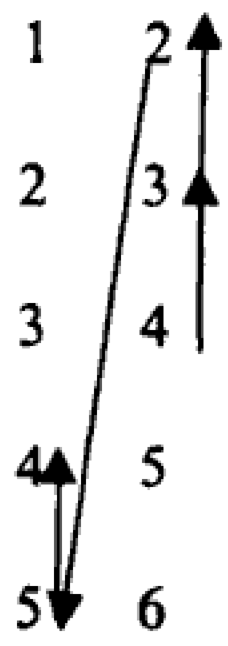
Step 3: Entering into 2nd column with 3 digits in the 1st column and 2 digits in the 2nd column and completing the permutations.

Status

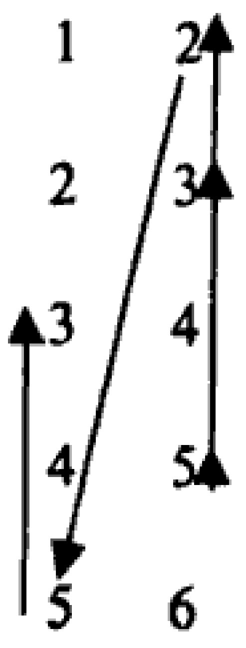
For Hundreds: $1 \times 1 \times 1 \times 10 \times 10 = 100$



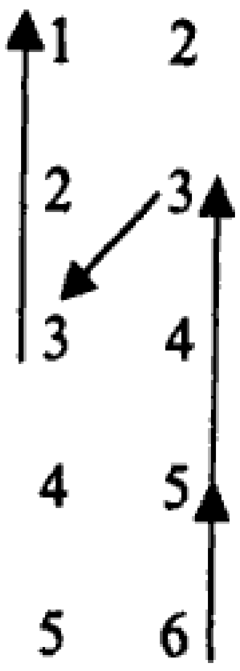
=240 +



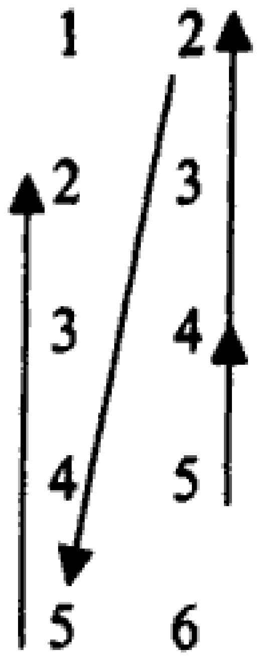
=480 +



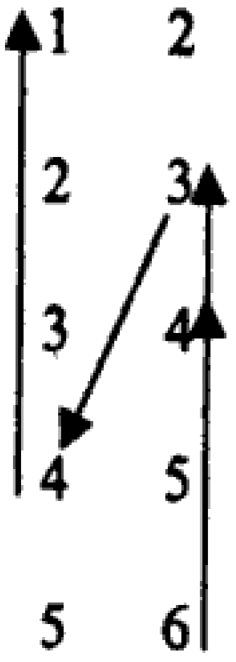
=450 +



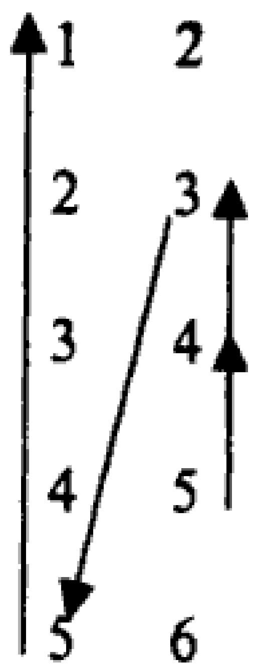
=270 +



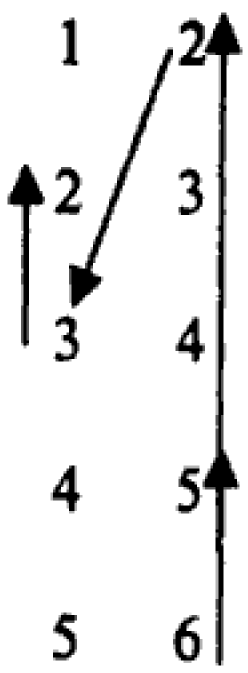
=400 +



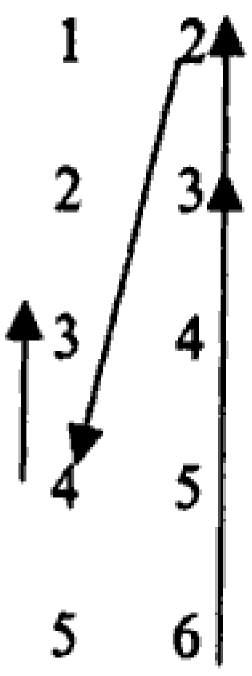
=288 +



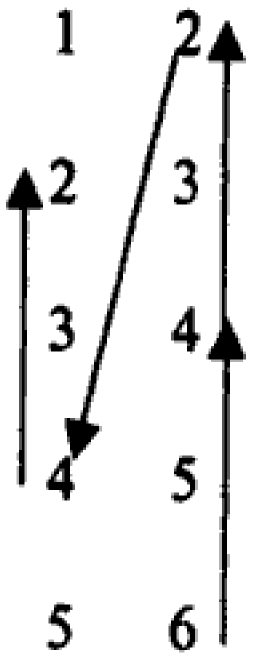
=300 +



=360 +



=432 +



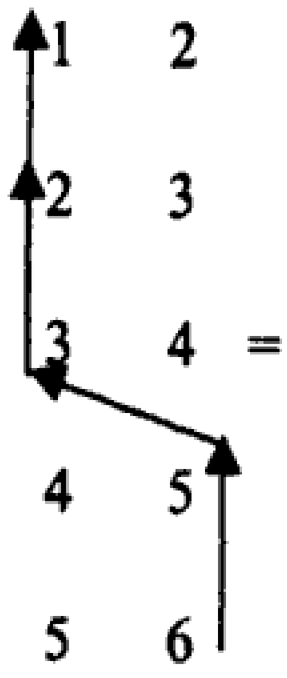
=384

240 + 480 + 450 + 270 + 400 + 288 + 300 + 360 + 432 + 384 = 3604

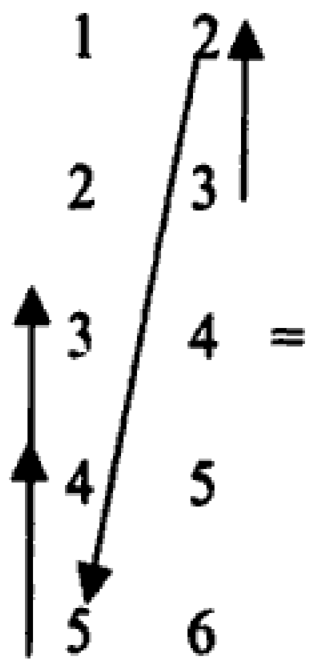
Step 4: Entering into 2nd column with two digits in the 1st column and three digits in the 2nd column and completing the permutations.

Status

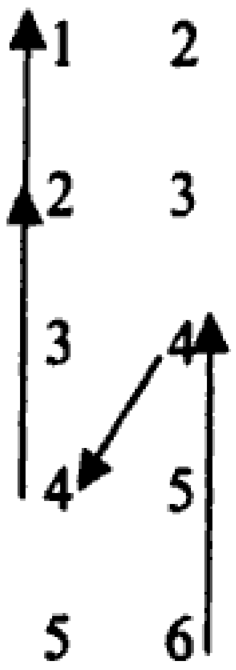
For Thousands: 1 × 1 × 10 × 10 × 10 = 1000



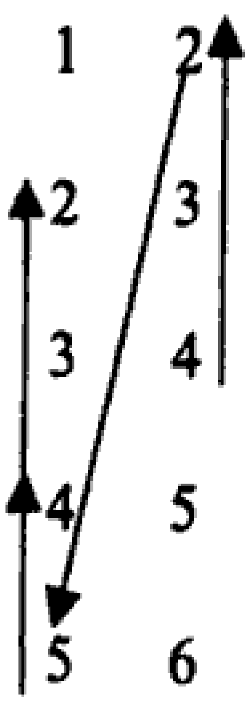
=180 +



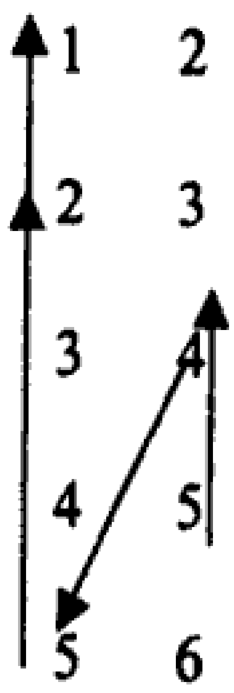
=360 +



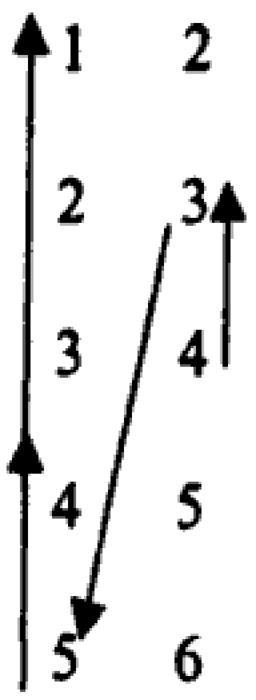
=192 +



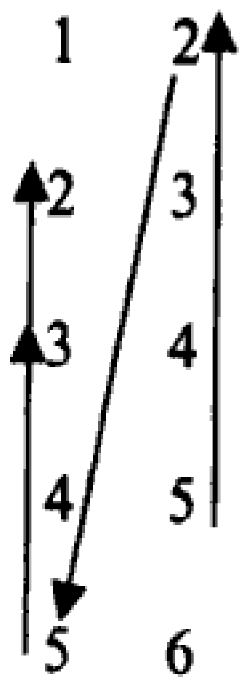
=320 +



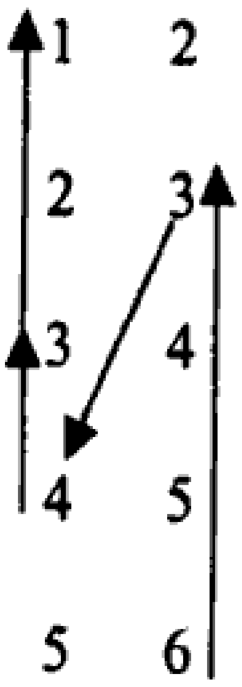
=200 +



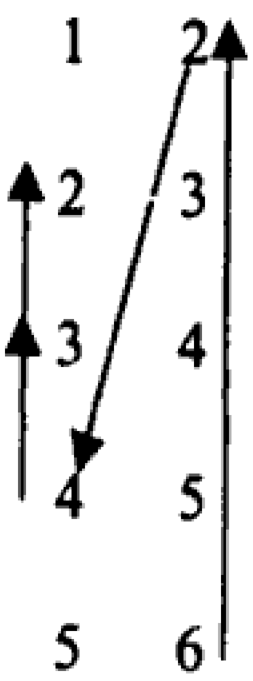
=240 +



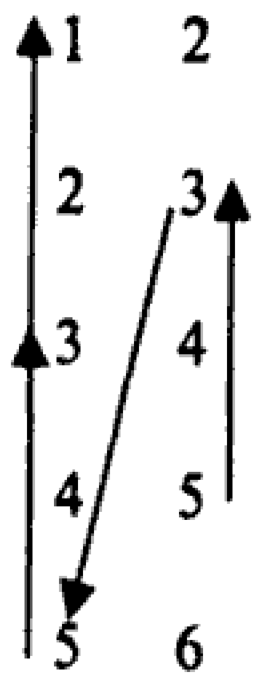
=300 +



=216 +



=288 +



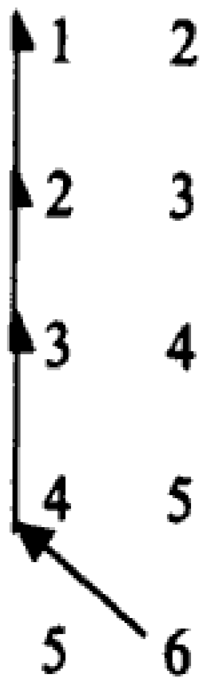
=225

180 + 360 + 192 + 320 + 200 + 240 + 300 + 216 + 288 + 225 = 2521

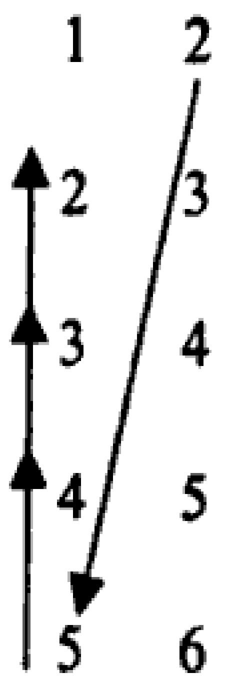
Step 5: Entering into 2nd column with 1 digit in the 1st column and 4 digits in the 2nd column and completing the permutations.

Status

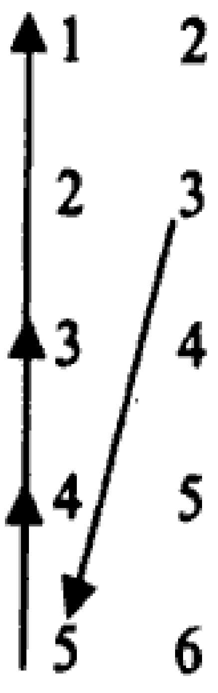
For Ten Thousands: $1 \times 10 \times 10 \times 10 \times 10 = 10000$



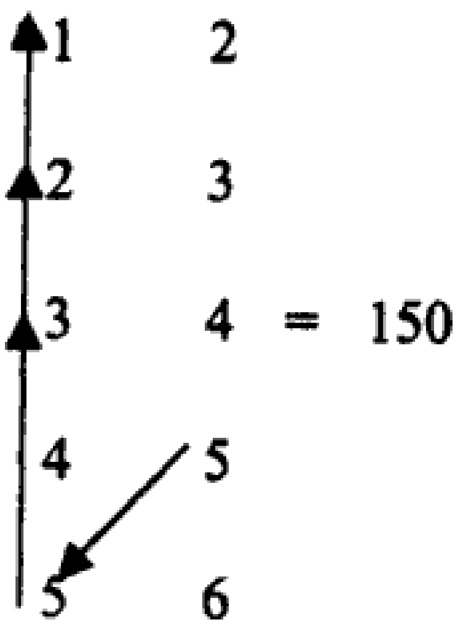
=144 +



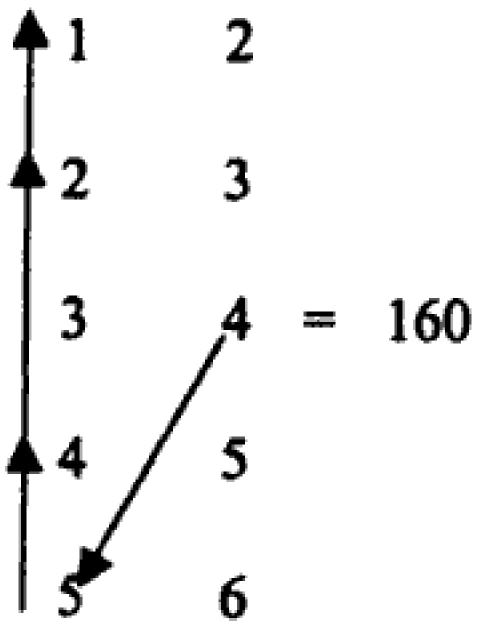
=240 +



=180 +



+

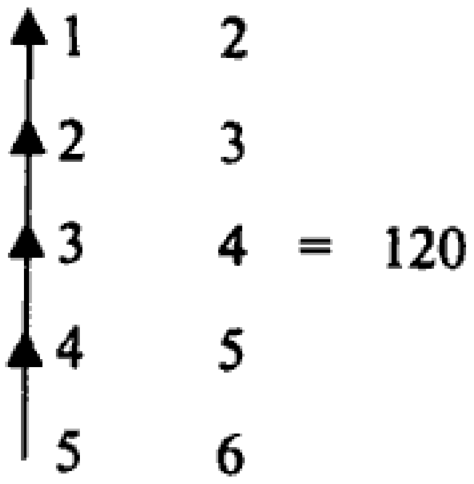


144 + 240 + 180 + 150 + 160 = 874

Step 6: All digits in the 2^{nd} column belonging to tens (Urdhva Only)

Status

For Lakhs: $10 \times 10 \times 10 \times 10 \times 10 = 100000$



Verification

720
25560
360400
2521000
8740000
12000000
23647680

Three-digit Numbers Multiplication:

Let us consider three number multiplication in series. For example:

a b c
d e f
g h k

This can be carried out in two ways applying Urdhva and Tiryag.

Right to left : Multiply each time three digits, one belonging to each of the given numbers, with no two digit lying in the same row of the above arrangement these are the i.e. allowed digits

Method I : By considering column wise multiplication either from right to left or left to right.

Step 1 : Multiply the first column numbers (Urdhva) i.e. k f c. this gives units and any remaining will go into either tens or hundreds as the case may be.

Step 2 : (i) Considering any two digits from the first column and multiplying them with the allowed digit of the second column (Urdhva and Tiryag). All such products are added to find place in 10's i.e. c f h + c k e + f k b

(ii) Considering any two digits from the first column first and multiplying them with the allowed digit of the third column. All such products are added to find the place in 100's i.e. c f g + c k d + f k a.

Step 3 : (i) Considering one digit in first column and multiplying with two allowed digits of the second column, the result gives 100's place. The procedure is continued till all the digits of the first column are exhausted. The sum of all these products gives a result in 100's i.e. k e b + f h b + c e h.

(ii) Considering one digit of the first column and multiplying it with two allowed digits of the third column. This procedure is exhausted with all the first column digits. The results so obtained are in 10,000's place i.e. k a d + f a g + c d g.

(iii) Multiplying one digit of the first column, one digit of the second column with the allowed digit of the third column. The results so obtained are in 1000's place. i.e., k c a + k b d + f b d + f h a + c e g + c h d.

Step 4 : Multiply the numbers of the second column (Urdhva) i.e. b e h. This gives digit of the 1000's place and any remaining will go into either 1000's or 10,000's place as the case may be.

Step 5 : Consider two digits from the second column and multiply them with the allowed digit of the first column. All such products are added to find the digit in 10,000's place i.e. b d g + e a g + h a d.

Step 6 : Multiplication each digit of the second column with two allowed digits of the first column to give lakhs value i.e., b e g + b h d + e h a.

Step 7 : The last step is to multiply all the digits of the first column (Urdhva) i.e. a d g. The result of which is in 10,00,000's place.

The final result is obtained by adding all the above individual values of different denominations.

Example 1: 312 × 231 × 123
Vedic Method (Right to Left)

				Hundreds	Tens	Units
				3	1	2
				2	3	1
				1	2	3
8	8	6	4	8	5	6
2	5	6				

Ans.: 8864856

The structural diagrams for column-wise multiplication are given here under, in which the first column is exhausted in the multiplication with itself and with the other columns. The same procedure is followed for the remaining columns. Care is taken to see that no repetition occurs. The same procedure is valid also from Left to Right.

Vedic method I (Right to left)

3 1 2
2 3 1
1 2 3
$\boxed{8\ 8\ 6\ 4\ 8\ 5\ 6}$
2 5 6 4 2

Current Method

```
   312               72072
   231                 123
   ---              ------
   312     and      216216
  936              144144
 624              72072
 -----            -------
 72072            8864856
```

Multiplication is always carried out by taking one digit from each number. The complete parts of the multiplication is worked out as follows:

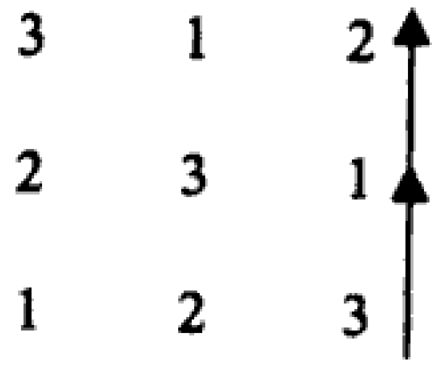
1) When all the digits belong to one column then only Urdhva operation is applied.

2) When digit(s) from one column are to be multiplied with digit(s) of the other columns, then the Urdhva with Tiryak or Tiryak alone are adopted depending on the multiplication of the digits from other columns.

The indication of such operations are clearly shown in diagrams and are to be understood as combination of two different operations belonging to different columns.

Diagrams are self-explanatory.

Step 1 : First Column (Urdhva only): (Status is given in brackets)

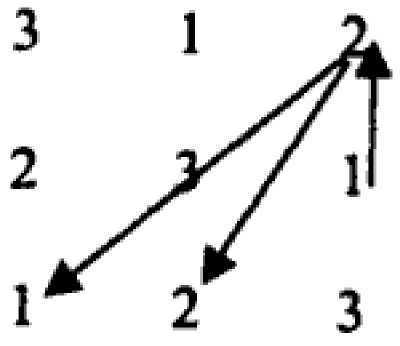


= (1)
6

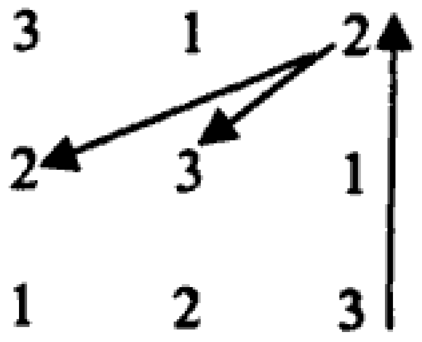
Current Method

```
   312               72072
   231                 123
   312              216216
  936              144144
 624              72072
 72072            8864856
```

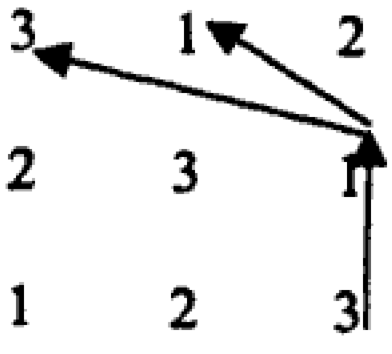
Step 2 (i) : First Column with two digits:



= (10), (100)
4, 2

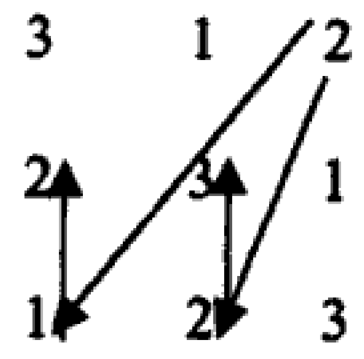


= (10), (100)
18, 12

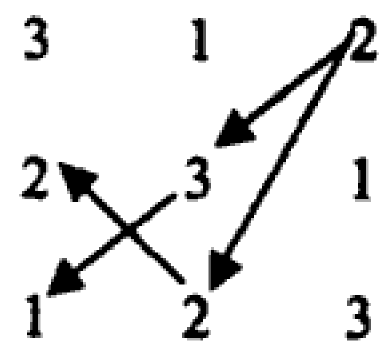


= (10), (100)
3, 9

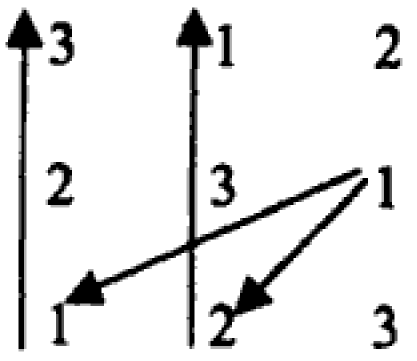
Step 3(i), (ii), (iii) : First Column with one digit:



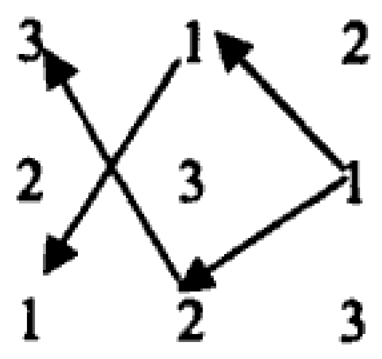
(100), (10,000)
= 12, 4



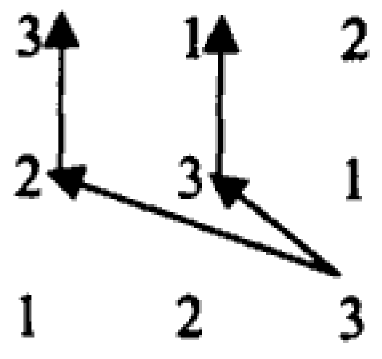
(1000), (1000)
= 6, 8



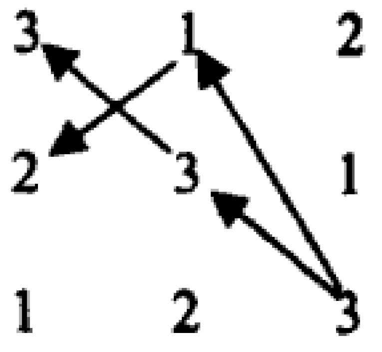
(100), (10,000)
= 2, 3



(1000), (1000)
= 1, 6



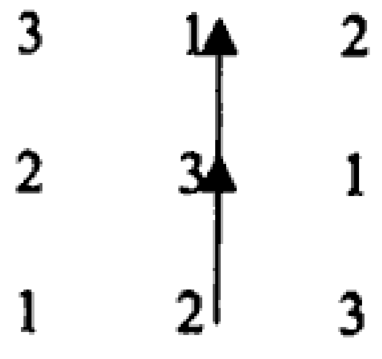
(100), (10000)
= 9, 18



(1000), (1000)
= 27, 6

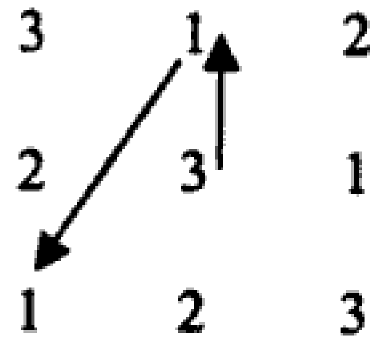
With this First column is exhausted.

Step 4 : Second Column (Urdhva Only):

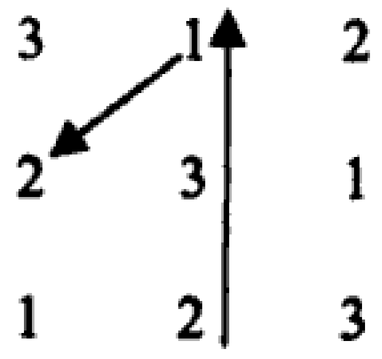


(1000)
= 6

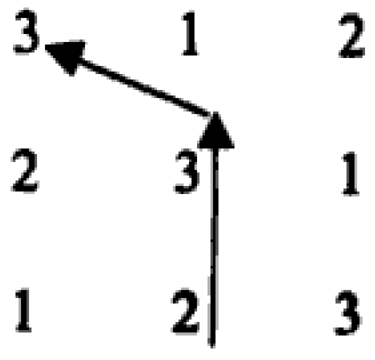
Step 5 : Second Column with two digits:



(10,000)
= 3

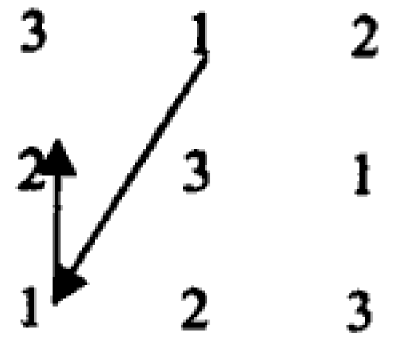


(10,000)
= 4

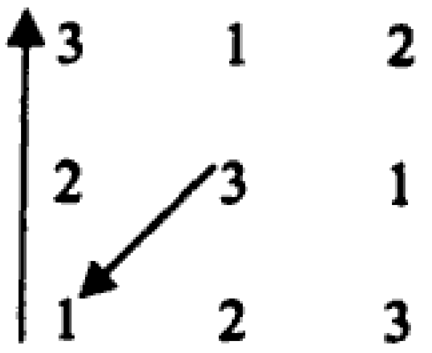


(10,000)
= 18

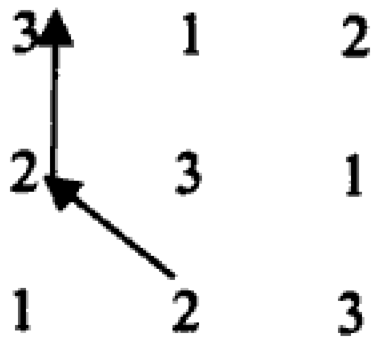
Step 6 : Second Column with one digit:



(10,0000)
= 2



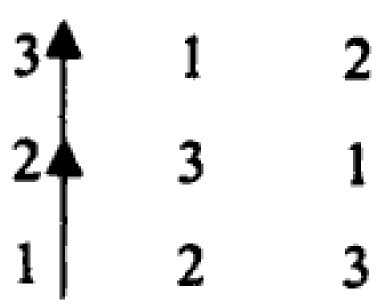
(10,0000)
= 9



(10,0000)
= 12

With this second column is exhausted.

Third Column (Urdhva Only):



(1000000)
= 6

For Units = 6
For Tens = 4 + 18 + 3 = 25
For Hundreds = 2 + 12 + 9 + 12 + 2 + 9 = 46
For Thousands = 6 + 8 + 1 + 6 + 6 + 27 + 6 = 60
For Ten Thousands = 4 + 3 + 18 + 3 + 4 + 18 = 50
For Lakhs = 2 + 9 + 12 = 23
For Ten Lakhs = 6

Answer is
6
250
4600
60000
500000
2300000
<u>6000000</u>
<u>8864856</u>

Another interesting feature in applying this method is that one need not be afraid of bigger digits than 5. In all such cases the vinculum method solves the problem. This is also shown by many examples in the next chapter.

Method II : To collect the different denominations from different multiplication results of units, tens, hundred etc. in the series multiplication :

a b c
d e f
g h k

Here, the method is to collect different multiplications using different columns and finally arriving at the same denominations such as units or tens or hundreds etc. through searching for such combinations of multiplications.

The combination multiplication c f h + c e k + k f b gives 10's and the combination multiplication giving rise to 100's can be obtained by multiplying two digits in the second column with one digit in first column (k e b + f b h + c e h). One digit of the first column is multiplied with two allowed digits in last column to get the digit in hundreds (i.e. units column). (k f a + k c d + f c g) i.e. searching for all possible different combinations of multiplications of unit digits, 10's digits and 100's digits and so on.

The authors find that the first method is more systematic because one may not miss the like denominations in the final result. Also one may try left to right multiplication to obtain the same results using any one of the above two methods.

Example 1 : 312 x 231 x 123
Vedic method II (Right to left)

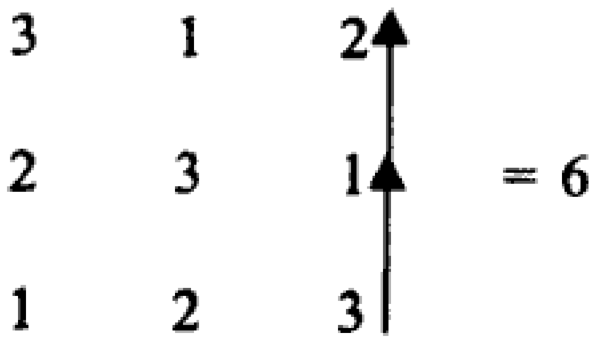
				Hundreds	Tens	Units
				3	1	2
				2	3	1
				1	2	3
8	8	6	4	8	5	6
2	5	6	4	2		

Answer : 8864856

Step 1: 1st column Urdhva Only.

Status

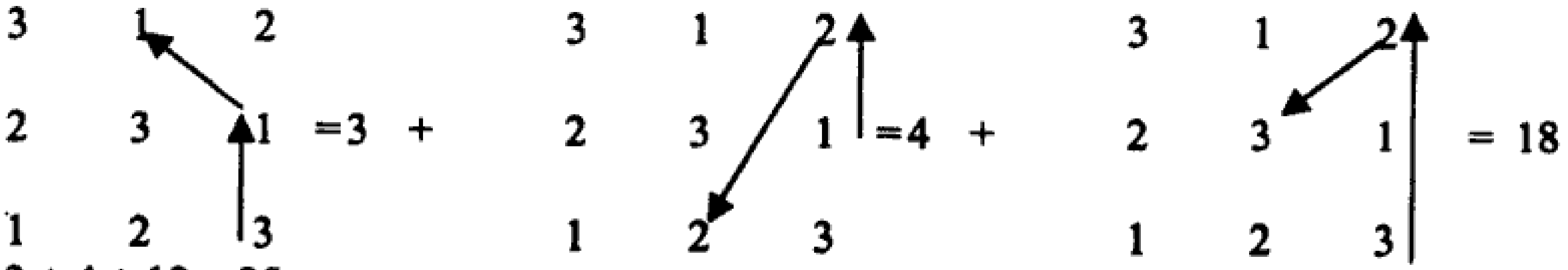
For Units: 1 × 1 × 1 = 1 (all digits in the units place)



Step 2: Entering into the 2nd column with two digits in the 1st column and one digit in the 2nd column

Status

For Tens: 1 × 1 × 10 = 10(two digits from units and one digit from tens place)



3 + 4 + 18 = 25

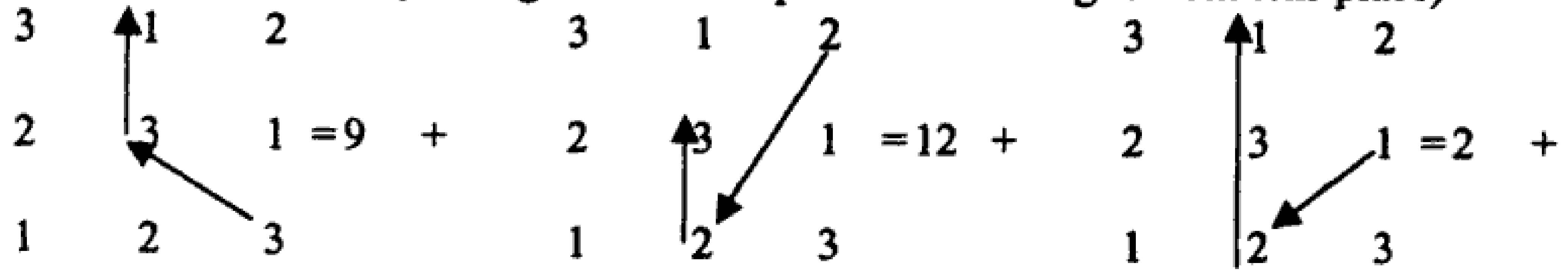
Step 3:

For Hundreds:

(a) Entering into 2nd column from 1st column with two digits in the 2nd column and one digit in the 1st column.

Status

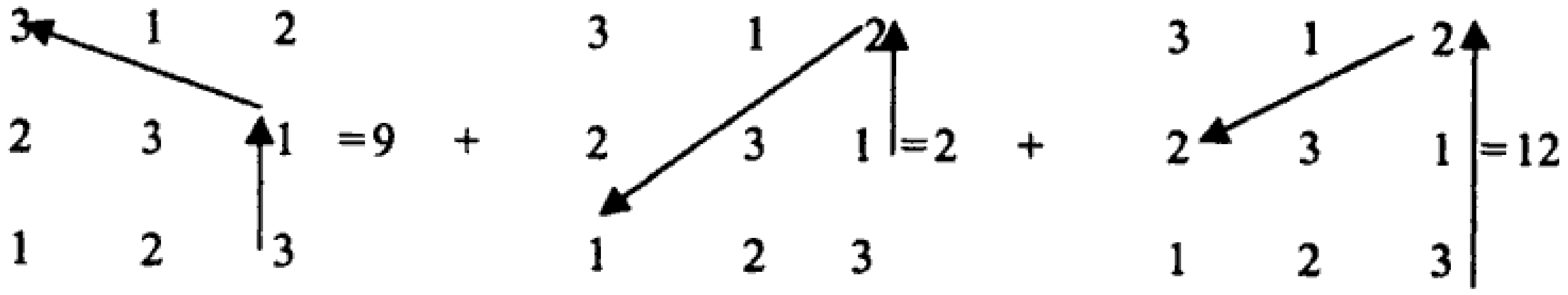
$1 \times 10 \times 10 = 100$ (one digit from units place and two digits from tens place)



(b) Entering into 3rd column from 1st column with one digit in the 3rd column and two digits in the 1st column.

Status

$1 \times 1 \times 100 = 100$ (two digits from units place and one digit from hundreds place)



(a) + (b) = 9 + 12 + 2 + 9 + 2 + 12 = 46

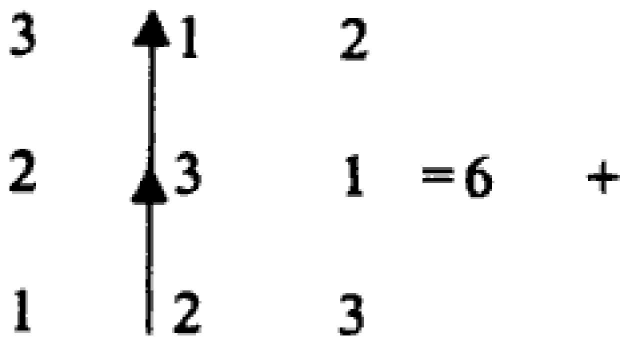
Step 4:

For Thousands:

(a) 2nd column (Urdhva Only)

Status

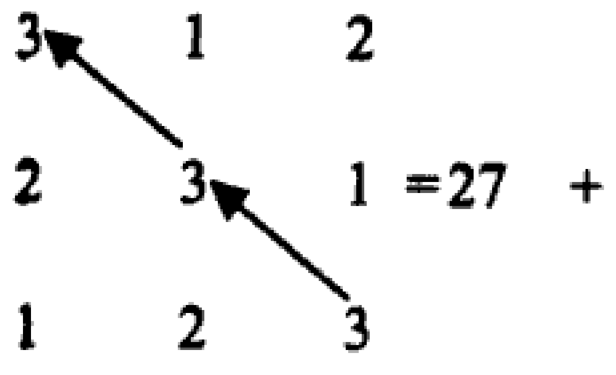
$10 \times 10 \times 10 = 1000$ (all digits from tens place)

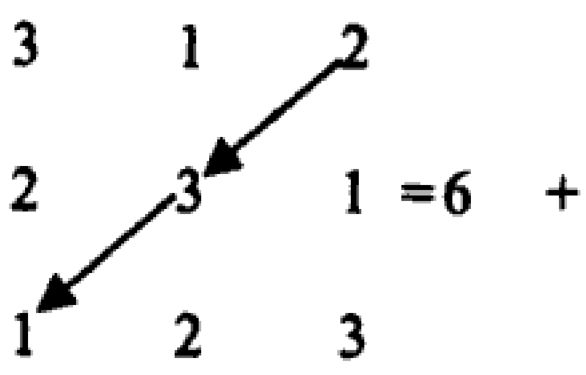


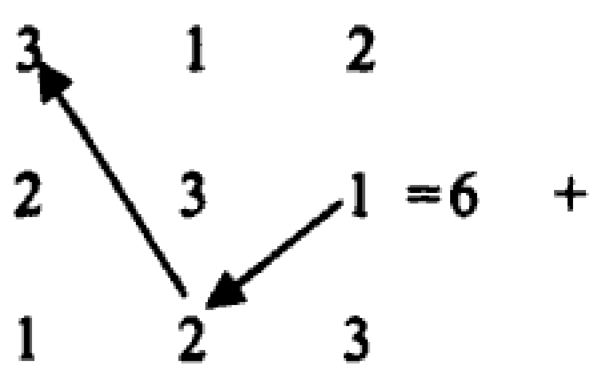
(b) Entering into 3rd and 2nd columns from 1st column taking one digit from each column.

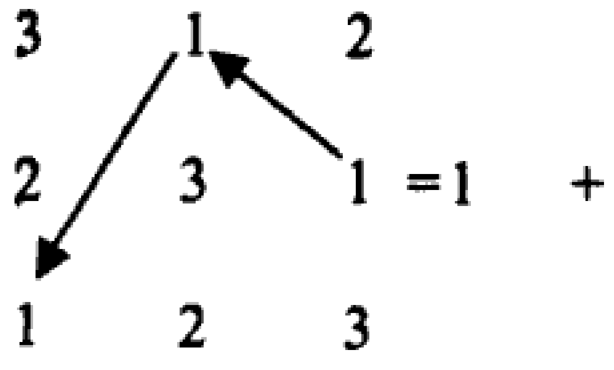
Status

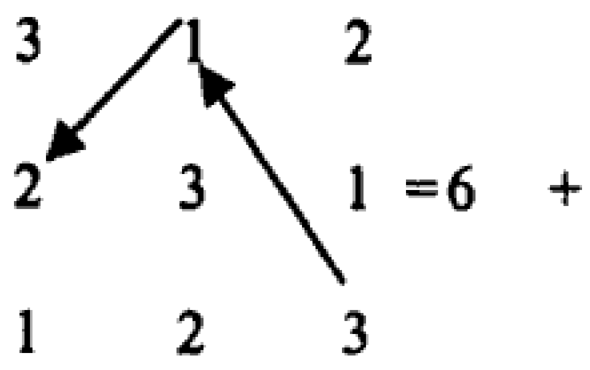
1 × 10 × 100 = 1000(taking one digit from units place, one digit from tens place and one digit from hundreds place)

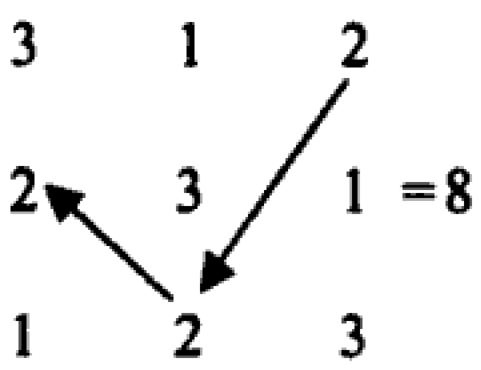












(a) + (b) = 6 + 27 + 6 + 6 + 1 + 6 + 8 = 60

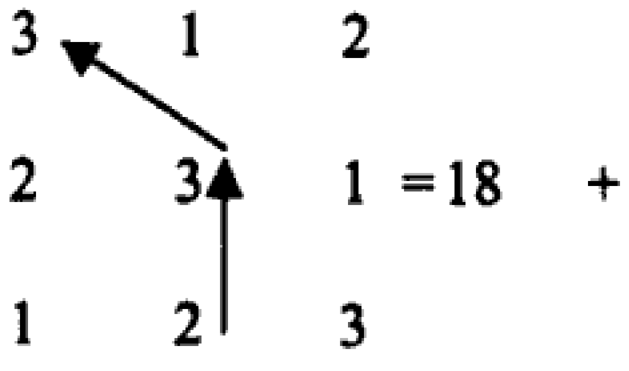
Step 5:

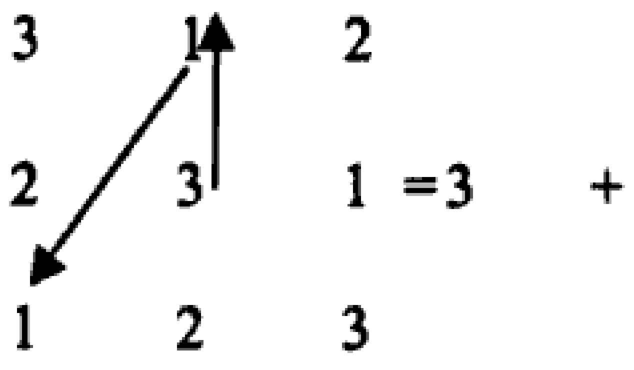
For Ten Thousands:

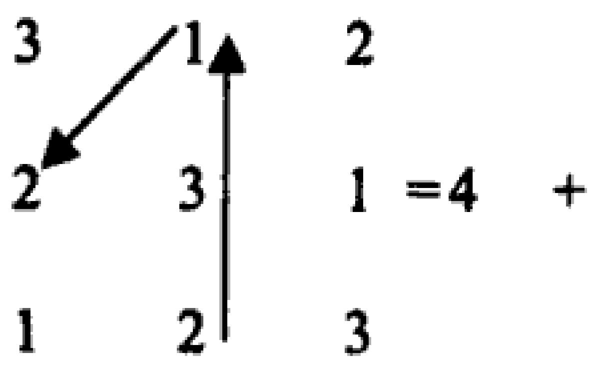
(a) Entering into 3rd column from 2nd column with one digit in the 3rd column and two digits in the 2nd column

Status

10 × 10 × 100 = 10000 (two digits from tens place and one digit from hundreds place)



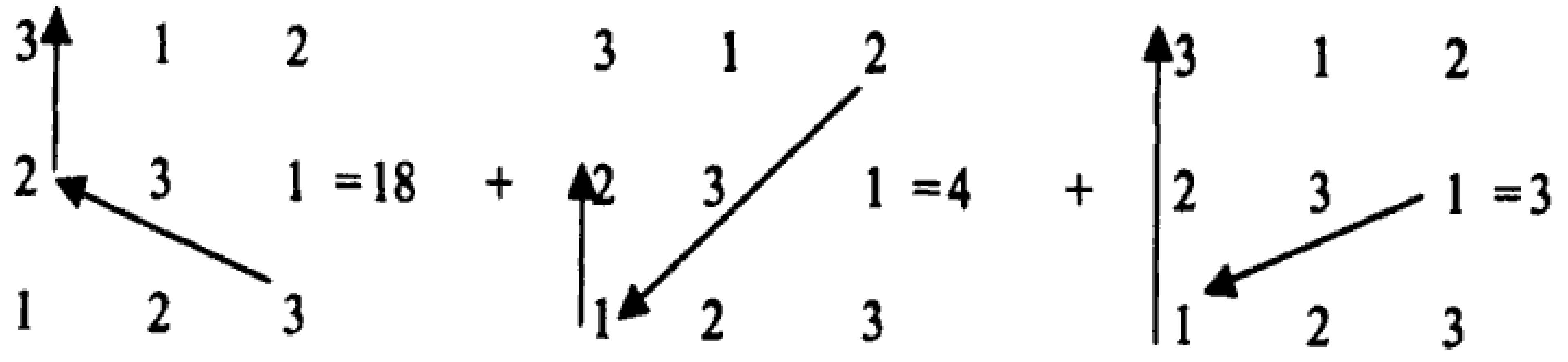




(b) Entering into 3rd column from 1st column with two digits in the 3rd column and one digit in the 1st column.

Status

1 × 100 × 100 = 10000 (one digit form units place and two digits from hundreds place)

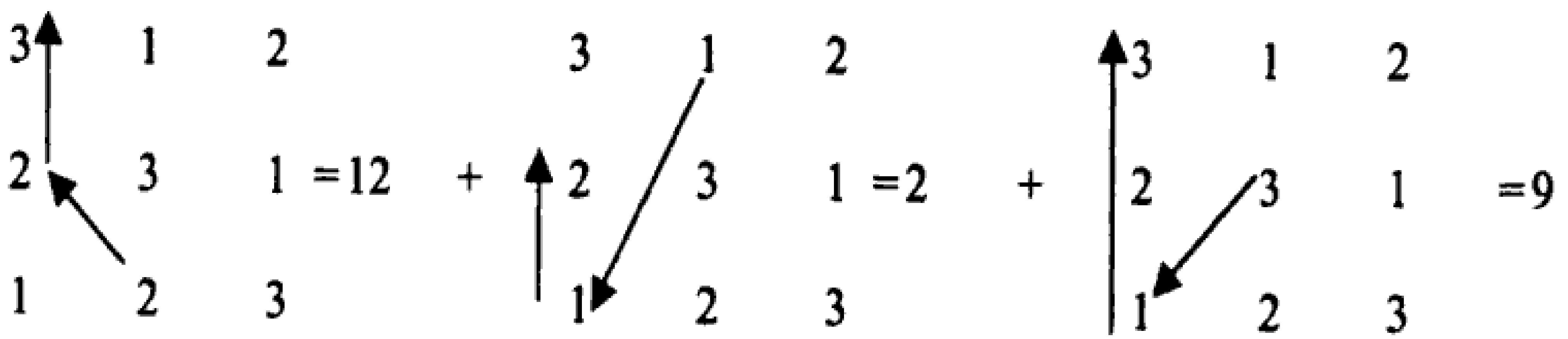


(a) + (b) = 18 + 3 + 4 + 18 + 4 + 3 = 50

Step 6: Entering into 3rd column from 2nd column with two digits in the 3rd column and one digit in the 2nd column.

Status

For Lakhs: 10 × 100 × 100 = 100000 (One digit from tens place and two digits from hundreds place)

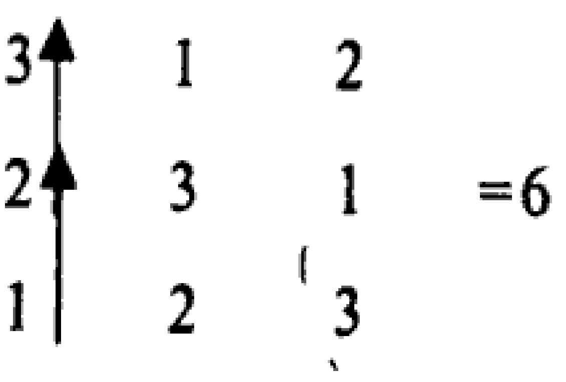


12 + 2 + 9 = 23

Step 7: 3rd column (Urdhva Only).

Status

For Ten Lakhs: 100 × 100 × 100 = 1000000 (all the digits in the hundreds place)



Verification

6
250
4600
60000
500000
2300000
6000000
8864856

For a Title practice and more familiarity the multiplication details of five numbers with three digits each is given in the appendix as an exercise.

(viii) Application of Urdhva Tiryagbhyam in multiplying two or more number of Polynomials (V.M.):

In this problem, the terms of the various degrees occurring in the polynomials are to be considered for multiplication. The procedure is same as applied in multiplication between any two numbers.

The same Urdhva Tiryak Multiplication that is applicable to numbers can be extended to polynomial multiplication. The difference is that there is no carrying over of the result of one step to other step, as it is very clear that the status of each step is different from the status of the other step. It is also seen that the multiplication is carried out in a single line and mentally, as such this method is definitely more elegant when compared with the current method. This can be easily worked out form left to right also. Various examples worked out clearly indicate the ease of Vedic Multiplication for polynomials.

This is extendable to multiplication of any number of polynomials by the above method.

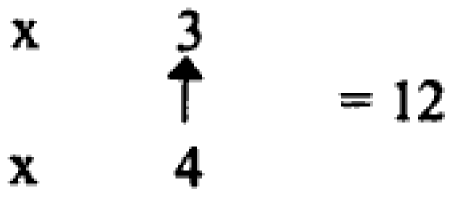
Current Method **Vedic Method**

1) $(x + 3)(x + 4)$
$= x(x + 4) + 3(x + 4)$
$= x^2 + 4x + 3x + 12$
$= x^2 + 7x + 12$

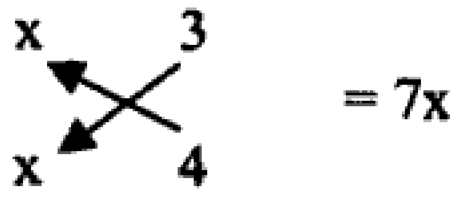
1)
$$\begin{array}{r} x + 3 \\ \underline{x + 4} \\ x^2 + 7x + 12 \end{array}$$

Vedic Method Steps:

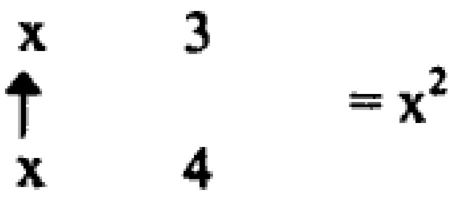
Step 1:



Step 2:



Step 3:



2) $(x - 2)(x + 7)$
$= x(x + 7) - 2(x + 7)$
$= x^2 + 7x - 2x - 14$
$= x^2 + 5x - 14$

2)
$$\begin{array}{r} x - 2 \\ \underline{x + 7} \\ x^2 + 5x - 14 \end{array}$$

3) $(x - 12)(x - 3)$
$= x(x - 3) - 12(x - 3)$
$= x^2 - 3x - 12x + 36$
$= x^2 - 15x + 3$

3)
$$\begin{array}{r} x - 12 \\ \underline{x - 3} \\ x^2 - 15x + 36 \end{array}$$

4) $(-x+5)(-x-5)$

$= -x(-x-5)+5(-x-5)$
$= x^2+5x-5x-25$
$= x^2-25$

4)
$$\begin{array}{r} -x+\ 5 \\ \underline{-x-\ 5} \\ x^2-25 \end{array}$$

5) $(2x-3)(x+8)$

$= 2x(x+8)-3(x+8)$
$= 2x^2+16x-3x-24$
$= 2x^2+13x-24$

5)
$$\begin{array}{r} 2x-\ 3 \\ \underline{x+\ 8} \\ 2x^2+13x-\ 24 \end{array}$$

6) $(5x-6)(2x+3)$

$= 5x(2x+3)-6(2x+3)$
$= 10x^2+15x-12x-18$
$= 10x^2+3x-18$

6)
$$\begin{array}{r} 5x-\ 6 \\ \underline{2x+\ 3} \\ 10x^2+3x\ -18 \end{array}$$

7) $(3x-5y)\ (3x+5y)$

$=3x(3x+5y)-5y(3x+5y)$
$= 9x^2+15xy-15xy-25y^2$
$= 9x^2-25y^2$

7)
$$\begin{array}{l} 3x-\ 5y \\ \underline{3x+\ 5y} \\ 9x^2-25y^2 \end{array}$$

8) $(a-2b)(a-8b)$
$= a(a-8b)-2b(a-8b)$
$= a^2-8ab-2ab-16b^2$
$= a^2-10ab+16b^2$

8)
$$\begin{array}{r} a-\ 2b \\ \underline{\quad\quad a-\ 8b} \\ a^2-10ab+16b^2 \end{array}$$

9) $(xy-ab)(xy+ab)$
$= xy(xy+ab)-ab(xy+ab)$
$= x^2y^2+xyab-xyab-a^2b^2$
$= x^2y^2-a^2b^2$

9)
$$\begin{array}{r} xy-\ ab \\ \underline{xy+\ ab} \\ x^2y^2-a^2b^2 \end{array}$$

10) $(a+b+c)(a+b-c)$

$= (a+b)^2-c^2$
$= a^2+2ab+b^2-c^2$

10)
$$\begin{array}{r} a\ +b\ +c \\ \underline{\quad\quad a\ +b\ -c} \\ a^2+2ab+b^2-c^2 \end{array}$$

11) $(x^2+5x+6)(x^2+3x\ +2)$

$= x^4+3x^3+2x^2+5x^3+15x^2+10x$
$\ \ +6x^2+18x+12$
$= x^4+8x^3+23x^2+28x+12$

11)
$$\begin{array}{r} x^2+\ 5x+\ \ 6 \\ \underline{x^2+\ 3x+\ \ 2} \\ x^4+8x^3+23x^2+28x+12 \end{array}$$

Current Method

12) $(x^3 + 6x^2 + 11x + 6)(x^3 + 9x^2 + 26x + 24)$

$= x^6 + 9x^5 + 26x^4 + 24x^3 + 6x^5 + 54x^4 + 156x^3 + 144x^2$

$+ 11x^4 + 99x^3 + 286x^2 + 264x + 6x^3 + 54x^2 + 156x + 144$

$= x^6 + 15x^5 + 91x^4 + 285x^3 + 484x^2 + 420x + 144$

13) $(x^4 + 2x^3 + 3x^2 + 4x + 5)(2x^4 + 3x^3 + 2x^2 + x - 10)$

$= 2x^8 + 3x^7 + 2x^6 + x^5 - 10x^4 + 4x^7 + 6x^6 + 4x^5 + 2x^4 - 20x^3$

$+ 6x^6 + 9x^5 + 6x^4 + 3x^3 - 30x^2 + 8x^5 + 12x^4 + 8x^3 + 4x^2 -$

$40x + 10x^4 + 15x^3 + 10x^2 + 5x - 50$

$= 2x^8 + 7x^7 + 14x^6 + 22x^5 + 20x^4 + 6x^3 - 16x^2 - 35x - 50$

14) $(x^5 + 2x^4 + 4x^3 + x^2 + 2x + 1)(3x^5 + x^4 + x^3 + 2x^2 + 3x + 5)$

$= 3x^{10} + x^9 + x^8 + 2x^7 + 3x^6 + 5x^5 + 6x^9 + 2x^8 + 2x^7 + 4x^6 +$

$6x^5 + 10x^4 + 12x^8 + 4x^7 + 4x^6 + 8x^5 + 12x^4 + 20x^3 + 3x^7 + x^6$

$+ x^5 + 2x^4 + 3x^3 + 5x^2 + 6x^6 + 2x^5 + 2x^4 + 4x^3 + 6x^2 + 10x +$

$3x^5 + x^4 + x^3 + 2x^2 + 3x + 5$

$= 3x^{10} + 7x^9 + 15x^8 + 11x^7 + 18x^6 + 25x^5 + 27x^4 + 28x^3 +$

$13x^2 + 13x + 5$

15) $(3x^2 - 2x - 5)(8x - 10)$

$= 8x(3x^2 - 2x - 5) - 10(3x^2 - 2x - 5)$

$= 24x^3 - 16x^2 - 40x - 30x^2 + 20x + 50$

$= 24x^3 - 46x^2 - 20x + 50$

Vedic Method

12)

$$\begin{array}{r} x^3 + \ \ 6x^2 + \ 11x + \ 6 \\ \underline{x^3 + \ \ 9x^2 + \ 26x + \ 24} \\ x^6 + 15x^5 + 91x^4 + 285x^3 + 484x^2 + 420x + 144 \end{array}$$

13)

$$\begin{array}{r} x^4 + 2x^3 + 3x^2 + \ 4x + \ 5 \\ \underline{2x^4 + 3x^3 + 2x^2 + \ \ x - 10} \\ 2x^8 + 7x^7 + 14x^6 + 22x^5 + 20x^4 + 6x^3 - 16x^2 - 35x - 50 \end{array}$$

14)

$$\begin{array}{r} x^5 + 2x^4 + 4x^3 + \ \ x^2 + 2x + 1 \\ \underline{3x^5 + \ x^4 + \ x^3 + 2x^2 + 3x + 5} \\ 3x^{10} + 7x^9 + 15x^8 + 11x^7 + 18x^6 + 25x^5 + 27x^4 + 28x^3 + 13x^2 + 13x + 5 \end{array}$$

15)

$$\begin{array}{r} 3x^2 - 2x - \ 5 \\ \underline{0 \ + \ 8x - 10} \\ 24x^3 - 46x^2 - 20x + 50 \end{array}$$

Current Method	Vedic Method
$x^5+2x^3+2x+1)(x^4+x^2+5x+3$ $x^9+x^7+5x^6+3x^5+2x^7+2x^5+10x^4+6x^3+$ $2x^5+2x^3+10x^2+6x+x^4+x^2+5x+3$ $x^9+3x^7+5x^6+7x^5+11x^4+8x^3+11x^2+11x+3$	$x^5+0+2x^3+0+2x+1$ $\underline{0+x^4+0+x^2+5x+3}$ $x^9+3x^7+5x^6+7x^5+11x^4+8x^3+11x^2+11x+3$
$(x^2-xy+x+y^2+y+1)(x+y-1)$ $=x^3-x^2y+x^2+xy^2+xy+x+yx^2-xy^2+xy+y^3$ $+y^2+y-x^2+xy-x-y^2-y-1$ $=x^3+y^3+3xy-1$	17) $x^2-xy+y^2+x+y+1$ $\underline{0+0+0+x+y-1}$ $x^3-x^2y+x^2y+xy^2-xy^2-x^2+x^2+y^3+xy+2xy-y^2+y^2-1$ $=x^3+y^3+3xy-1$
$(27x^3-36ax^2+48a^2x-64a^3)(3x+4a)$ $=81x^4-108ax^3+144a^2x^2-192a^3x+108ax^3-$ $144a^2x^2+192a^3x-256a^4$ $=81x^4-256a^4$	18) $27x^3-36ax^2+48a^2x-64a^3$ $\underline{0+0+3x+4a}$ $81x^4-256a^4$
$(-3a^2b^2+4ab^3+15a^3b)(5a^2b^2+ab^3-3b^4)$ $=-15a^4b^4-3a^3b^5+9a^2b^6+20a^3b^5+4a^2b^6-12ab^7$ $+75a^5b^3+15a^4b^4-45a^3b^5$ $=75a^5b^3-12ab^7+13a^2b^6-28a^3b^5$	19) $-3a^2b^2+4ab^3+15a^3b$ $\underline{5a^2b^2+ab^3-3b^4}$ $-15a^4b^4+20a^3b^5-3a^3b^5+4a^2b^6+75a^5b^3+9a^2b^6+15a^4b^4+12ab^7-45a^3b^5$ $=75a^5b^3-12ab^7+13a^2b^6-28a^3b^5$

Current Method

20) $(a^2x - ax^2 + x^3 - a^3)(x + a)$

$= a^2x^2 - ax^3 + x^4 - a^3x + a^3x - a^2x^2 + ax^3 - a^4$

$= x^4 - a^4$

21) $(-x^3y + y^4 + x^2y^2 + x^4 + xy^3)(x + y)$

$= -x^4y + xy^4 + x^3y^2 + x^5 + x^2y^3 - x^3y^2 + y^5 + x^2y^3 + x^4y + xy^4$

$= x^5 + y^5 + 2x^2y^3 + 2xy^4$

Vedic Method

20)

$$\begin{array}{r} a^2x - ax^2 + x^3 - a^3 \\ 0 + 0 + x + a \\ \hline a^2x^2 - ax^3 + a^3x + x^4 - a^2x^2 - a^3x + ax^3 + a^4 \end{array}$$

$= x^4 - a^4$

21)

$$\begin{array}{r} -x^3y + y^4 + x^2y^2 + x^4 + xy^3 \\ 0 + 0 + 0 + x + y \\ \hline -x^4y + xy^4 - x^3y^2 + x^3y^2 + y^5 + x^5 + x^2y^3 + x^2y^3 + x^4y + xy^4 \end{array}$$

$= x^5 + y^5 + 2x^2y^3 + 2xy^4$

Current Method

22) $(-2x^3y + y^4 + 3x^2y^2 + x^4 - 2xy^3)(x^2 + 2xy + y^2)$

$= -2x^5y + x^2y^4 + 3x^4y^2 + x^6 - 2x^3y^3 - 4x^4y^2 + 2xy^5 + 6x^3y^3 + 2x^5y - 4x^2y^4 - 2x^3y^3 + y^6 + 3x^2y^4 + x^4y^2 - 2xy^5$

$= x^6 + y^6 + 2x^3y^3$

Vedic Method

22)

$$\begin{array}{r} -2x^3y + y^4 + 3x^2y^2 + x^4 - 2xy^3 \\ 0 + 0 + x^2 + 2xy + y^2 \\ \hline -2x^5y + x^2y^2 - 4x^4y^2 + 3x^4y^2 + 2xy^5 - 2x^3y^3 + x^6 + 6x^3y^3 + y^6 + 2x^5y - 2x^3y^3 + 3x^2y^4 - 4x^2y^4 + x^2y^4 - 2xy^5 \end{array}$$

$= x^6 + y^6 + 2x^3y^3$

When in case of multiplication of number of polynomials in a single stretch, the Vedic method definitely show its elegance. Its working method is adopted from the working of series multiplication of many numbers in a single step, which is already explained earlier. An example of multiplication of three polynomials is given below.

The coefficients of x^2, x and the constant term of the three polynomials are written in order. The method of multiplication is exactly same as is explained in series multiplication. The multiplication is carried out by collecting the x coefficients and x^2 coefficients, etc separately.

Current Method

23) $(2x^2 + 3x + 5)(x^2 + 5x + 1)(3x^2 + x + 6)$ First multiply any two and the result is multiplied by the 3rd

$$(2x^2 + 3x + 5)(x^2 + 5x + 1)$$
$$= 2x^4 + 3x^3 + 5x^2 + 10x^3 + 15x^2 + 25x + 2x^2 + 3x + 5$$
$$= 2x^4 + 13x^3 + 22x^2 + 28x + 5$$
$$(2x^4 + 13x^3 + 22x^2 + 28x + 5)(3x^2 + x + 6)$$
$$= 6x^6 + 39x^5 + 66x^4 + 84x^3 + 15x^2 + 2x^5 + 13x^4 + 22x^3 + 28x^2 + 5x + 12x^4 + 78x^3 + 132x^2 + 168x + 30$$
$$= 6x^6 + 41x^5 + 91x^4 + 184x^3 + 175x^2 + 173x + 30$$

Vedic Method

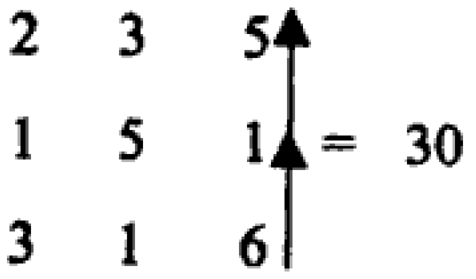
23)

$$\begin{array}{r} 2x^2 + 3x + 5 \\ x^2 + 5x + 1 \\ 3x^2 + \ x + 6 \\ \hline 6x^6 + 41x^5 + 91x^4 + 184x^3 + 175x^2 + 173x + 30 \end{array}$$

				Coeff. of x^2 term	Coeff. of x term	Constant term
				2	3	5
				1	5	1
				3	1	6
6	41	91	184	175	173	30
x^6	x^5	x^4	x^3	x^2	x	constant

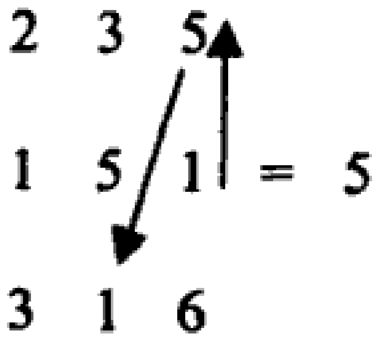
By using the symmetrical diagrams, of the series multiplication described earlier.

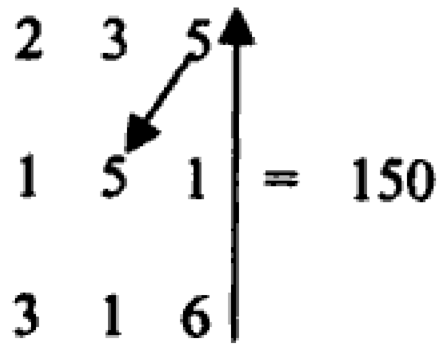
Step 1:
Constant term:

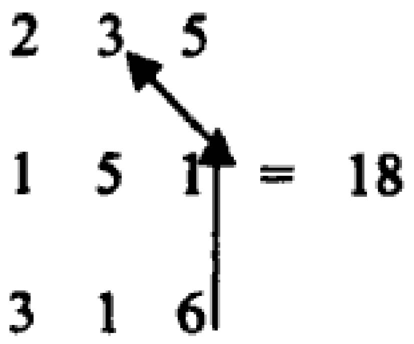


Step 2:

x term:



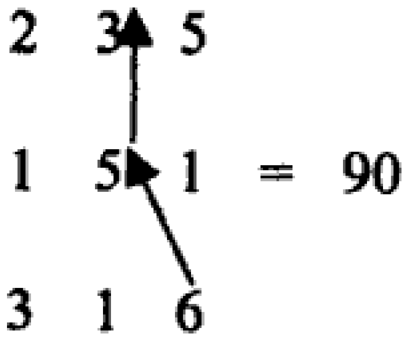


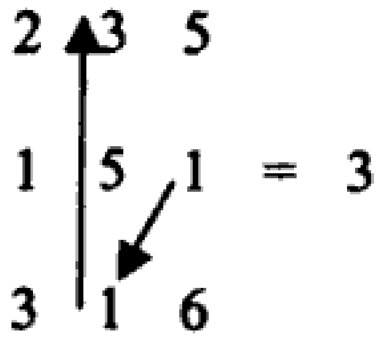


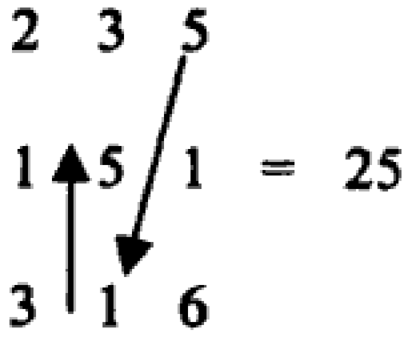
5 + 150 + 18 = 173

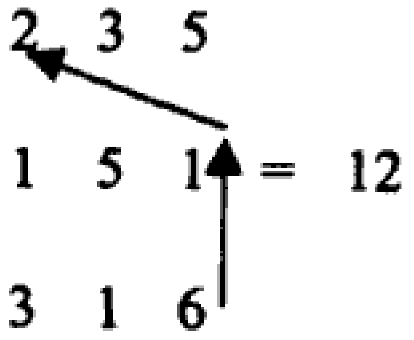
Step 3:

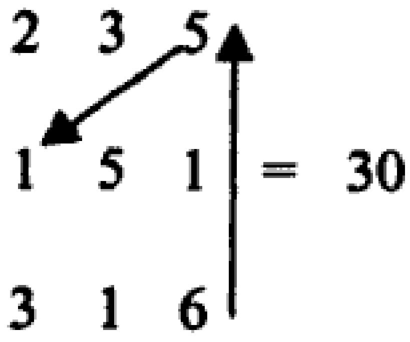
x^2 term:

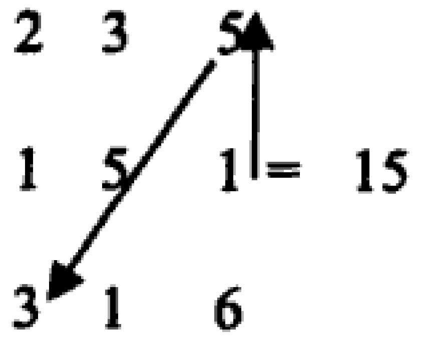






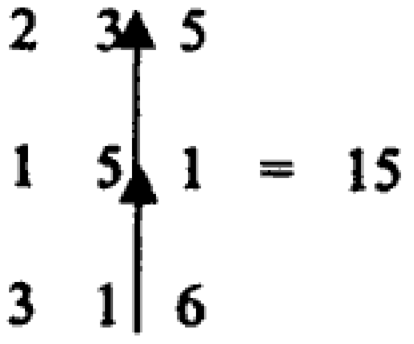


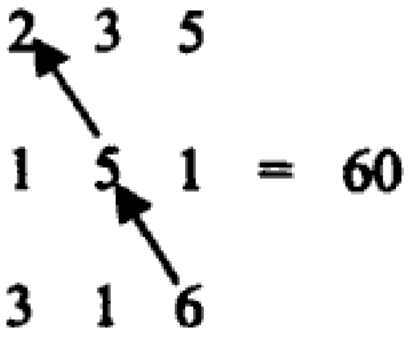


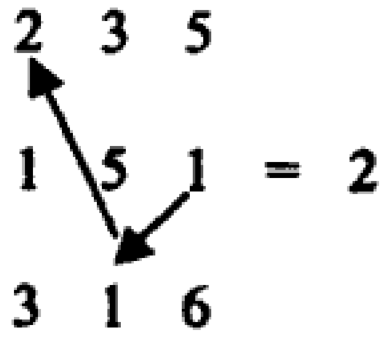


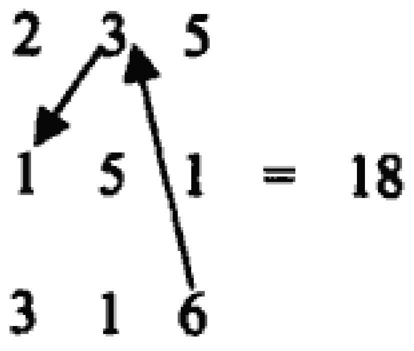
90 + 3 + 25 + 12 + 30 + 15 = 175

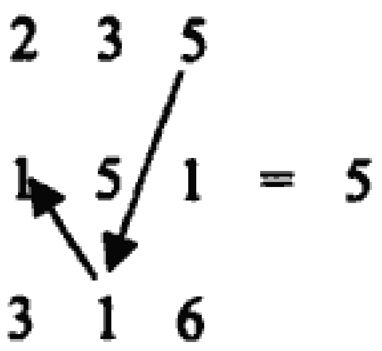
Step 4:
x^3 term:

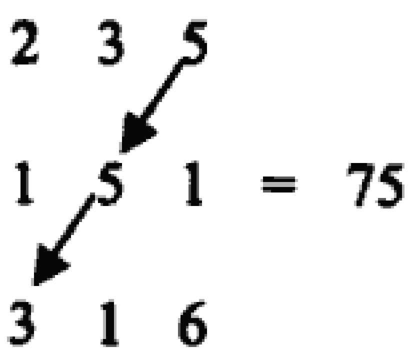


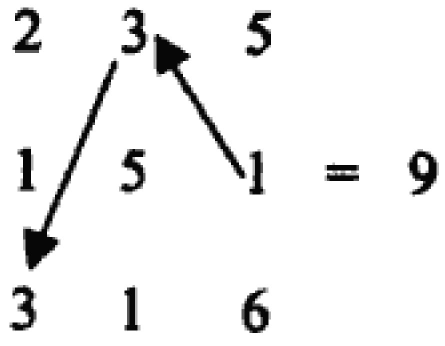








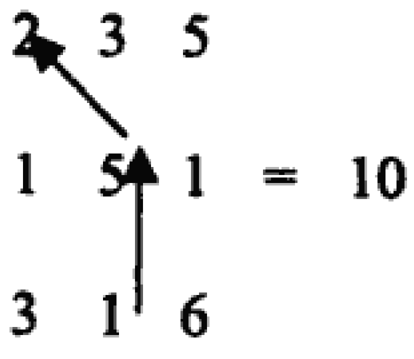


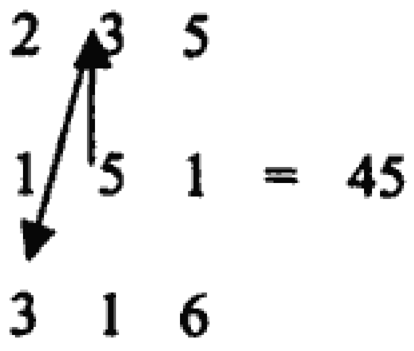


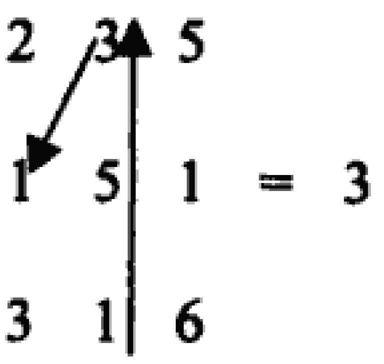
15 + 60 + 2 + 18 + 5 + 75 + 9 = <u>184</u>

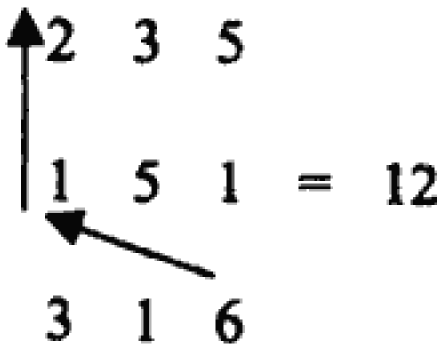
Step 5:

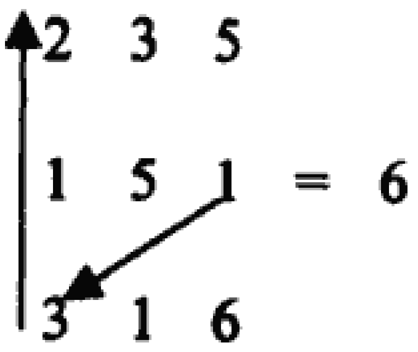
x^4 term:

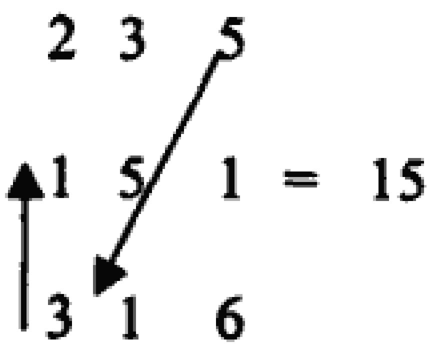








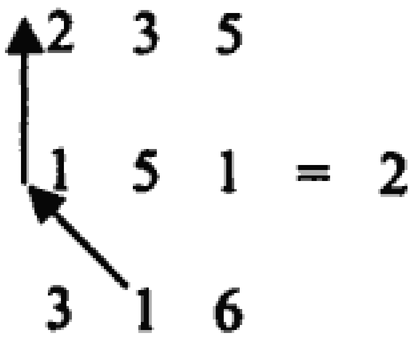


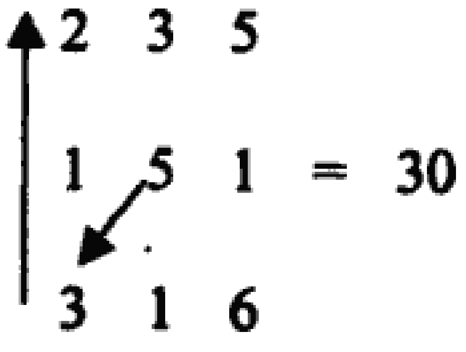


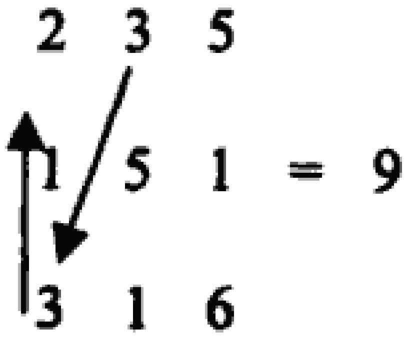
10 + 45 + 3 + 12 + 6 + 15 = <u>91</u>

Step 6:

x^5 term:







2 + 30 + 9 = <u>41</u>

Step 7:

x^6 term:

$$\begin{array}{|lll}
\uparrow 2 & 3 & 5 \\
\uparrow 1 & 5 & 1 = 6 \\
3 & 1 & 6
\end{array}$$

On addition of the steps, the result is $6x^6 + 41x^5 + 91x^4 + 184x^3 + 175x^2 + 173x + 30$.

(ix) Multiplication with groups (V.M.):

Still there is another way of multiplying two numbers by splitting each number into smaller groups. Multiplication is carried out using Urdhava Tiryak Sutra.

1) For Example in the usual Vedic Multiplication (Right to Left)

$$\begin{array}{r}
1\ 2\ 4 \\
1\ 1\ 3 \\
\boxed{1\ 4\ 0\ 1\ 2} \\
0\ 1\ 1\ 1
\end{array}$$

The group formation in the multiplicand can be (12) 4; 1 (24); 1 2 4 (which is the original), similarly for the multiplier (11) 3; 1 (13); 1 1 3 (which is the original).

There are 9 possibilities of multiplication for the above example. The result is independent of the groups symmetry or asymmetry in comparison between the multiplier and multiplicand and also it is independent of the number of digits in each group.

Here we have considered four such possibilities of grouping.

1. 1 (24), 1 (13)
2. (12) 4, (11) 3
3. 1 (24), (11) 3
4. (12) 4, 1 (13)

1) Considering the first grouping, multiplication of 1 (24) and 1 (13) is carried out as follows:

1 (24)

1 (13)

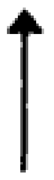
We can consider the first groups (from right end) as unit groups the second groups will be in hundreds as we have used units place and tens place under the units group. With this concept, multiplication is carried out using Urdhva Tiryak.

Hundreds	Units
1	(24)
1	(13)

First Step:

(Urdhva)

(24)



= 312 units

(13)

For placement, one can use a single digit placement under units, tens, hundreds, etc for the result. Hence first step gives 2 under the unit digit and 31 to be carried over.

1 (24)

1 (13)

$/_{31}{}^{2}$

Second Step: Tiryak

1 (24)

= 37 (hundreds)

1 (13)

2

We have to fill tens digit of the answer, after carrying over 31 tens.

Second step gives the value 3700 or 370 tens. To this 31 tens are carried over from the previous step, which becomes 401 tens. We keep under tens column 1 and represent 40 as carrying over.

1 (24)

1 (13)

$/_{40}\,{}^{1}/_{31}\,{}^{2}$

Third Step:

(Urdhva)

$$\begin{array}{l} 1 \\ \uparrow \quad = 1 \text{ (Ten thousand)} \\ 1 \end{array}$$

To this 10,000, which is 100 hundreds, one has to carry over 40 hundreds from the previous step. Thus it becomes 140 hundreds. Therefore, 140 is kept under hundreds as the third step value.

$$\begin{array}{lll} 1 & (24) & \\ 1 & (13) & \\ \hline \boxed{140 \quad 1 \quad 2} & \longrightarrow & \text{Answer} \\ /40 & /31 \longrightarrow & \text{Carrying over} \end{array}$$

Answer = 14012

We give the working details of the other three combinations. Same procedure is followed with status being specified for each grouping. Specification of status is as follows.

2) Tens Units

$$\begin{array}{lll} & \text{Tens} & \text{Units} \\ & (12) & 4 \\ & (11) & 3 \\ \hline 140 & /_{8} \; 1 & /_{1} \; 2 \end{array}$$

3) Hundreds Units

$$\begin{array}{ll} \text{Hundreds} & \text{Units} \\ 1 & (24) \\ \text{Tens} & \text{Units} \\ (11) & 3 \\ \hline 140 /_{30} \; 1 & /_{7} \; 2 \end{array}$$

4)	Tens	Units
	(12)	*4*
	Hundreds	*Units*
	1	*(13)*

140 / 1 / 2
/ 20 / 5

Following same argument as given above, one can see different grouping multiplications give the same result when the status is taken into consideration, such as

1) Status of the first group in units. If the first group consists of more than one digit, then the second group will be in hundreds or thousands etc. depending on the number of digits in the first group. More generally, the statuses of the other groups are dependent successively on the immediate previous group and so on.

We can also multiply the groups by re-coursing to polynomial conversions and by applying Urdhva Tiryagbhyam Sutram.

a) Let us consider first grouping, i.e., 1(24), 1(13)

Now treating one group as one unit, we can write 1(24) as $x + 24$ where $x = 100$. Similarly the multiplier 1(13) can also be written as $x + 13$. Now applying Urdhava Tiryagbhyam, we get the following result: (polynomial multiplication).

$$\begin{array}{r} x + 24 \\ \underline{x + 13} \\ x^2 + 37x + 312 \end{array}$$

Substituting for $x = 100$, we get 14012 as result.

b) Consider another grouping (Second grouping) (12)4, (11)3

We can write (12)4 as $12x + 4$ where $x = 10$. Similarly (11)3 as $11x + 3$,

$$\begin{array}{r} 12x + 4 \\ \underline{11x + 3} \\ 132x^2 + 80x + 12 \end{array}$$

Substituting for $x = 10$, we get 14012 as result.

b) When grouping is asymmetrical between multiplier and multiplicand, for example (Third grouping) 1(24), (11)3.

We can write (1)24 as $x^2 + 24$ and similarly (11)3 as $11x + 3$, where $x = 10$.

$$\begin{array}{r} x^2 + \quad 0 + 24 \\ \underline{\qquad\quad 0 + 11x + \ \ 3} \\ 11x^3 + 3x^2 + 264x + 72 \end{array}$$

Substituting for x, we get 14012.

d) Consider (12) 4, and 1 (13) (Fourth grouping):

One can represent (12) 4 as $12x + 4$ and 1 (13) as $x^2 + 13$. Obviously $x = 10$.

Applying Urdhava Tiryagbhyam for the multiplication

$$\begin{array}{r} 0 + \ 12x + \ \ 4 \\ \underline{\qquad\quad x^2 + \quad 0 + \ 13} \\ 12x^3 + 4x^2 + 156x + 52 \end{array}$$

Substituting for x, we get 14012

Similarly we can do grouping and multiplication as follows:

[1 (24), (11) 3]; [1 24, (11) 3]; [124, 1(13)]; [(12) 4, 113]; [1 (24), 113] etc.

2) Consider another example **12411 × 13515** using polynomial conversion of groups.

We can group multiplicand and multiplier as (12) 4 (11) and (13) 5 (15). First write

down in the form of a polynomial considering group as a unit.

We can write (12) 4 (11) as $12x^3 + 4x^2 + 11$ and (13) 5 (15) as $13x^3 + 5x^2 + 15$ where $x = 10$ and then carry out the Vedic method of polynomial multiplication.

$$\begin{array}{r} 12x^3 + \quad 4x^2 + 11 \\ \underline{\qquad\qquad\qquad\qquad 13x^3 + \quad 5x^2 + 15} \\ 156x^6 + 112x^5 + 20x^4 + 323x^3 + 115x^2 + 165 \end{array}$$

Substituting for x we get 167734665 as result.

$$\begin{array}{r} 156000000 \\ 11200000 \\ 200000 \\ 323000 \\ 11500 \\ \underline{\qquad 165} \\ 167734665 \end{array}$$

Vedic Method without Grouping

$$\begin{array}{ccccccccc} & & & & 1 & 2 & 4 & 1 & 1 \\ & & & & 1 & 3 & 5 & 1 & 5 \\ \hline \multicolumn{9}{|c|}{1\ \ 6\ \ 7\ \ 7\ \ 3\ \ 4\ \ 6\ \ 6\ \ 5} \\ \hline & 0 & 1 & 2 & 3 & 2 & 2 & & \end{array}$$

Whenever the group contains more than two digits, the individual multiplication concerned with such groups are to be calculated by Urdhava Tiryagbhyam separately, if necessary.

In general, in grouping method one can write down all possible groupings and proceed directly or converting them to polynomial forms. But, in the conversion to the polynomial also we can consider each group as one entity and the first group on the right extreme side as under units. For the other groups one should consider as from tens or hundreds or thousands, etc according as the value of the number. These are clear in the following groups where x takes different values of powers of 10. By vilokanam it suggests that 2 or 3 possibilities are comparatively simpler than the others and hence one can prefer to work with such groupings.

When we consider multiplication of 12411 × 13515 with groupings as 1 (2411), 1(3515). This can be written as $x + 2411$ and $x + 3515$ where $x = 10000$.

The groupings can also be done as follows:

1. (1241) 1
   (1351) 5

   This can be written as

   $1241x + 1$

   $1351x + 5$

   where $x = 10$

2. (12)(411)
   (13)(515)

   This can be written as

   $12x + 411$

   $13x + 515$ where $x = 1000$

3. (124)(11)
   (135)(15)

   This can be written as

   $124x + 11$

   $135x + 15$ where $x = 100$

4. (12)(41) 1
   (13)(51) 5

   This can be written as

   $12x^3 + 41x + 1$

   $13x^3 + 51x + 5$ where $x = 10$.

5. 1 (24)(11)
   1 (35)(15)

   This can be written as

   $x^2 + 24x + 11$

   $x^2 + 35x + 15$ where $x = 100$

6. (124)(11)
   (13)(515)

   This can be written as

   $124x^2 + 11$

   $13x^3 + 515$ where $x = 10$.

7. 1 (241) 1
   1 (351) 5

   This can be written as

   $x^4 + 241x + 1$

   $x^4 + 351x + 5$ where $x = 10$.

8. (12)(41) 1
1 (35)(15)

This can be written as

$12x^3 + 41x + 1$

$x^4 + 35x^2 + 15$ where $x = 10$.

9. 1 (2411)
(1351) 5

This can be written as

$x^4 + 2411$

$1351x + 5$ where $x = 10$

10. 1 2 (411)
1 3 (515)

This can be written as

$x^4 + 2x^3 + 411$

$x^4 + 3x^3 + 515$ where $x = 10$.

11. (124) 1 1
(135) 1 1

This can be written as

$124x^2 + x + 1$

$135x^2 + x + 5$ where $x = 10$.

12. 1 2 4 (11)
1 3 5 (15)

This can be written as

$x^4 + 2x^3 + 4x^2 + 11$

$x^4 + 3x^3 + 5x^2 + 15$ where $x = 10$.

13. (12) 4 1 1
(13) 5 1 5

This can be written as

$12x^3 + 4x^2 + x + 1$

$13x^3 + 5x^2 + x + 5$ where $x = 10$.

14. 1 (24) 1 1
1 (35) 1 5

This can be written as

$x^4 + 24x^2 + x + 1$

$x^4 + 35x^2 + x + 5$ where $x = 10$.

15. 1 2 (41) 1
1 3 (51) 5

This can be written as

$x^4 + 2x^3 + 41x + 1$

$x^4 + 3x^3 + 51x + 5$ where $x = 10$.

Following the procedure that is worked for three digited multiplication, this also can be worked out without re-coursing to polynomials, i.e., using groups of numbers only.

This method can be extended to combined operations as well.

This grouping reduces the number of steps in multiplication.

This method is considered to be simple than when considering multiplication with total numbers as it is.

The grouping method is simpler in general. But in particular, when the groups can be possible with numbers, which can be multiplied mentally, then the working is much easier.

In addition to this, one can contemplate on the use of reduced base such as 2,3,4,5, etc and also reduction/enhancement from power of base as well. Such reduction/enhancement of bases may find application in the computer hardware usage. A few reductions are shown below. If the base is reduced from 10 to 2, its Anrupyena is 5, i.e., 10/2. With this ratio the given multiplicand and the multiplier can be constructed as follows.

Multiplicand $= 124$

$= 12x + 4$ where $x = 10$

Multiplier $= 113$

$= 11x + 3$ where $x = 10$

In the reduced base the multiplicand becomes $60x + 4$, where $x = 2$. Similarly the multiplier becomes $55x + 3$. Now the multiplication is between $60x + 4$ and $55x + 3$, which gives the same result. One can try with base 3, 4, 5, and 6. But with base 7, the coefficient of x is a fraction, which is not desirable for easy working. On the other hand if the base is increased, the multiplication may be simpler because of reduction in coefficients. If the base is 20, then

$124 = 6x + 4$

$113 = 5x + 13$

This also gives the same result.

8. CHAPTER II

VINCULUM

(i) Use of Vinculum (V.M.):

Use of vinculum in multiplication is one of the novelties of the Vedic calculations. Any digit, greater than 5, can be written in the Vinculum form as given below.

Table

The Vinculum forms for digits 5 to 9 are as follows:

$5 = 1\bar{5} = 10 - 5$

$6 = 1\bar{4} = 10 - 4$

$7 = 1\bar{3} = 10 - 3$

$8 = 1\bar{2} = 10 - 2$

$9 = 1\bar{1} = 10 - 1$

Coming out of Vinculum:

$\bar{1} = \bar{1}9 = -10 + 9$

$\bar{2} = \bar{1}8 = -10 + 8$

$\bar{3} = \bar{1}7 = -10 + 7$

$\bar{4} = \bar{1}6 = -10 + 6$

$\bar{5} = \bar{1}5 = -10 + 5$

$\bar{6} = \bar{1}4 = -10 + 4$

$\bar{7} = \bar{1}3 = -10 + 3$

$\bar{8} = \bar{1}2 = -10 + 2$

$\bar{9} = \bar{1}1 = -10 + 1$

Where'−' means literal negative. Taking complements from base 10 makes use of a lesser digit and hence the mental multiplication will be easier. It is definitely simpler while working with smaller values.

The same rules of multiplication, addition and subtraction between the positive and negative numbers will apply here also.

Making use of the above table of Vinculum concept, we can write down a number in its vinculum form as follows. Consider 29. We can write 29 as $21\bar{1} = 3\ \bar{1}$. Instead of the intermediate step, we can directly write down $3\ \bar{1}$. Starting from 9 its Vinculum is $1\bar{1}$. 1 is added to 2. which results in 3. This is a direct and mental working. In general, in order to write or express a number in the vinculum form, starting from right end, we can proceed to write down the first digit in the vinculum form if necessary, then add 1 to the previous digit and consider its vinculum again, if necessary, and continue the procedure till the last digit of the number is reached.

For example, let us consider a number 25909535575456.

From right to left $3\ \bar{4}\ \bar{1}\ 1\ \bar{1}\ 5\ 4\ \bar{4}\ \bar{4}\ \bar{3}\ 5\ 5\ \bar{4}\ \bar{4}$ *(5 is not put in vinculum form)*

One can also write down the given number in the Vinculum form starting from left.

In this case also, one has to remember that the Vinculum effects the previous digit or sometimes successively to other digits as well. Again one has to bear in mind, the digit, which need not be written in vinculum form, remains as it is. But it may be effected by the successive digits, which may be written in vinculum form. Bearing these points one can write the vinculum form of entire number from left to right as well.

For example consider 25909535575456.

The details are shown in steps:

$$2\quad 5\quad 9\quad 0\quad 9\quad 5\quad 3\quad 5\quad 5\quad 7\quad 5\quad 4\quad 5\quad 6$$

$$= 2\quad \underbrace{5\ 1}\bar{1}\quad \underbrace{0\ 1}\bar{1}\quad 5\quad 3\quad 5\quad \underbrace{5\ 1}\bar{3}\quad 5\quad 4\quad \underbrace{5\ 1}\bar{4}$$

$$= 2\quad 6\quad \bar{1}\quad 1\quad \bar{1}\quad 5\quad 3\quad 5\quad 6\quad \bar{3}\quad 5\quad 4\quad 6\quad \bar{4}$$

$$= \underbrace{2\ 1}\bar{4}\quad \bar{1}\quad 1\quad \bar{1}\quad 5\quad 3\quad \underbrace{5\ 1}\bar{4}\quad \bar{3}\quad 5\quad 4\ 1\bar{4}\quad \bar{4}$$

$$= 3\quad \bar{4}\quad \bar{1}\quad 1\quad \bar{1}\quad 5\quad 3\quad 6\quad \bar{4}\quad \bar{3}\quad 5\quad 5\quad \bar{4}\quad \bar{4}$$

$$= 3\quad \bar{4}\quad \bar{1}\quad 1\quad \bar{1}\quad 5\quad \underbrace{3\ 1}\bar{4}\quad \bar{4}\quad \bar{3}\quad 5\quad 5\quad \bar{4}\quad \bar{4}$$

$$= 3\quad \bar{4}\quad \bar{1}\quad 1\quad \bar{1}\quad 5\quad 4\quad \bar{4}\quad \bar{4}\quad \bar{3}\quad 5\quad 5\quad \bar{4}\quad \bar{4}$$

Vilokanam* can work out all these steps in one step with the help of above considerations and hence to write down in single step, it can be seen easily by the following method.

By Vilokanam*, we can go up to the point where the vinculum starts. Here in this case 9 are equal to $1\bar{1}$. The previous digit 5 is made 6 by 1. When written in

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* Vilokanam by supervision and inspection

vinculum form is $1\bar{4}$. 1 is added to 2. Therefore, 2 5 1 $\bar{1}$ becomes $3\bar{4}\bar{1}$, which is found in the last step. Consider 3 5 5 7 (part of the number). Proceeding up to 7, which is written in the vinculum form $1\bar{3}$. Now this effects the previous digit 5 and successively up to 3. As 5 becomes 6, which is $1\bar{4}$. 1 is added to the next 5 and again becomes $1\bar{4}$. This 1 is added to 3 becoming 4. Thus 3 5 5 7 = $4\ \bar{4}\ \bar{3}$ and so on. Let us consider $3\ \bar{4}\ \bar{1}\ 1\ \bar{1}\ 5\ 4\ \bar{4}\ \bar{4}\ \bar{3}\ 5\ 5\ \bar{4}\ \bar{4}$.

To come out of Vinculum from right to left, let us start from extreme $\bar{4}$, which is equal to $\bar{1}6$. This $\bar{1}$ is added to next $\bar{4}$ on the left becoming $\bar{5}$. $\bar{5}$ is $\bar{1}5$. This $\bar{1}$ is added to 5, which becomes 4. Now $\bar{3}$ is $\bar{1}7$. $\bar{1}$ is added to $\bar{4}$ becoming $\bar{5}$, which is $\bar{1}5$ and it is thus continued.

$$\begin{array}{cccccccccccccccc} & 3 & \bar{4} & \bar{1} & 1 & \bar{1} & 5 & 4 & \bar{4} & \bar{4} & \bar{3} & 5 & 5 & \bar{4} & \bar{4} \\ = & 2 & 5 & 9 & 0 & 9 & 5 & 3 & 5 & 5 & 7 & 5 & 4 & 5 & 6 \end{array}$$

It can also be carried out stepwise as follows. To come out of the Vinculum, the Vinculum of each digit is first noted down (Refer 2nd row). This is followed by the addition of each Vinculum digit (part of Vinculum form) to the previous vinculum form. This is continued. But if successive numbers do not contain Vinculum forms, then this addition will not take place among themselves (for example 2nd row between 5 and 5, 4 and 5).

$$\begin{array}{cccccccccccccccc} & 3 & \bar{4} & \bar{1} & 1 & \bar{1} & 5 & 4 & \bar{4} & \bar{4} & \bar{3} & 5 & 5 & \bar{4} & \bar{4} \\ = & 3 & \bar{1}6 & \bar{1}9 & 1 & \bar{1}9 & 5 & 4 & \bar{1}6 & \bar{1}6 & \bar{1}7 & 5 & 5 & \bar{1}6 & \bar{1}6\text{(2}^{\text{nd}}\text{ Row)} \\ & \llcorner & \lrcorner\llcorner & \lrcorner & \llcorner & \lrcorner & & \llcorner & \lrcorner\llcorner & \lrcorner\llcorner & \lrcorner & & \llcorner & \lrcorner\llcorner & \lrcorner \\ = & 2 & 5 & 9 & 0 & 9 & 5 & 3 & 5 & 5 & 7 & 5 & 4 & 5 & 6 \end{array}$$

We can come out of Vinculum from left to right

$$\begin{array}{cccccccccccccccc} & 3 & \bar{4} & \bar{1} & 1 & \bar{1} & 5 & 4 & \bar{4} & \bar{4} & \bar{3} & 5 & 5 & \bar{4} & \bar{4} \\ = & 2 & 5 & 9 & 0 & 9 & 5 & 3 & 5 & 5 & 7 & 5 & 4 & 5 & 6 \end{array}$$

We can come out from Vinculum, from left to right by mental adoption of Vinculum additions. We start with the Vinculum form from left to right ($\bar{4}$) and write down its equivalent as $\bar{1}6$. This $\bar{1}$, when added to 3 gives the value 2. 6 replaces $\bar{4}$. Coming to the next digit $\bar{1}$, its equivalent is $\bar{1}9$. $\bar{1}$ and 6 are added together to give 5. Effectively the second digit becomes 5. Third digit is 9. Fourth digit is effected by the fifth one, which is $\bar{1}$. Its equivalent is $\bar{1}9$. $\bar{1}$ is added to 1 (fourth digit) and becomes 0. Effectively fourth digit becomes zero and so on. This process is continued till the last.

One can also write down step-wise as follows. Put down the equivalent vinculum form of each digit while coming out. This is followed by the addition (from left) of each digit or Vinculum form to the vinculum digit in the next vinculum form.

$$\begin{array}{llllllllllllllll} & 3 & \bar{4} & \bar{1} & 1 & \bar{1} & 5 & 4 & \bar{4} & \bar{4} & \bar{3} & 5 & 5 & \bar{4} & \bar{4} \\ = & 3 & \overline{16} & \overline{19} & 1 & \overline{19} & 5 & 4 & \overline{16} & \overline{16} & \overline{17} & 5 & 5 & \overline{16} & \overline{16} \\ = & 2 & 5 & 9 & 0 & 9 & 5 & 3 & 5 & 5 & 7 & 5 & 4 & 5 & 6 \end{array}$$

(ii) Right to Left Multiplication:

a) Two-digit Numbers Multiplication:

For example, consider multiplication of 29 by 38.

9 can be written as $1\bar{1}$

$29 = 2 \quad 1\bar{1} = 3\bar{1}$

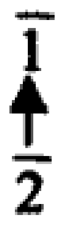
Similarly, $38 = 3 \quad 1\bar{2} = 4\bar{2}$

Using Urdhva Tiryagbhyam sutram in the usual manner, steps are given below.

Current Method	**Vedic Method (Right to Left)**	**Vinculum Method (Right to Left)**
29	2 9	$3\ \bar{1}$
38	3 8	$4\ \bar{2}$
232	$\boxed{1\,1\,0\,2}$	$\boxed{1\,1\,0\,2}$
87	5 7	$\bar{1}$
1102		

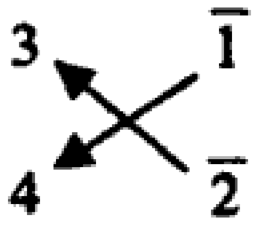
Step Diagram :

Step 1 :



= 2

Step 2:



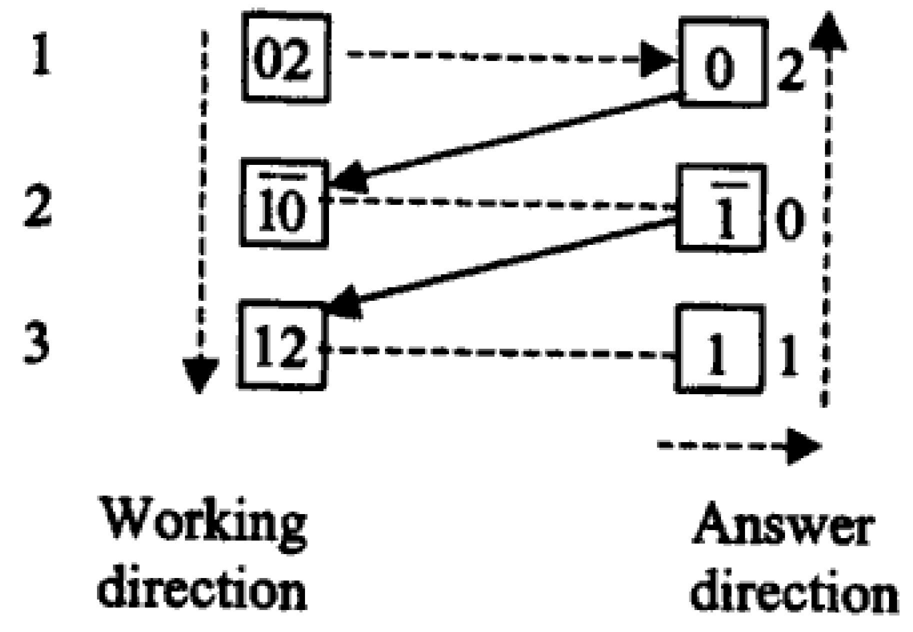
$= \bar{4} + \bar{6} = \bar{1}\ \bar{0}$

Step 3:

= 12

Step values constitute the answer and are shown in the result.

Step **Step Value** **Computed Value of the steps**



Ans.: 1102.

If the answer contains Vinculum, then coming out of vinculum is explained from the following examples:

b) Three-digit Numbers Multiplication

Current Method

$$\begin{array}{r} 387\\ \underline{492}\\ 774\\ 3483\\ \underline{1548}\\ 190404 \end{array}$$

Vedic Method
(Right to Left)

$$\begin{array}{ccccc} & & 3 & 8 & 7\\ & & 4 & 9 & 2\\ \multicolumn{5}{c}{\boxed{19\ 0\ \ 4\ \ 0\ \ 4}}\\ & 7 & 11 & 8 & 1 \end{array}$$

Vinculum Method
(Right to Left)

$$\begin{array}{cccccc} & & & 4 & \bar{1} & \bar{3}\\ & & & 5 & \bar{1} & 2\\ \hline & 20 & \bar{9} & \bar{6} & 1 & \bar{6}\\ = & 20 & \bar{1}1 & \bar{1}4 & 1 & \bar{1}4\\ = & 2\bar{1} & 0 & 4 & 0 & 4\\ = & \multicolumn{5}{c}{\boxed{19\quad 0\quad 4\quad 0\quad 4}} \end{array}$$

Ans.: 190404

c) Four-digit Numbers Multiplication:

Current Method

$$\begin{array}{r} 9684\\ \underline{2756}\\ 58104\\ 48420\\ 67788\\ \underline{19368}\\ 26689104 \end{array}$$

Vedic Method
(Right to Left)

$$\begin{array}{ccccccc} & & & 9 & 6 & 8 & 4\\ & & & 2 & 7 & 5 & 6\\ \multicolumn{7}{c}{\boxed{26\ 6\ \ 8\ \ 9\ 1\ \ 0\ \ 4}}\\ & 8 & 11 & 15 & 11 & 7 & 2 \end{array}$$

Vinculum Method
(Right to Left)

$$\begin{array}{cccccccc} & & & 1 & 0 & \bar{3} & \bar{2} & 4\\ & & & 0 & 3 & \bar{2} & \bar{4} & \bar{4}\\ \multicolumn{8}{c}{\boxed{3\ \ \bar{3}\ \ \bar{3}\ \ \bar{2}\ 9\ 2\ \ \bar{9}\ \ \bar{6}}}\\ & 0 & \bar{1} & 0 & 2 & 1 & 0 & \bar{1} \end{array}$$

Ans.: $\boxed{26689104}$

d) Five-digit Numbers Multiplication:

Current Method

$$\begin{array}{r} 92486 \\ \underline{35678} \\ 739888 \\ 647402 \\ 554916 \\ 462430 \\ \underline{277458} \\ 3299715508 \end{array}$$

Vedic Method (Right to Left)

$$\begin{array}{r} 9\ 2\ 4\ 8\ 6 \\ 3\ 5\ 6\ 7\ 8 \\ \boxed{32\ 9\ 9\ 7\ 1\ 5\ 5\ 0\ 8} \\ 5\ 8\ 13\ 18\ 13\ 13\ 11\ 4 \end{array}$$

Vinculum Method (Right to Left)

$$\begin{array}{r} 1\ \bar{1}\ 2\ 5\ \bar{1}\ \bar{4} \\ 0\ 4\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{2} \\ \hline \boxed{4\ \bar{7}\ 0\ 0\ \bar{2}\ \bar{9}\ 5\ 5\ 0\ 8} \\ 1\ 1\ \bar{3}\ \bar{2}\ \quad 1 \quad \end{array}$$

Ans.: $\boxed{3299715508}$

e) Six-digit Numbers Multiplication:

Current Method

$$\begin{array}{r} 634869 \\ \underline{721685} \\ 3174345 \\ 5078952 \\ 3809214 \\ 634869 \\ 1269738 \\ \underline{4444083} \\ 458175434265 \end{array}$$

Vedic Method (Right to Left)

$$\begin{array}{r} 6\ 3\ 4\ 8\ 6\ 9 \\ 7\ 2\ 1\ 6\ 8\ 5 \\ \boxed{45\ 8\ 1\ 7\ 5\ 4\ 3\ 4\ 2\ 6\ 5} \\ 3\ 5\ 11\ 14\ 17\ 13\ 14\ 15\ 10\ 4 \end{array}$$

Vinculum Method (Right to Left)

$$\begin{array}{r} 1\bar{4}\ 3\ 5\ \bar{1}\ \bar{3}\ \bar{1} \\ 1\bar{3}\ 2\ 2\ \bar{3}\ \bar{2}\ 5 \\ \boxed{1\ \bar{6}\ 6\ \bar{1}\ \bar{9}\ 7\ 6\ \bar{6}\ 3\ 4\ 3\ \bar{3}\ \bar{5}} \\ 1\ \ \bar{1}\bar{1}\ 2\ 1\ \bar{4}\ 0\ \ 3\ \ \ \bar{1}\ \quad \end{array}$$

Ans.: $\boxed{458175434265}$

(iii) Left to Right Multiplication:

(a) Two-digit Numbers Multiplication: consider 29 x 38

$29 = 3\bar{1}$ and $38 = 4\bar{2}$

Vinculum Method 1

(Left to Right)

$$\begin{array}{r} 3\ \bar{1} \\ 4\ \bar{2} \\ \hline 1\ 2\ 0\ 2 \\ \bar{1}\ 0\ \ \\ \hline 1\ 1\ 0\ 2 \\ \hline \end{array}$$

Vinculum Method 2

(Left to Right)

$$\begin{array}{l} 3\ \bar{1} \\ 4\ \bar{2} \\ \boxed{1\ 1\ 0\ 2} \\ \ \ 2\ 0 \end{array}$$

Step Diagrams:

Step 1:

$$\begin{tikzcd} 3 \\ 4 \arrow[u] \end{tikzcd} = 12$$

Step 2:

$$\begin{tikzcd} 3 \arrow[dr] & \bar{1} \\ 4 \arrow[ur] & \bar{2} \end{tikzcd} = \bar{6} + \bar{4} = \bar{1}0$$

Step 3:

$$\begin{tikzcd} \bar{1} \\ \bar{2} \arrow[u] \end{tikzcd} = 2$$

(b) Three-digit Numbers Multiplication: Consider 387 x 492 where the vinculum forms are 387 = $4\bar{1}\bar{3}$ and 492 = $5\bar{1}2$

Vinculum Method 1
(Left to Right)

$4 \ \bar{1} \ \bar{3}$

$5 \ \bar{1} \ 2$

$\boxed{20 \ \bar{9} \ \bar{6} \ 1 \ \bar{6}}$

Ans: 190404

Vinculum Method 2
(Left to Right)

$4 \ \bar{1} \ \bar{3}$

$5 \ \bar{1} \ 2$

$\boxed{2 \ 0 \ \bar{9} \ \bar{6} \ 1\bar{6}}$

$0 \ \bar{9} \ \bar{6} \ 1$

Ans: 190404

(c) Four-digit Numbers Multiplication: consider 9684 x 2756 where 9684 = $100\bar{3}\bar{2}4$ and 2756 = $03\bar{2}\bar{4}\bar{4}$

Vinculum Method 1
(Left to Right)

$1 \ 0 \ \bar{3} \ \bar{2} \ 4 1 \ 0 \ \bar{3} \ \bar{2} \ 4$

$\underline{0 \ 3 \ \bar{2} \ \bar{4} \ \bar{4}}$

$0 \ 3 \ \bar{2} \ \bar{3} \ \bar{4} \ 8 \ 2 \ \bar{8} \ \bar{6}$

$0 \ 0 \ \bar{1} \ 0 \ 2 \ 1 \ 0 \ \bar{1}$

$\boxed{0 \ 3 \ \bar{3} \ \bar{3} \ \bar{2} \ 9 \ 2 \ \bar{9} \ \bar{6}}$

Ans: 26689104

Vinculum Method 2
(Left to Right)

$0 \ 3 \ \bar{2} \ \bar{4} \ \bar{4}$

$0 \ 3 \ \bar{2} \ \bar{4} \ \bar{4}$

$\boxed{0 \ 0 \ 3 \ \bar{3} \ \bar{3} \ \bar{2} \ 9 \ 2 \ \bar{9} \ \bar{6}}$

$3 \ \bar{2} \ \bar{3} \ \bar{4} \ 8 \ 2 \ \bar{8}$

Ans: 26689104

(d) Five-digit Numbers Multiplication: Consider 92486 x 35678 where 92486 = $1\bar{1}25\bar{1}\bar{4}$ and 35678 = $4\bar{4}\bar{3}\bar{2}\bar{2}$

Vinculum Method 1
(Left to Right)

$1 \ \bar{1} \ 2 \ 5 \ \bar{1} \ \bar{4}$

$\underline{0 \ 4 \ \bar{4} \ \bar{3} \ \bar{2} \ \bar{2}}$

$0 \ 4 \ \bar{8} \ 9 \ \bar{7} \ 0 \ \bar{9} \ 5 \ 4 \ 0 \ 8$

$0 \ 0 \ 0 \ 2 \ \bar{3} \ \bar{2} \ 0 \ 0 \ 1$

$\boxed{0 \ 4 \ \bar{8} \ 11 \ \bar{1}0 \ \bar{2} \ \bar{9} \ 5 \ 5 \ 0 \ 8}$

Vinculum Method 1
(Left to Right)

$1 \ \bar{1} \ 2 \ 5 \ \bar{1} \ \bar{4}$

$0 \ 4 \ \bar{4} \ \bar{3} \ \bar{2} \ \bar{2}$

$\boxed{0 \ 0 \ 4 \ \bar{8} \ 11 \ \bar{1}0 \ \bar{3} \ 1 \ 5 \ 5 \ 0 \ 8}$

$4 \ \bar{8} \ 9 \ \bar{7} \ 0 \ 1 \ 5 \ 4 \ 0$

Ans: 3299715508

(e) Six-digit Numbers Multiplication : Consider 634869 x 7216855 where $634869 = 1\bar{4}351\bar{1}\bar{3}\bar{1}$ and $721685 = 1\bar{3}22\bar{3}\bar{2}5$

Vinculum Method I
(Left to Right)

$$\begin{array}{l}
1\ \bar{4}\ 3\ 5\ \bar{1}\ \bar{3}\ \bar{1} \\
\underline{1\ \bar{3}\ 2\ 2\ \bar{3}\ \bar{2}\ 5\qquad\qquad} \\
1\ \bar{7}\ 7\ 0\ \bar{1}\ 6\ 0\ \bar{6}\ 0\ 4\ 4\ \bar{3}\ \bar{5} \\
0\ 1\ \bar{1}\ \bar{2}\ 2\ 2\ \bar{4}\ 0\ 3\ 0\ \bar{1}\ 0 \\
\boxed{1\ \bar{6}\ 6\ \bar{2}\ 1\ 8\ \bar{4}\ \bar{6}\ 3\ 4\ 3\ \bar{3}\ \bar{5}}
\end{array}$$

Vinculum Method II
(Left to Right)

$$\begin{array}{l}
1\ \bar{4}\ 3\ 5\ \bar{1}\ \bar{3}\ \bar{1} \\
1\ \bar{3}\ 2\ 2\ \bar{3}\ \bar{2}\ 5 \\
\boxed{0\ 1\ \bar{6}\ 6\ \bar{2}\ 1\ 8\ \bar{4}\ \bar{6}\ 3\ 4\ 3\ \bar{3}\ \bar{5}} \\
1\ \bar{7}\ 7\ 0\ \bar{1}\ 6\ 0\ \bar{6}\ 0\ 4\ 4\ \bar{3}
\end{array}$$

Ans: 458175434265

(iv) Series Multiplication:

Example 1: 98 × 76 × 59

Vedic Method (Right to Left)

$$\begin{array}{rrrr}
 & & 9 & 8 \\
 & & 7 & 6 \\
 & & 5 & 9 \\
\hline
\boxed{439} & 4 & 3 & 2 \\
 & 124 & 127 & 43
\end{array}$$

Ans.: 439432

Vinculum Method (Right to Left)

$$\begin{array}{l}
1\ 0\ \bar{2} \\
1\ \bar{2}\ \bar{4} \\
1\ \bar{4}\ \bar{1} \\
\boxed{1\ \bar{6}\ 4\ \bar{1}\ 5\ \bar{6}\ \bar{8}} \\
3\ \bar{1}\ \bar{3}
\end{array}$$

Ans: $1\bar{6}4\bar{1}5\bar{6}\bar{8} = \boxed{43\ 9472}$

Step 1:

<u>Status</u>

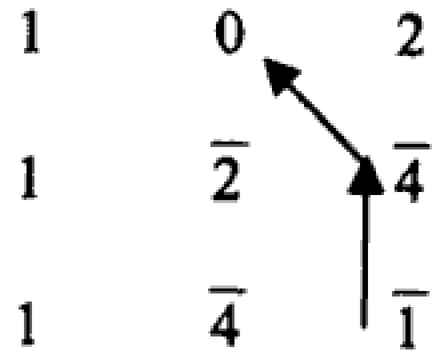
For Units: 1×1×1 = 1

$$\begin{array}{llll}
1 & 0 & \bar{2}\uparrow & \\
1 & \bar{2} & \bar{4}\uparrow & = \bar{8} \\
1 & \bar{4} & \bar{1} &
\end{array}$$

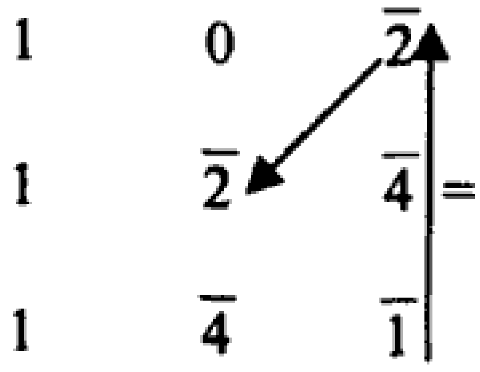
Step 2:

Status

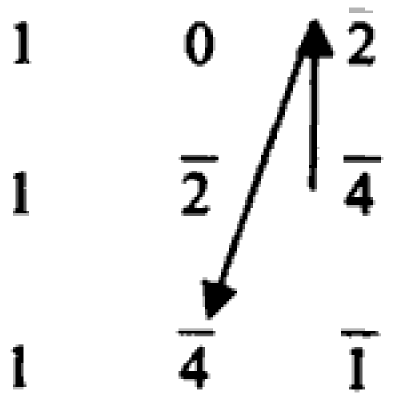
For Tens: $1 \times 1 \times 10 = 10$



$=0$ +



$= \bar{4}$ +



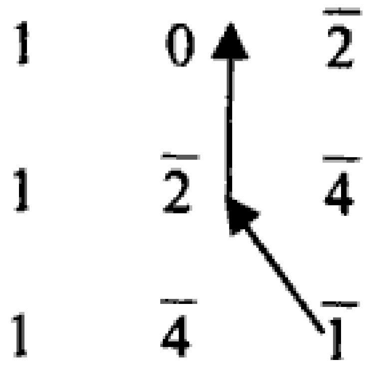
$= \overline{32}$

$\bar{4} + \overline{32} = \overline{36}$

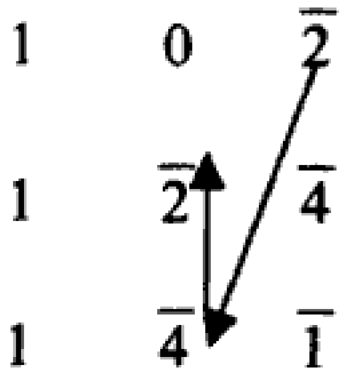
Step 3:

Status

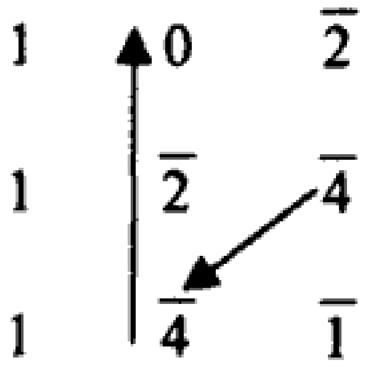
For Hundreds: $1 \times 10 \times 10 + 1 \times 1 \times 100 = 100$



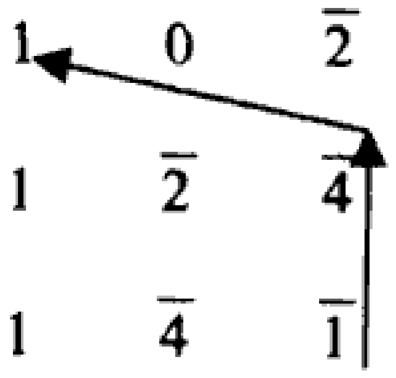
$=0$ +



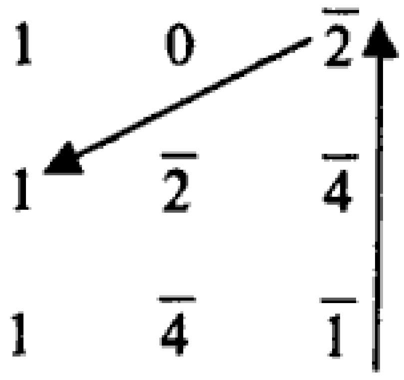
$= \bar{1}\bar{6}$ +



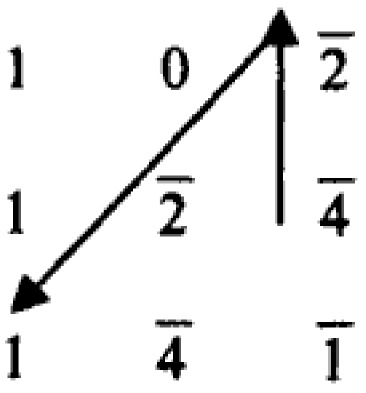
$=0$ +



$=4$ +



$= 2$ +



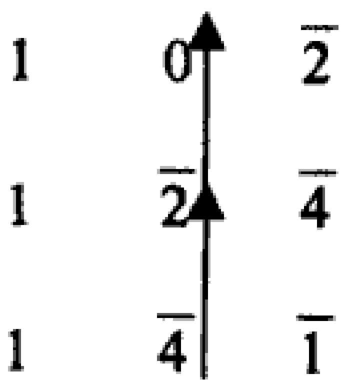
$= 8$

$0 + \bar{1}\bar{6} + 0 + 4 + 2 + 8 = \bar{1}8$

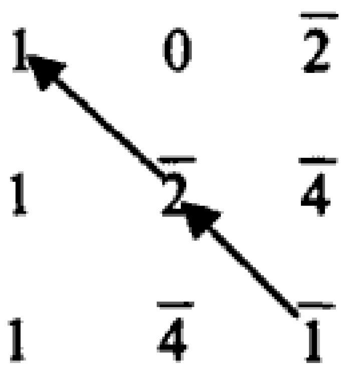
Step 4:

Status

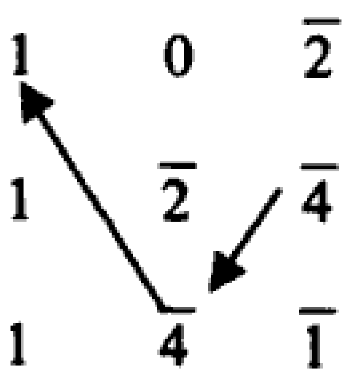
For Thousands: $10 \times 10 \times 10 + 1 \times 10 \times 100 = 1000$



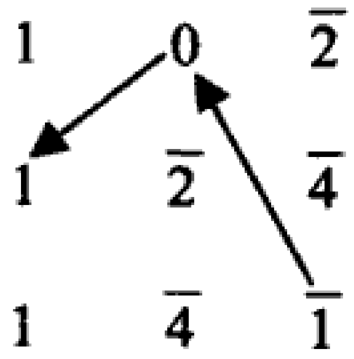
$=0$ +



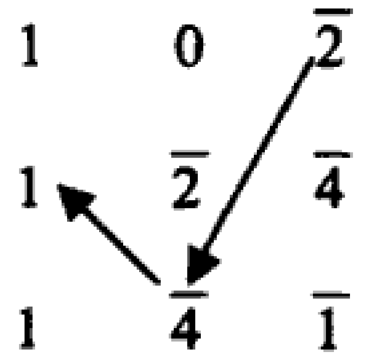
$=2$ +



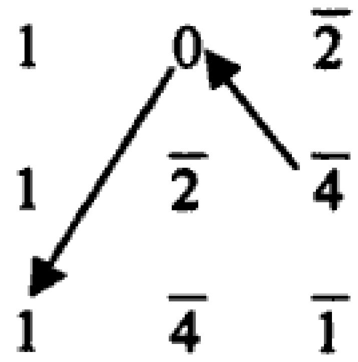
$=16$ +



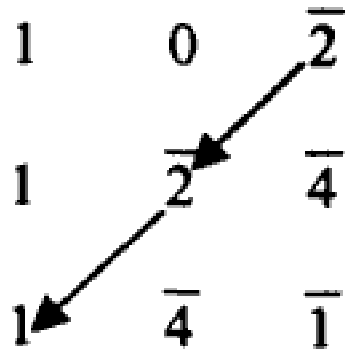
=0 +



=8 +



=0 +



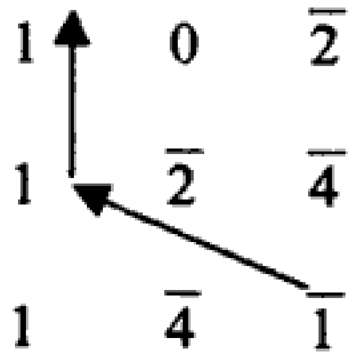
=4

0 + 2 + 16 + 0 + 8 + 0 + 4 = 30

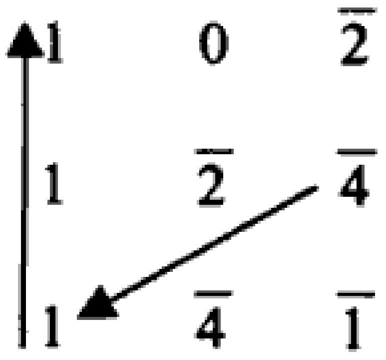
Step 5:

Status

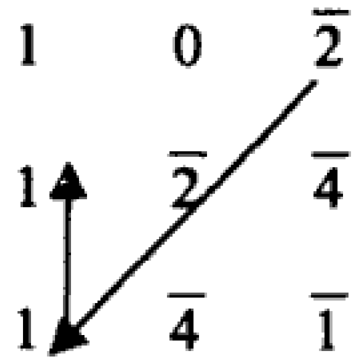
For Ten Thousands: 10×10×100 +1×100×100 = 10000



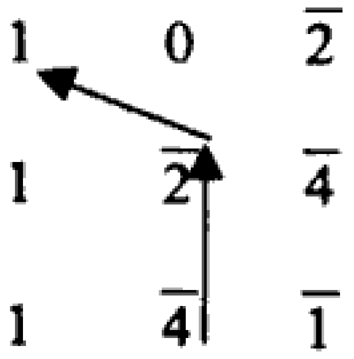
=1 +



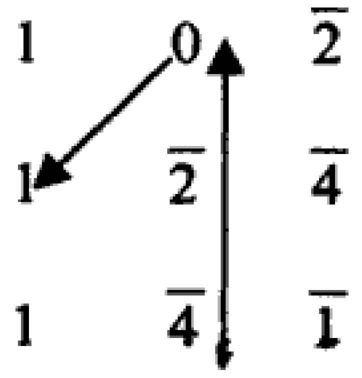
=1 +



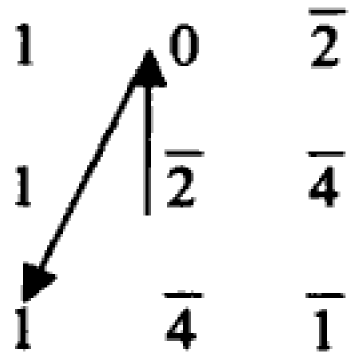
= $\bar{2}$ +



= 8 +



= 0 +



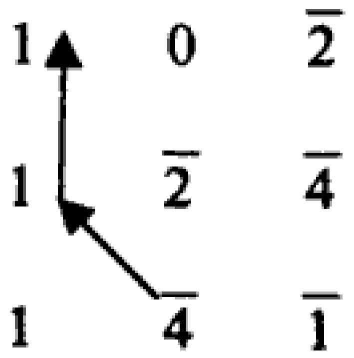
= 0

8 + 0 + 0 + $\bar{1}$ + $\bar{4}$ + $\bar{2}$ = 1

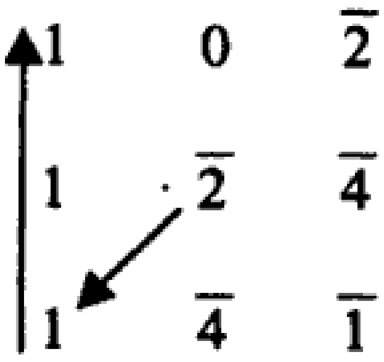
Step 6:

Status

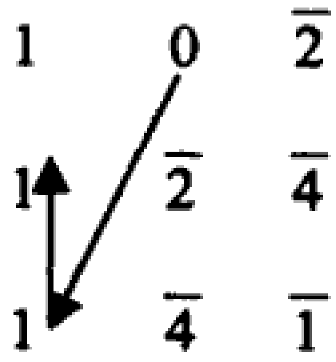
For Lakhs: 10×100×100 = 100000



= $\bar{4}$ +



= $\bar{2}$ +



=0

$\bar{4}$ + $\bar{2}$ + 0 = $\bar{6}$

Step 7:

Status

For Ten Lakhs: $100 \times 100 \times 100 = 1000000$

Answer:

$$\begin{array}{ccccc} 1\uparrow & 0 & \overline{2} & & \\ 1\uparrow & \overline{2} & \overline{4} & = & 1 \\ 1 & \overline{4} & \overline{1} & & \end{array}$$

$$\begin{array}{r} 1000000 \\ \overline{6}00000 \\ 10000 \\ 30000 \\ \overline{1}800 \\ \overline{36}0 \\ \overline{8} \\ \hline 1\overline{6}4\overline{1}5\overline{68} \\ \hline 439432 \end{array} \qquad \begin{array}{r} 1000000 \\ \overline{1}400000 \\ 10000 \\ 30000 \\ \overline{1}9800 \\ \overline{1}640 \\ \overline{1}2 \\ \hline 439432 \\ \hline \end{array}$$

Example 2: 99 × 99 × 99

Vedic Method (Right to Left)

$$\begin{array}{cccc} & & 9 & 9 \\ & & 9 & 9 \\ & & 9 & 9 \\ \hline \multicolumn{1}{|c}{970} & 2 & 9 & \multicolumn{1}{c|}{9} \\ \hline \end{array}$$

Ans.: 970299

Vinculum Method (Right to Left)

$$\begin{array}{ccccccc} & & & & 1 & 0 & \overline{1} \\ & & & & 1 & 0 & \overline{1} \\ & & & & 1 & 0 & \overline{1} \\ \hline \multicolumn{1}{|c}{1} & 0 & \overline{3} & 0 & 3 & 0 & \multicolumn{1}{c|}{\overline{1}} \\ \hline \end{array}$$

Step 1:

Status

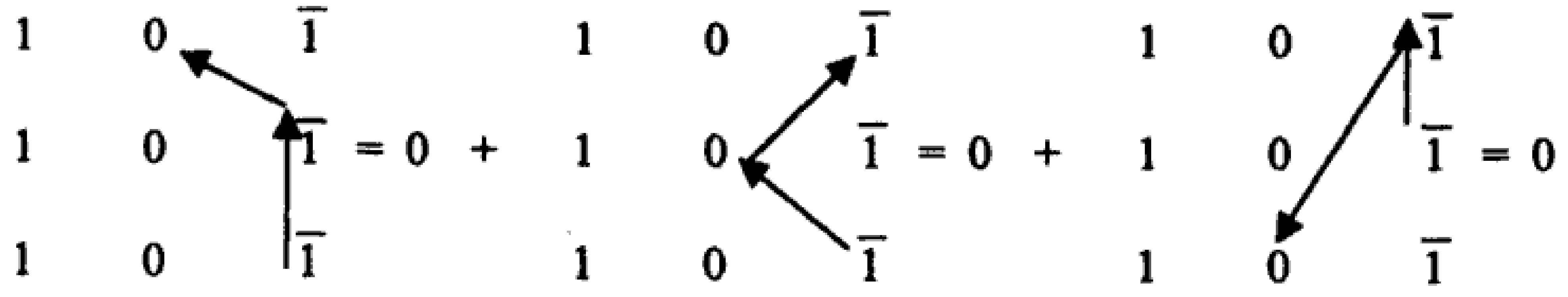
For Units: $1 \times 1 \times 1 = 1$

$$\begin{array}{ccccc} 1 & 0 & \overline{1}\uparrow & & \\ 1 & 0 & \overline{1}\uparrow & = & \overline{1} \\ 1 & 0 & \overline{1} & & \end{array}$$

Step 2:

Status

For Tens: 1×1×10 = 10

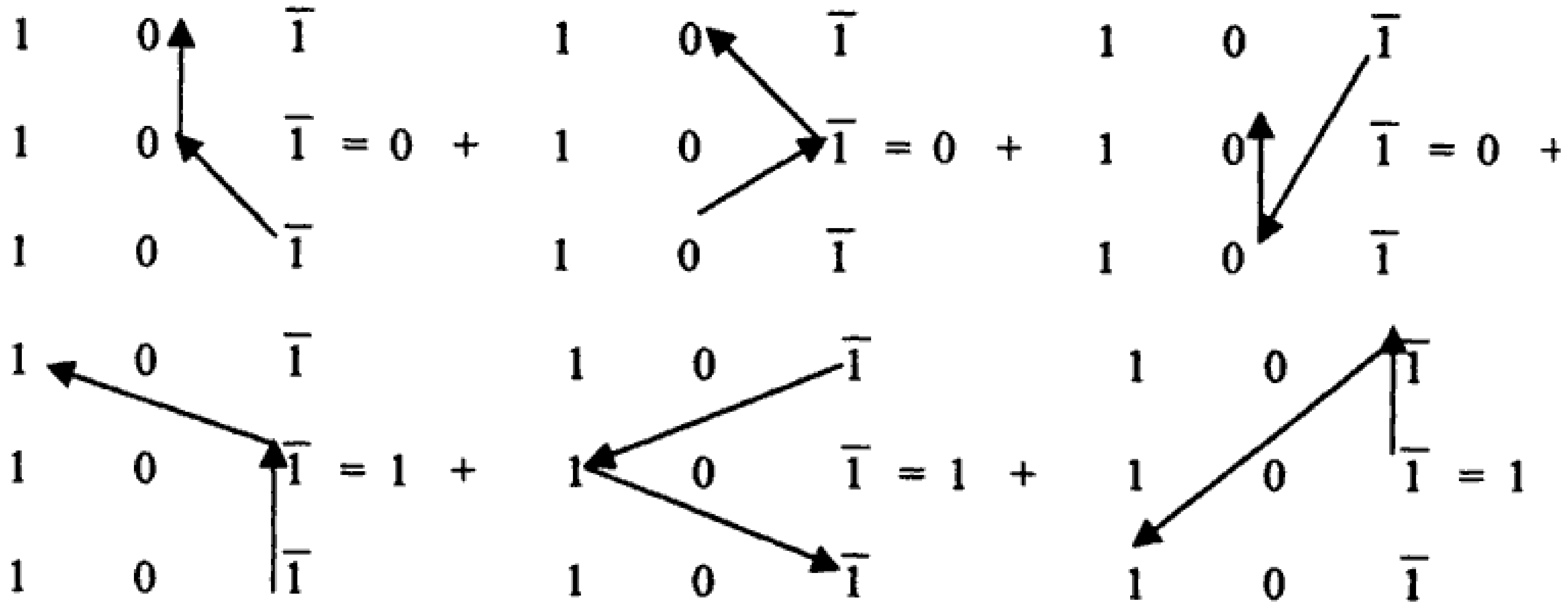


0 + 0 + 0 = 0

Step 3:

Status

For Hundreds: 1×10×10 + 1×1×100 = 100

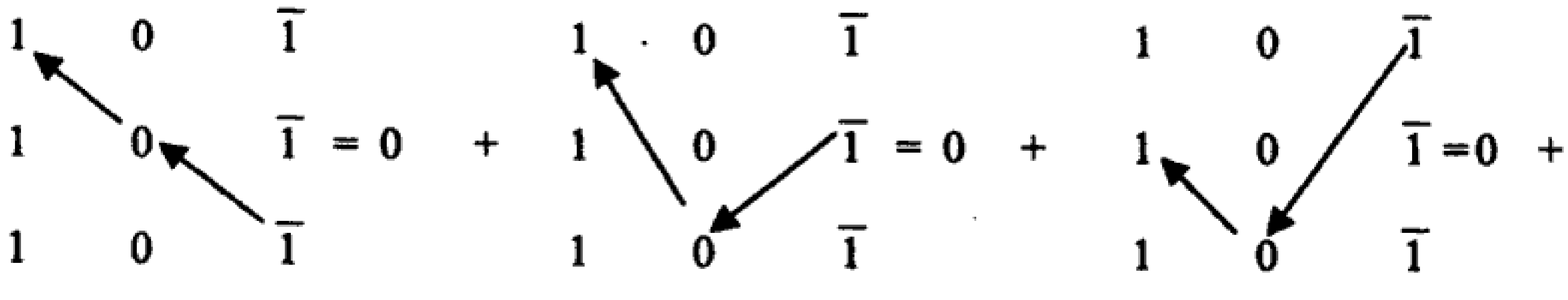


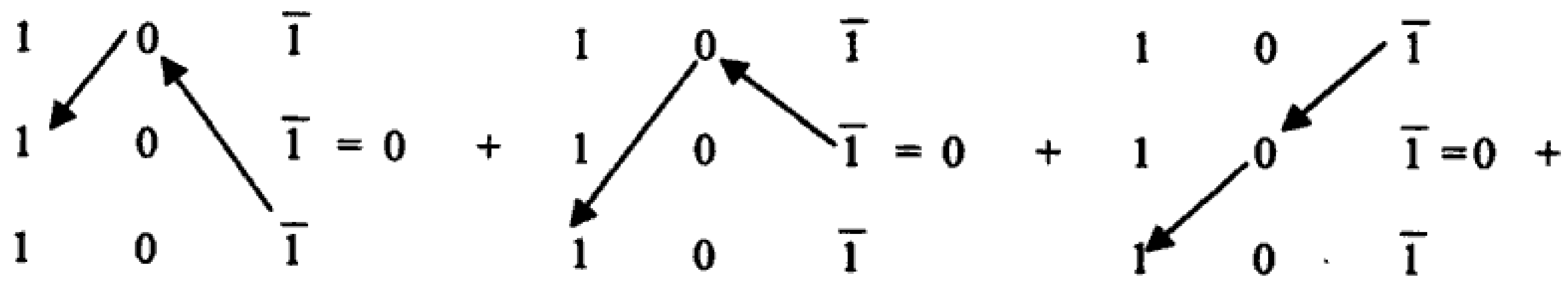
0 + 0 + 0 + 1 + 1 + 1 = 3

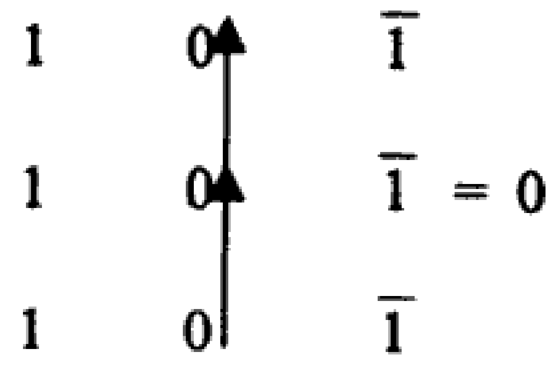
Step 4:

Status

For Thousands: 1×10×100 + 10×10×10 = 1000





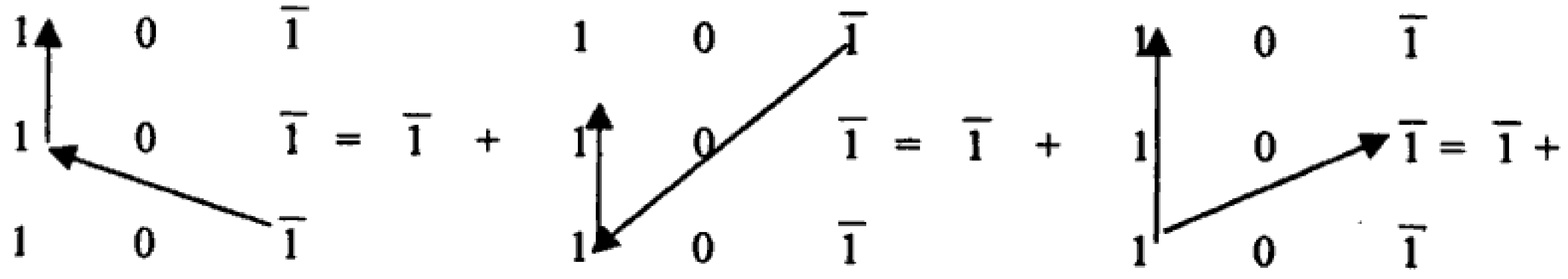


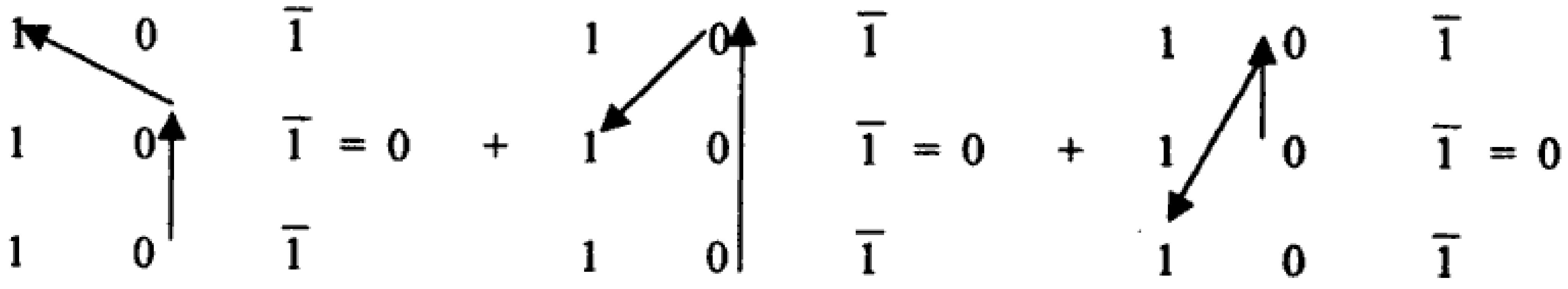
Total = 0

Step 5:

Status

For Ten Thousands: 1×100×100 + 10×10×100 = 10000



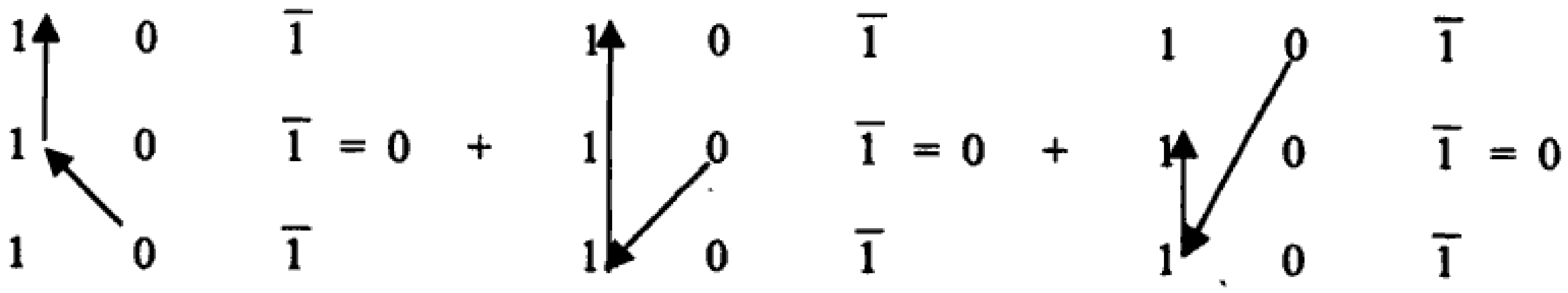


$\bar{1}+\bar{1}+\bar{1}+0+0+0=\bar{3}$

Step 6:

Status

For Lakhs: 10×100×100 = 100000



Total = 0

Step 7:

Status

For Ten Lakhs: 100×100×100 = 1000000

$$\begin{array}{cccc} 1\uparrow & 0 & \bar{1} & \\ 1\uparrow & 0 & \bar{1} & = 1 \\ 1| & 0 & \bar{1} & \end{array}$$

$$\begin{array}{r} 1000000 \\ +000000 \\ +\bar{3}0000 \\ +0000 \\ +\bar{3}00 \\ +00 \\ +\bar{1} \\ \hline 10\bar{3}0\bar{3}0\bar{1} = 970299 \end{array}$$

Example 3: 989 × 456 × 786

Vedic Method (Right to Left)

$$\begin{array}{ccccccc} & & & & 9 & 8 & 9 \\ & & & & 4 & 5 & 6 \\ & & & & 7 & 8 & 6 \\ \hline \multicolumn{1}{|c}{354} & 4 & 7 & 3 & 4 & 2 & \multicolumn{1}{c|}{4} \\ \hline & 102 & 197 & 235 & 200 & 102 & 32 \end{array}$$

Ans.: 354473424

Vinculum Method (Right to Left)

$$\begin{array}{ccccccccc} & & & & & 1 & 0 & \bar{1} & \bar{1} \\ & & & & & 0 & 5 & \bar{4} & \bar{4} \\ & & & & & 1 & \bar{2} & \bar{1} & \bar{4} \\ \hline 4 & \bar{4} & \bar{6} & 4 & 8 & \bar{6} & \bar{5} & \bar{7} & \bar{6} \\ \bar{1} & 0 & 0 & 3 & 3 & \bar{1} & \bar{3} & \bar{1} & \end{array}$$

Ans: $4\bar{4}\bar{6}48\bar{6}\bar{5}\bar{7}\bar{6} = \boxed{354473424}$

Step 1:

Status

For Units: 1×1×1 = 1

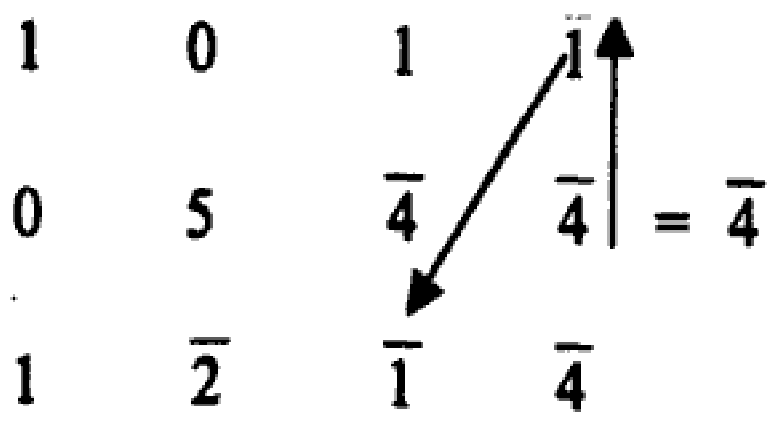
$$\begin{array}{ccccc} 1 & 0 & \bar{1} & \bar{1}\uparrow & \\ 0 & 5 & \bar{4} & \bar{4}\uparrow & = \bar{1}\bar{6} \\ 1 & \bar{2} & \bar{1} & \bar{4}| & \end{array}$$

Step 2:

Status

For Tens: 1×1×10 = 10

$$\begin{array}{ccccc} 1 & 0 & \bar{1}\nwarrow & \bar{1} & \\ 0 & 5 & \bar{4} & \bar{4}\uparrow & = \bar{1}\bar{6} \quad + \\ 1 & \bar{2} & \bar{1} & \bar{4}| & \end{array} \qquad \begin{array}{ccccc} 1 & 0 & \bar{1} & \bar{1}\uparrow & \\ 0 & 5 & \swarrow\bar{4} & \bar{4}| & = \bar{1}\bar{6} \quad + \\ 1 & \bar{2} & \bar{1} & \bar{4}| & \end{array}$$

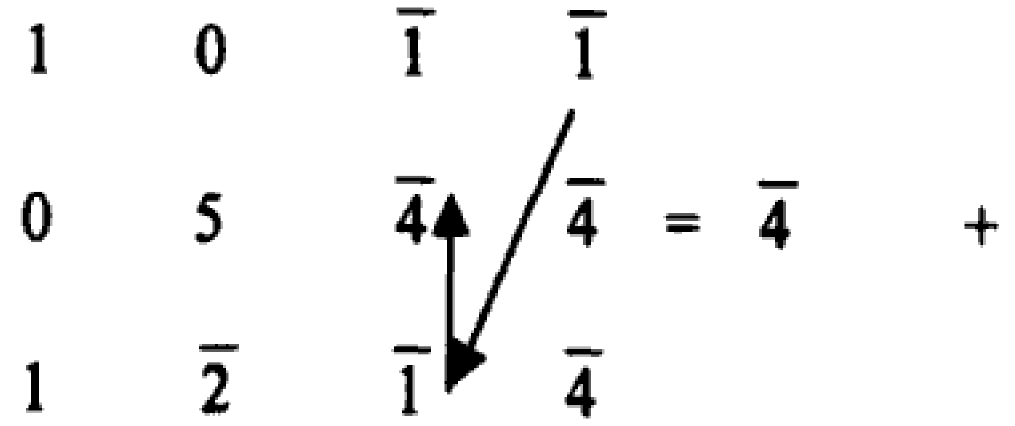


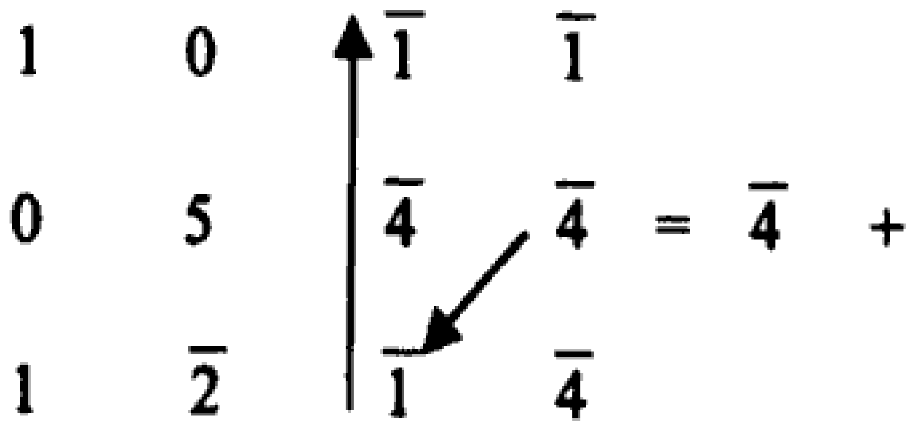
$\bar{1}\bar{6} + \bar{1}\bar{6} + \bar{4} = \bar{3}\bar{6}$

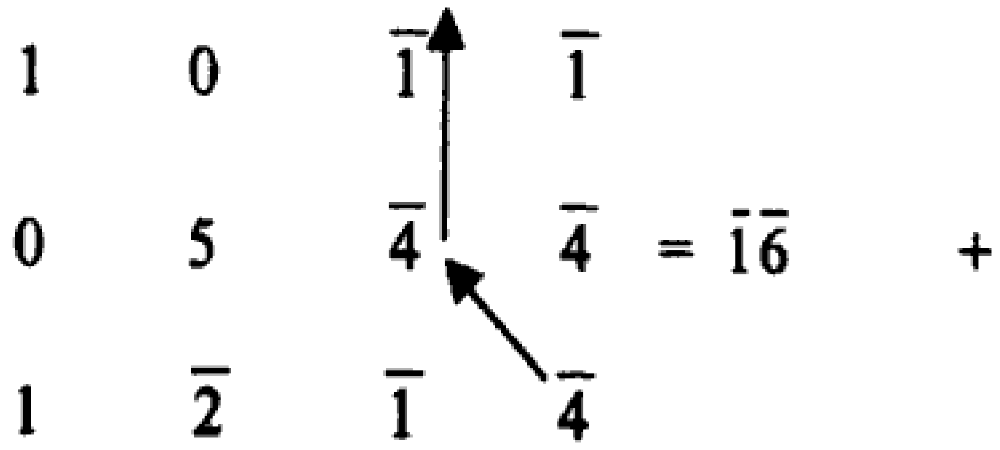
Step 3:

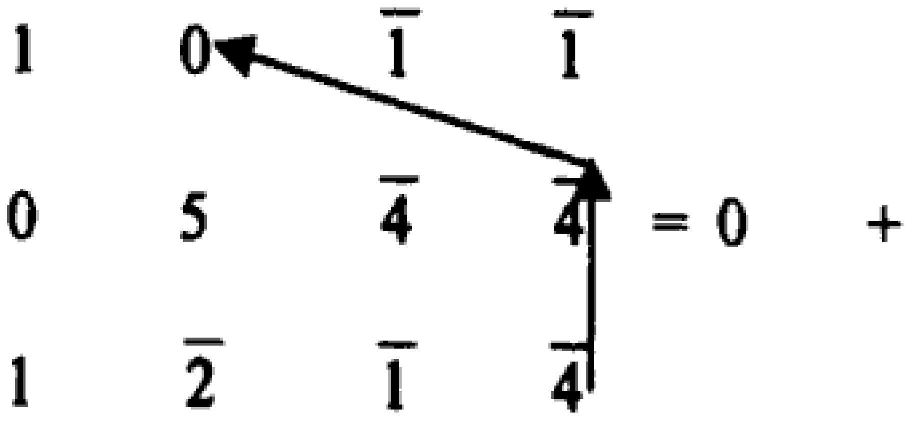
<u>Status</u>

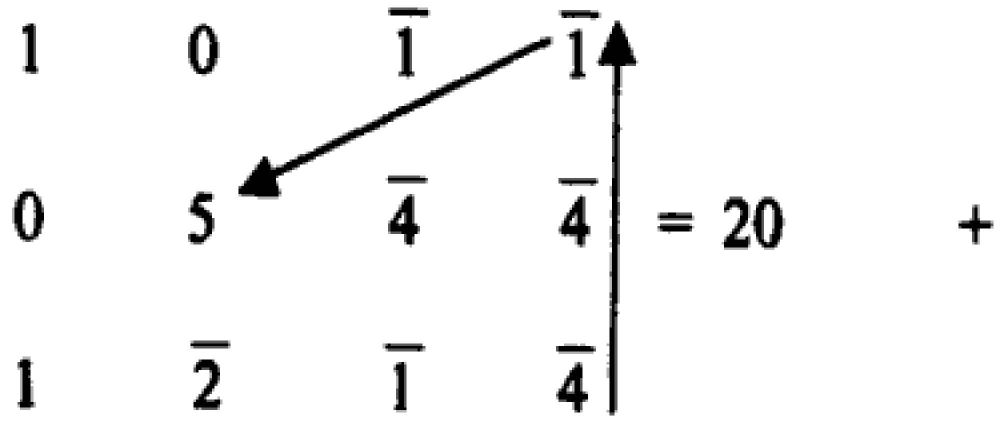
For Hundreds: 1×10×10 + 1×1×100 = 100

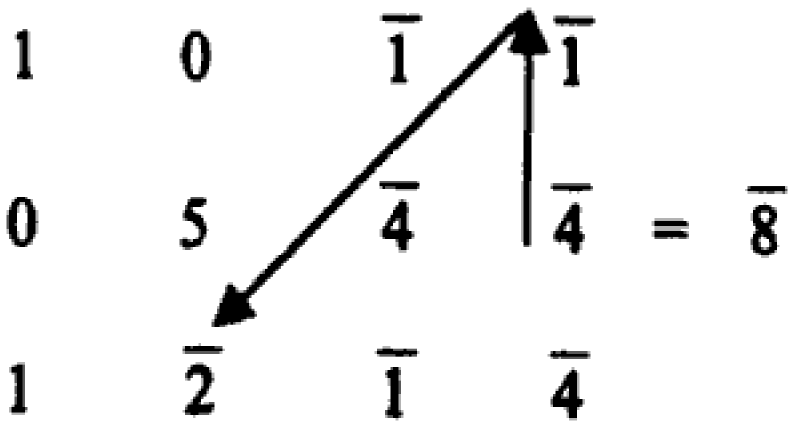










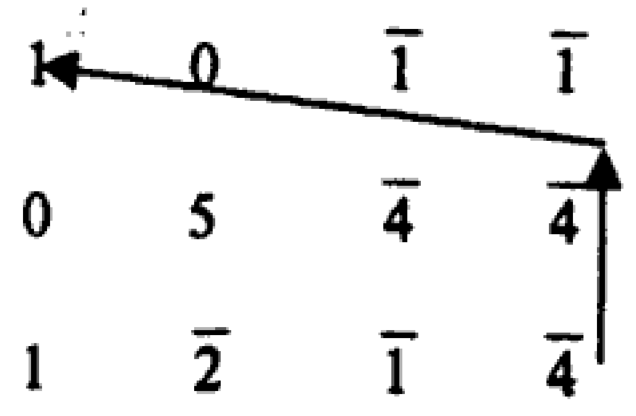


$\bar{4} + \bar{4} + \bar{1}\bar{6} + 0 + 20 + \bar{8} = \bar{1}\bar{2}$

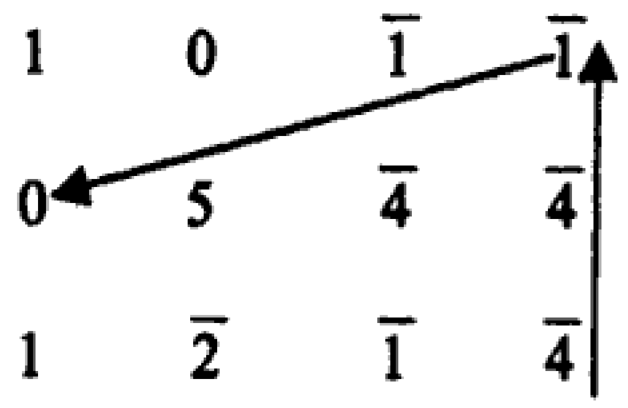
Step 4:

Status

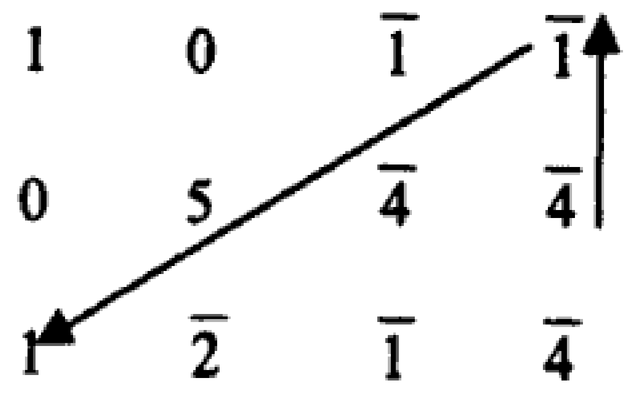
For Thousands: 1×1×1000 + 1×10×100 + 10×10×10 = 1000



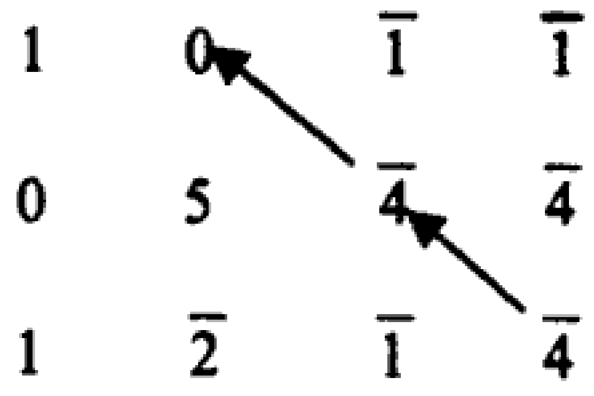
= 16 +



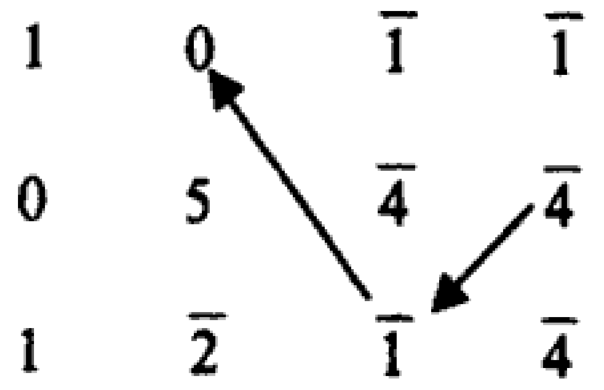
= 0 +



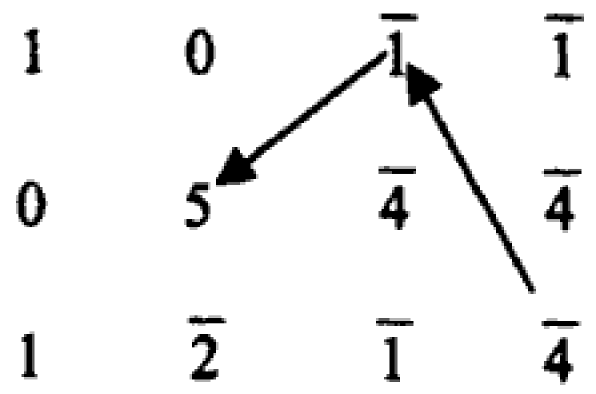
= 4 +



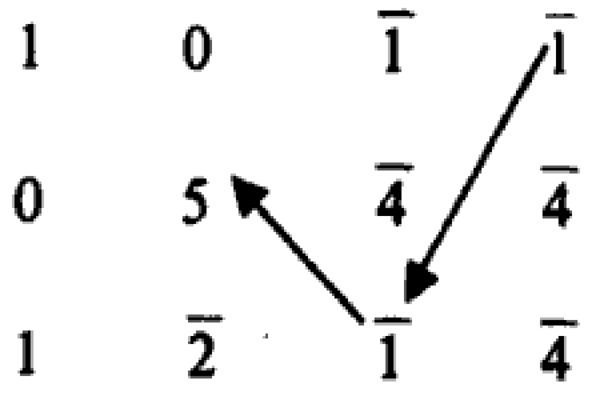
= 0 +



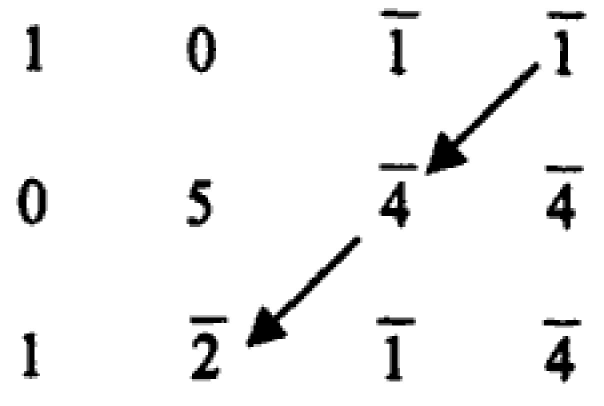
= 0 +



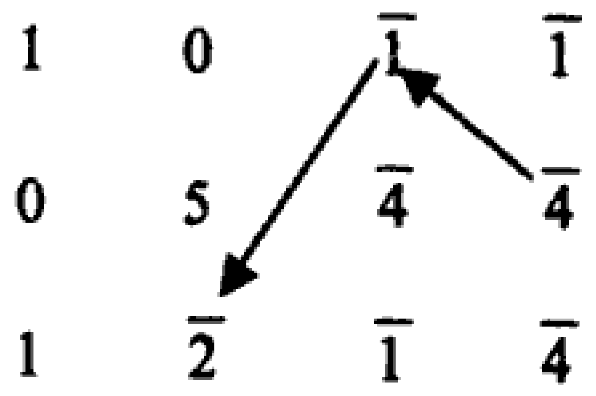
= 20 +



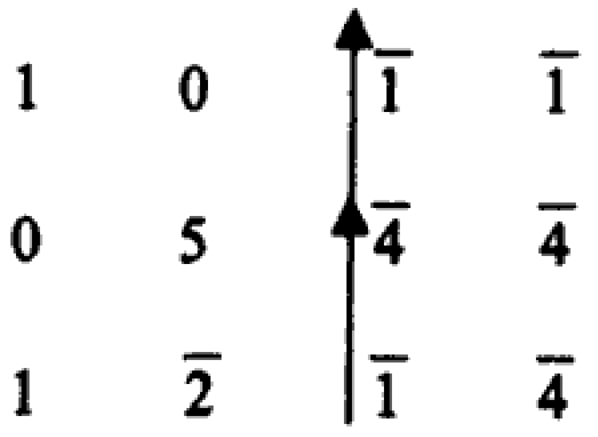
= 5 +



= $\bar{8}$ +



= $\bar{8}$ +



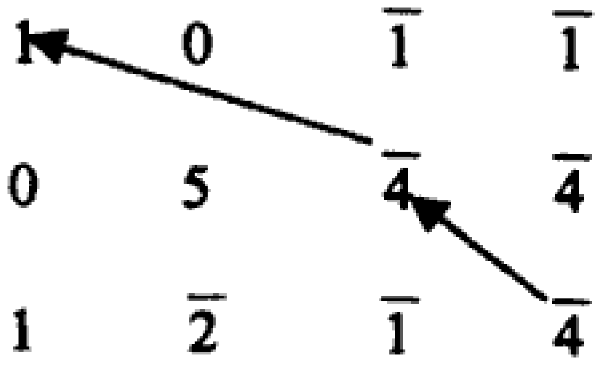
= $\bar{4}$

$16 + 4 + 20 + 5 + \bar{8} + \bar{8} + \bar{4} = 3\bar{5} = 25$

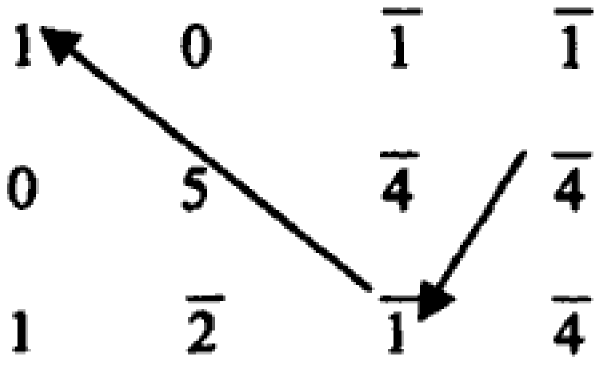
Step 5:

<u>Status</u>

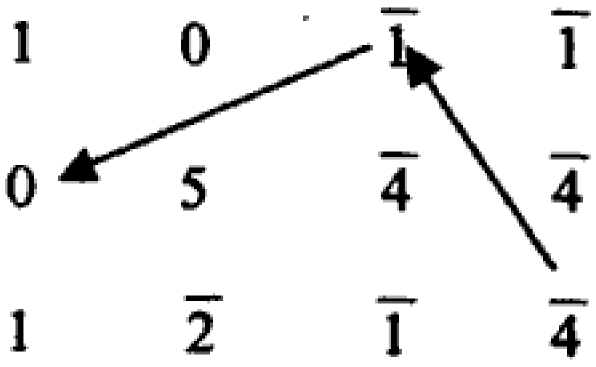
For Ten Thousands: $1\times10\times1000 + 1\times100\times100 + 10\times10\times100 = 10000$



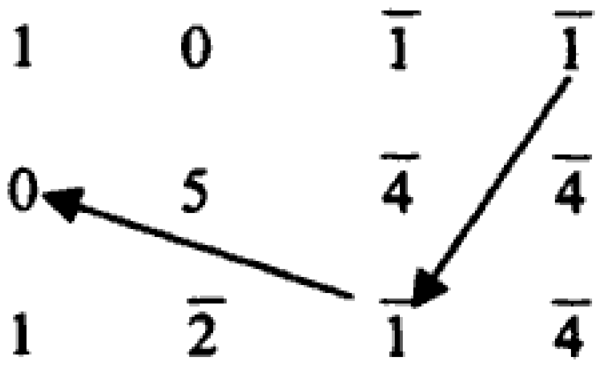
= 16 +



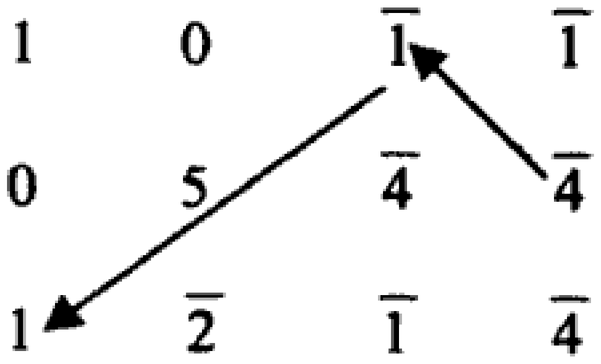
= 4 +



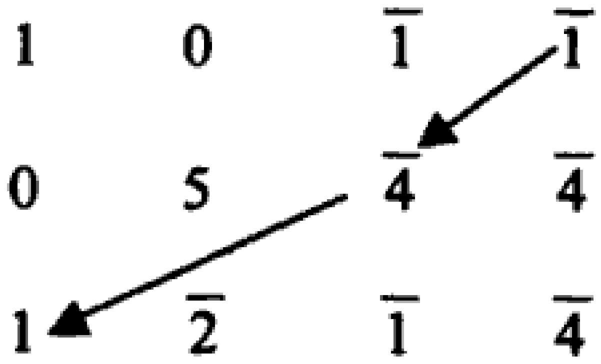
= 0 +



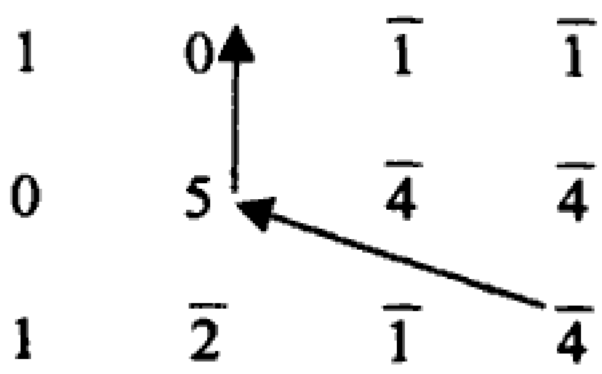
= 0 +



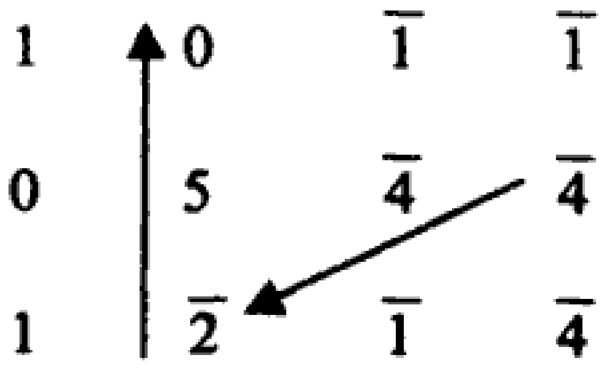
= 4 +



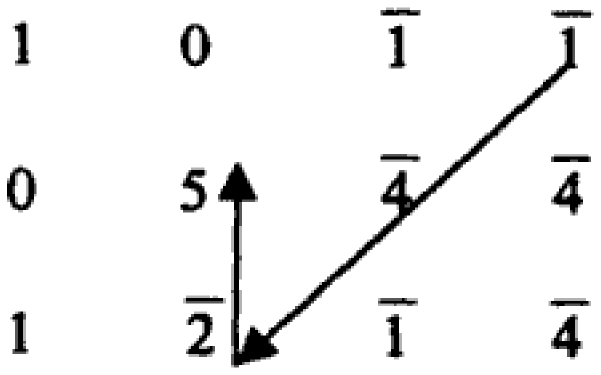
= 4 +



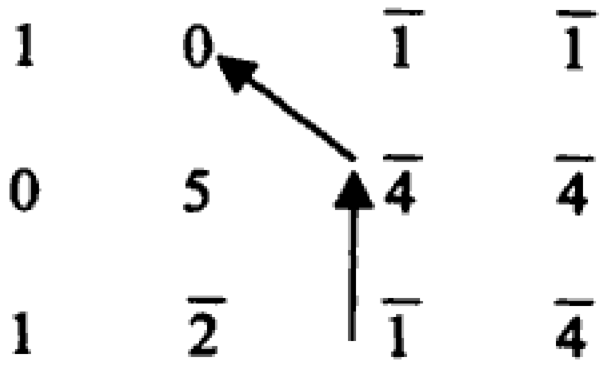
= 0 +



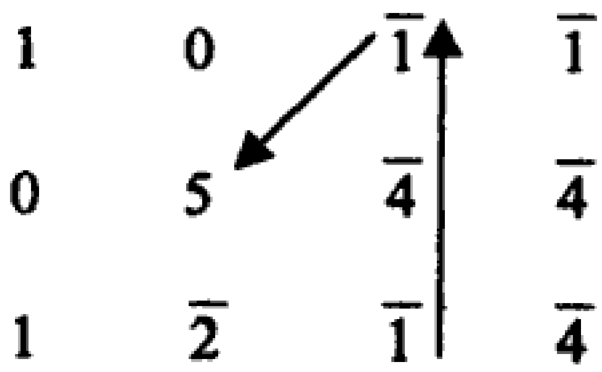
= 0 +



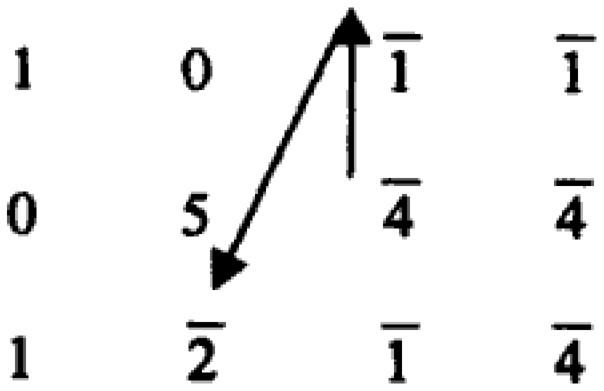
= 10 +



= 0 +



= 5 +



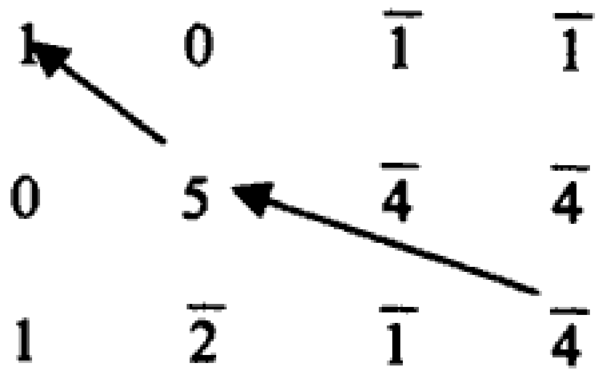
= $\bar{8}$

$16 + 4 + 4 + 4 + 10 + 5 + \bar{8} = 35$

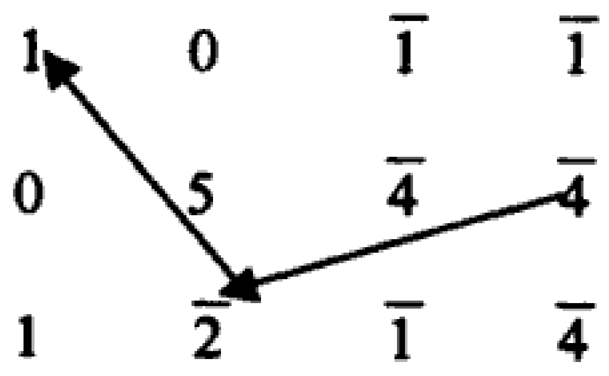
Step 6:

Status

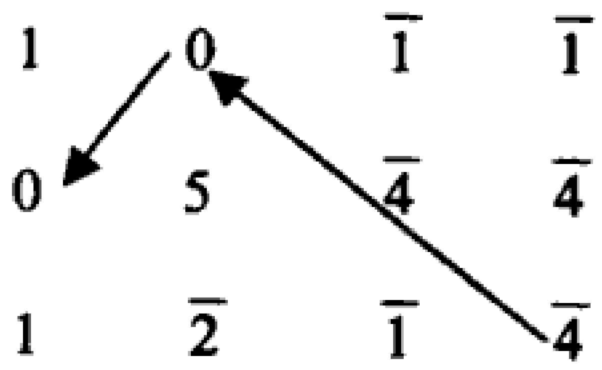
For Lakhs: $1 \times 100 \times 1000 + 10 \times 100 \times 100 + 10 \times 10 \times 1000 = 100000$



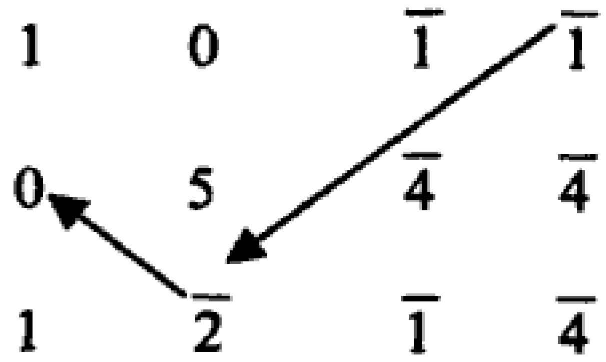
$= \overline{2}\overline{0}$ +



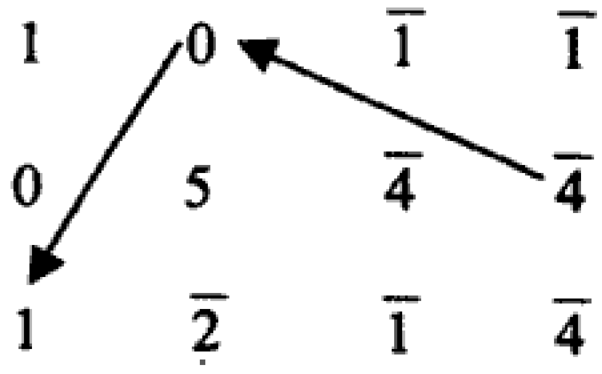
$= 8$ +



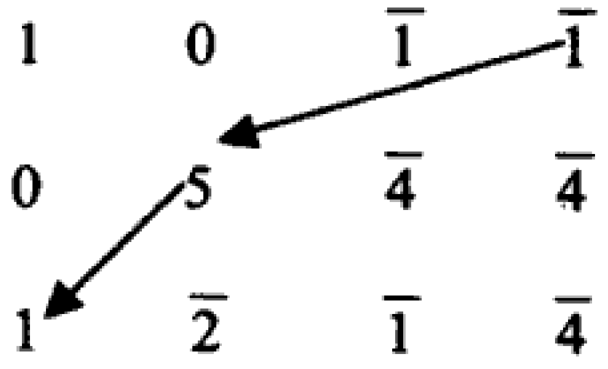
$= 0$ +



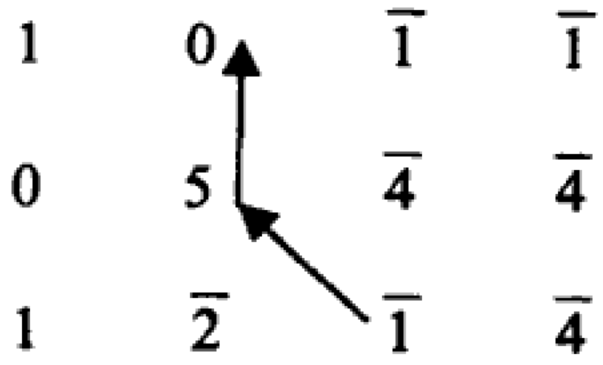
$= 0$ +



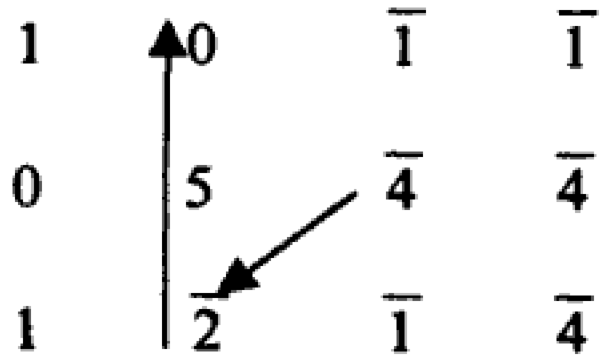
$= 0$ +



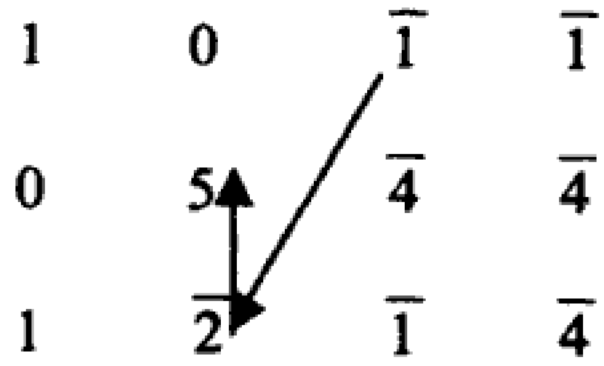
$= \overline{5}$ +



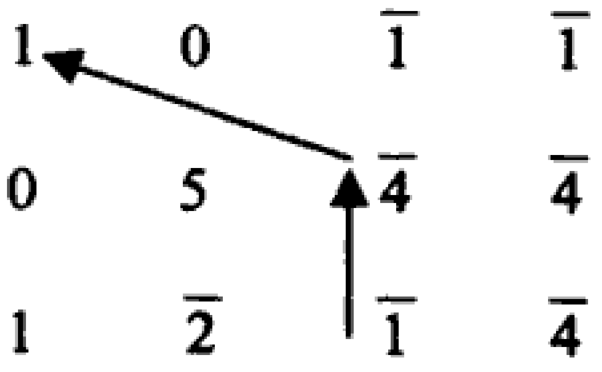
$= 0$ +



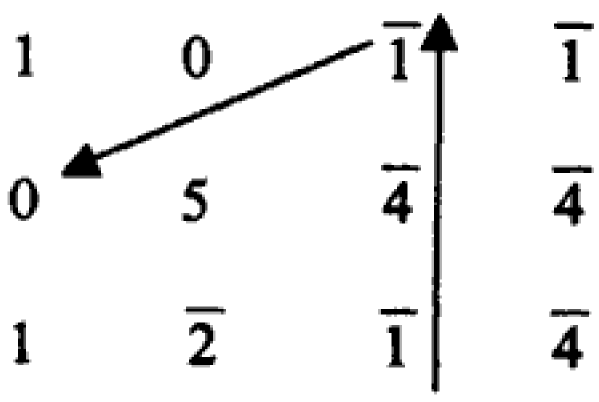
$= 0$ +



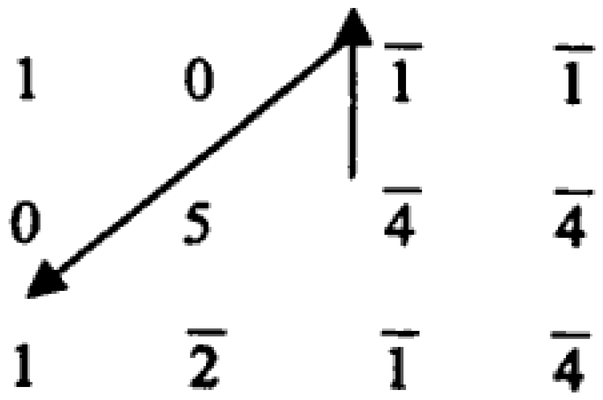
$= 10$ +



$= 4$ +



$= 0$ +



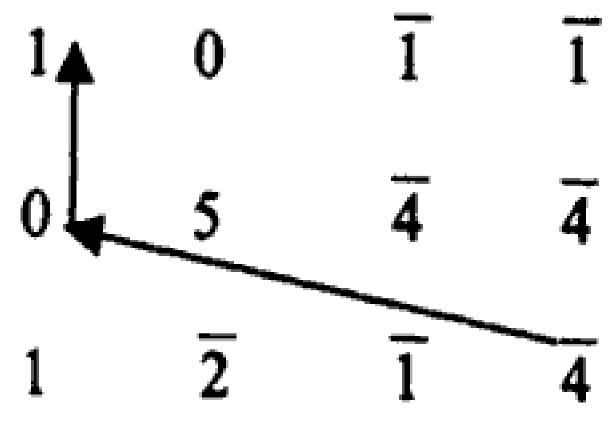
$= 4$

$+ 8 + \overline{5} + 10 + 4 + 4 = 1$

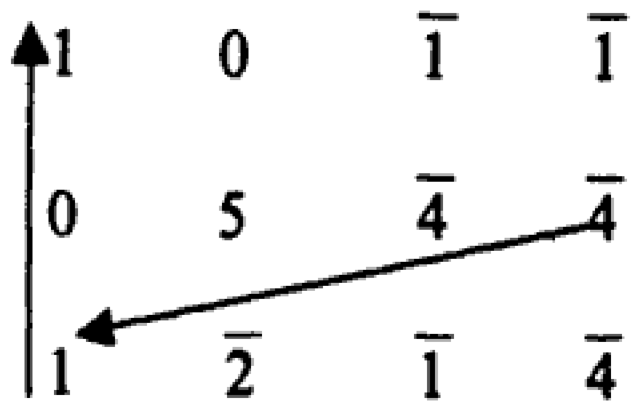
Step 7:

Status

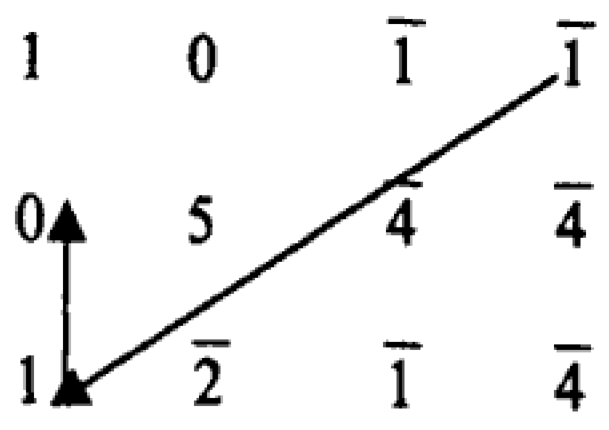
For Ten Lakhs: $1\times1000\times1000 + 10\times100\times1000 + 100\times100\times100 = 1000000$



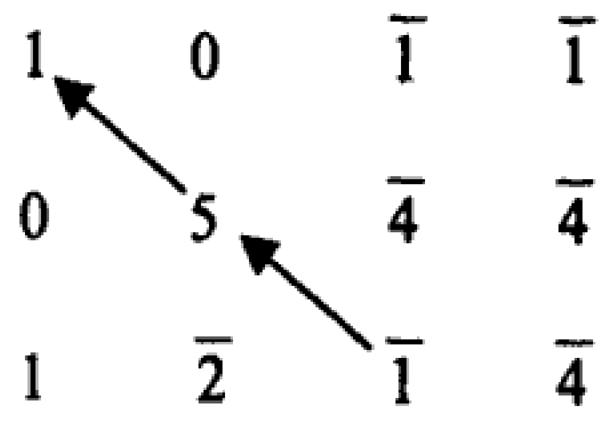
$=0$ +



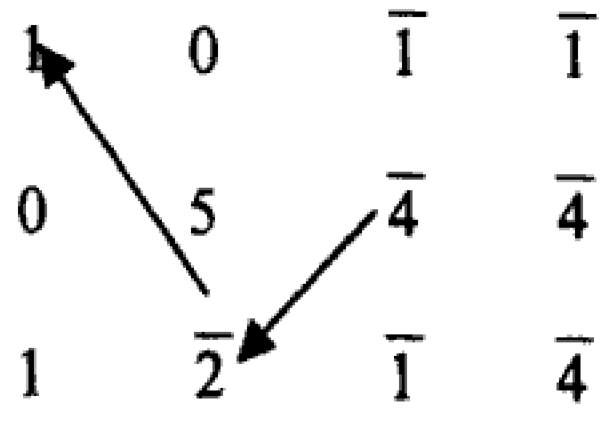
$= \bar{4}$ +



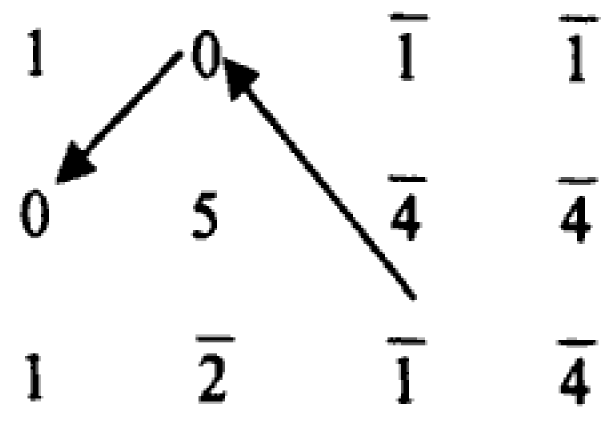
$=0$ +



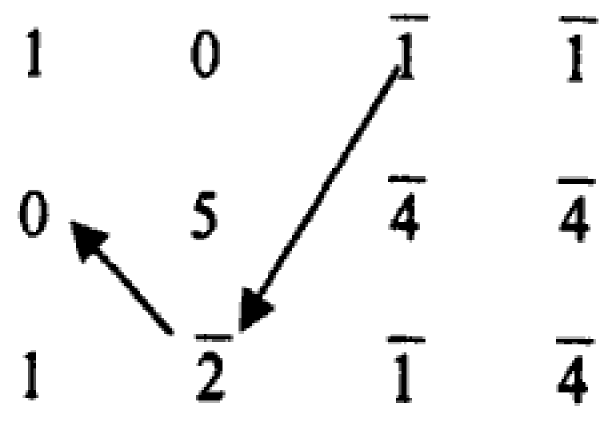
$= \bar{5}$ +



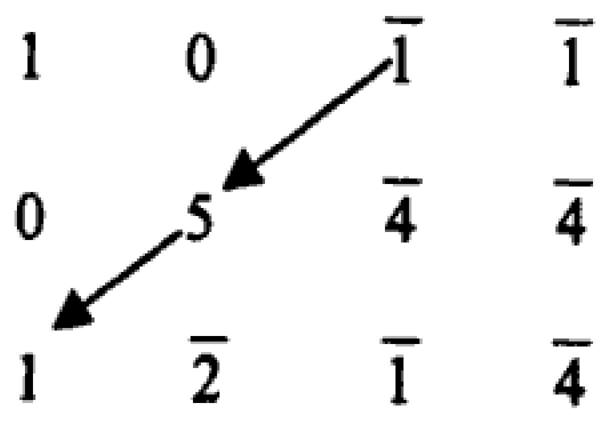
$=8$ +



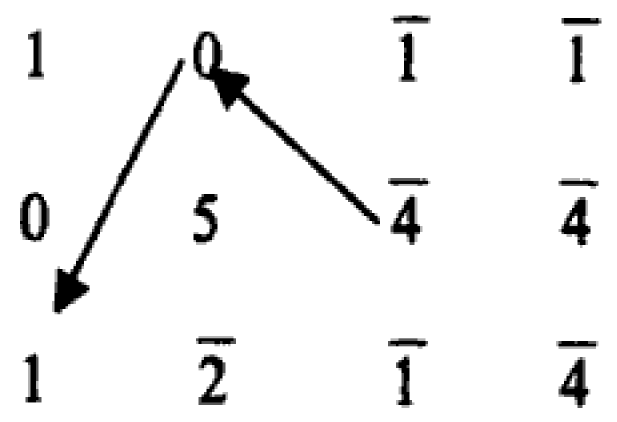
$=0$ +



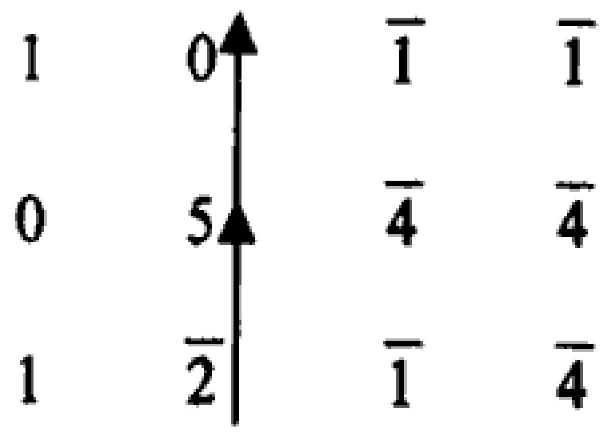
$=0$ +



$= \bar{5}$ +



$=0$ +



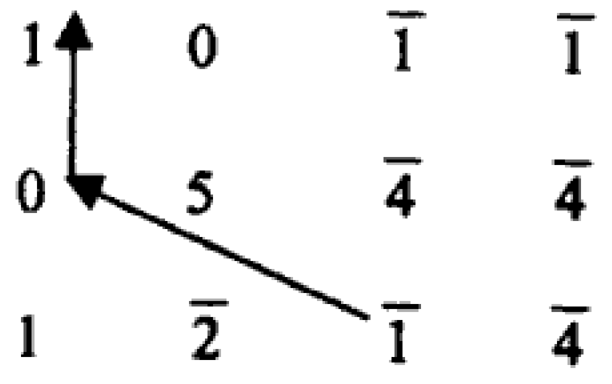
$=$ 0

$\bar{4}+\bar{5}+8+\bar{5}=\bar{6}$

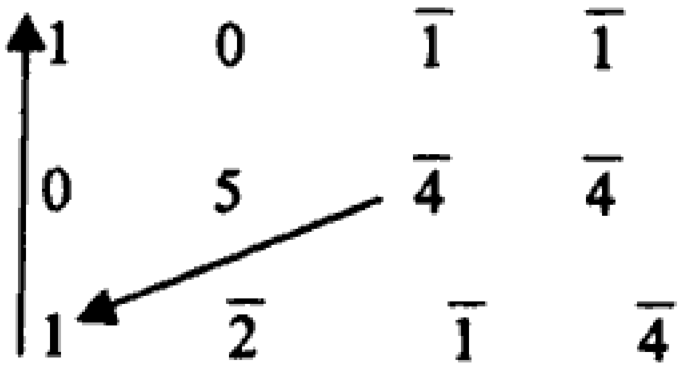
Step 8:

Status

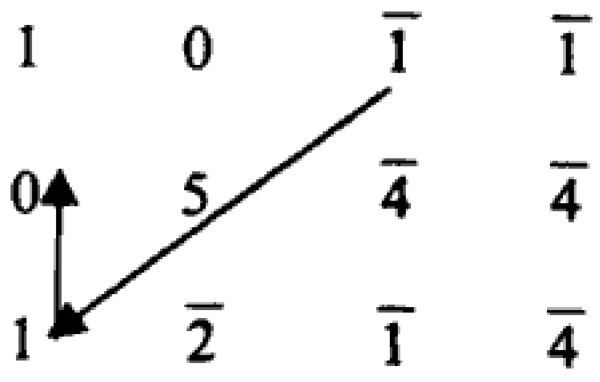
For Crores: 10×1000×1000 + 100×100×1000 = 10000000



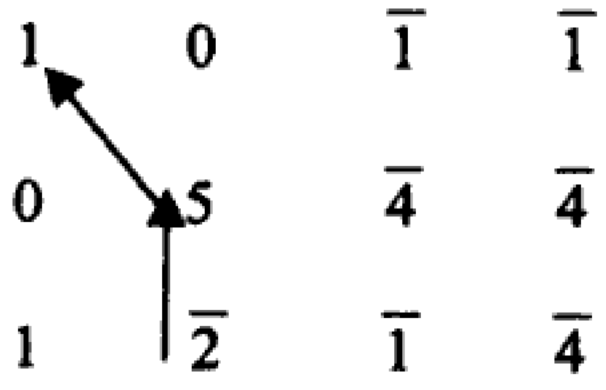
=0 +



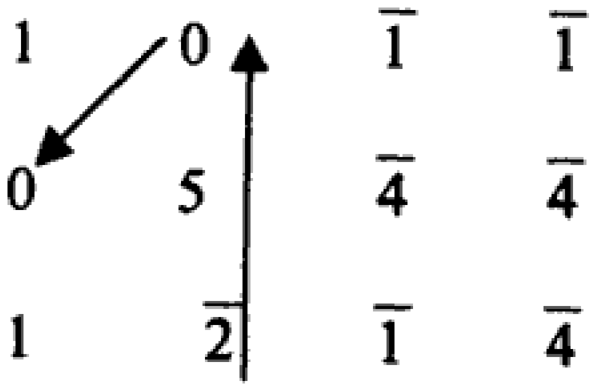
= $\bar{4}$



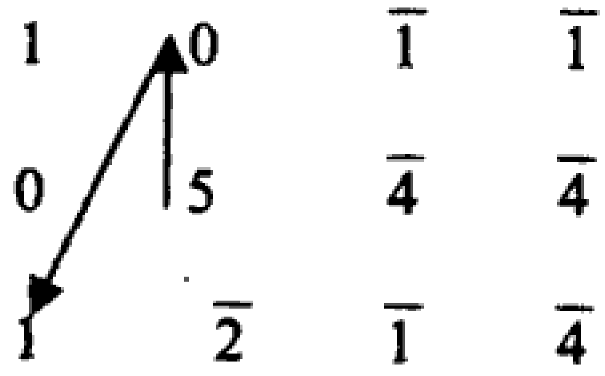
=0 +



= $\bar{1}\,\bar{0}$



= 0 +



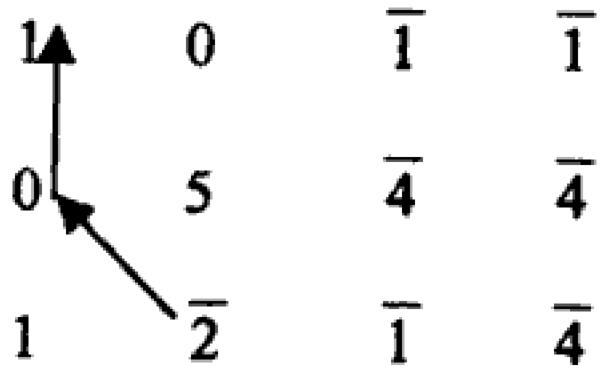
= 0

$\bar{4}$ + $\bar{1}$ $\bar{0}$ = $\bar{1}$ $\bar{4}$

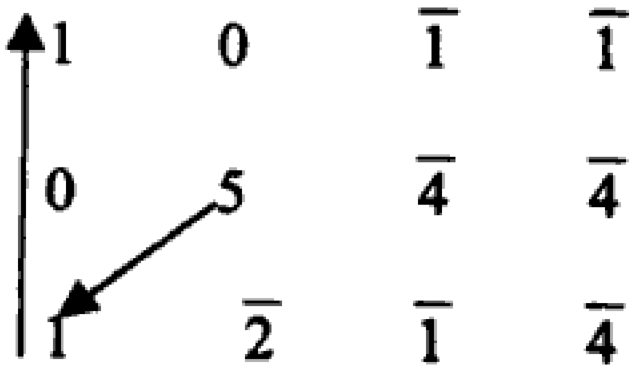
Step 9:

Status

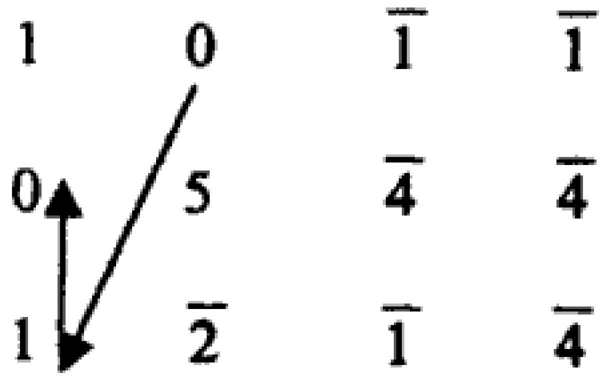
For ten Crorers : 100 x 1000 x 1000 = 100000000



=0 +



=5 +



=0

Step 10:

Status

For Hundred Crores : 1000 x 1000 x 1000 = 100000000

$$\begin{array}{lccccc} \uparrow & 1 & 0 & \bar{1} & \bar{1} & \\ \uparrow 0 & 5 & \bar{4} & \bar{4} & = & 0 \\ | 1 & \bar{2} & \bar{1} & \bar{4} & & \end{array}$$

Answer:

$$\begin{array}{r} 500000000 \\ \bar{1}\,\bar{4}0000000 \\ \bar{6}000000 \\ 100000 \\ 350000 \\ 3\,\bar{5}000 \\ \bar{1}\bar{2}00 \\ \bar{3}\,\bar{6}0 \\ \bar{1}\bar{6} \\ \hline 4\,\bar{4}\;\bar{6}48\,\bar{6}\bar{5}\bar{7}\bar{6} \\ \hline 35\;4473424 \end{array}$$

V) Group Multiplication using Vinculum is also given below:

Consider 98768 × 56879

Using Vinculum

$98768 = 1\,0\;\bar{1}\,\bar{2}\,\bar{3}\,\bar{2}$

$56879 = 1\;\bar{4}\,\bar{3}\,\bar{1}\,\bar{2}\,\bar{1}$

Taking groups as $(10)\,(\bar{1}\,\bar{2})\,(\bar{3}\,\bar{2})$

$(1\,\bar{4})\,(\bar{3}\,\bar{1})\,(\bar{2}\,\bar{1})$

$(10)\,(\bar{1}\,\bar{2})\,(\bar{3}\,\bar{2})$ can be written as $10x^2 + \bar{1}\bar{2}x + \bar{3}\bar{2}$ and

$(1\,\bar{4})\,(\bar{3}\,\bar{1})\,(\bar{2}\,\bar{1})$ can be written as $1\bar{4}x^2 + \bar{3}\bar{1}x + \bar{2}\bar{1}$ where x = 100

Applying Urdhava Tiryagbhyam Sutra

$$\begin{array}{r} 10x^2 + \;\bar{1}\bar{2}x + \bar{3}\bar{2} \\ 1\bar{4}x^2 + \;\bar{3}\bar{1}x + \bar{2}\bar{1} \\ \hline 1\bar{4}0x^4 + \bar{4}18x^3 + \;\bar{1}70x^2 + 1244x + 672 \end{array}$$

Substituting for x, we get the result:

$$\begin{array}{r} 1\;\bar{4}000000000 \\ \bar{4}18000000 \\ \bar{1}700000 \\ 124400 \\ 672 \\ \hline 1\;\bar{4}\;\bar{4}17825072 \\ \hline \end{array}$$

$1\;\bar{4}\;\bar{4}17825072 = 5617825072$

9. CHAPTER III

(I) APPLICATION OF NIKHILAM NAVATASCARAMAM DASATAH SUTRAM TO MULTIPLICATION (V.M.):

Nikhilam Navatascaramam Dasatah: (The last digit from ten, the others from nine)

The modus operandi of this sutram is to work out the deficit / excess of a number from its considered base, in powers of 10 and is nearest to the number. The last digit (caramam) is to be subtracted always from 10 (Dasatah) and all the remaining digits (Nikhilam) are to be subtracted from 9 (Navatah) (This is the meaning of the Sutram)

For example: The deficit of 799 from base 1000 is 201. Using this Nikhilam Sutram the deficit is worked out as follows: The last digit 9 is subtracted from ten; the result is 1. The remaining digits 9 and 7 are to be subtracted from 9 each. So the results are 0 and 2. Hence the deficit becomes 201.

Using this Sutram, the multiplication of two numbers can be explained as follows, considering the same base for both multiplicand and multiplier.

Consider the same base in powers of 10 which is nearest to the numbers to evaluate the deficit / excess of both the multiplier and the multiplicand. They play a very important role in the multiplication. The deficit is treated as negative and excess as positive. Now the answer consists of two parts.

The first part is obtained by multiplying the deficit / excess of the multiplicand and multiplier. While multiplying the deficit / excess as the first part of the answer, (a) one has to allot (n–1) digits space in it where 'n' is the number of digits in the base. If necessary, zeros are to be considered to make up (n–1) digits. (b) All other digits in excess to (n – 1) digits are to be carried over to the second part of the answer, which are shown in the working details. (c) In case the two numbers have different bases, the first part of the answer has (n–1) digits space in it, where n is the base of either of the two numbers. This is called **Placement.**

We may add here that whenever the multiplication of the first part of the answer is concerned with the multiplication of numbers consisting of more number of digits, one can resort to Urdhva Tiryak Sutram or may resort to Vinculum method of calculations or an iteration process with different bases can be tried.

The second part is worked out by considering the cross difference or sum (It is the sum of one number with deficit or excess of the other number) as the case may be. This is shown clearly in the examples.

The same rule applies for the multiplication between the numbers consisting of any number of digits.

In case of decimal multiplication, multiply the numbers without decimals, and finally decimals should be placed in the answer in the usual manner.

a) Deficit from base:

1) Single–digit Numbers (Multiplication of two numbers) :

i) 6 x 9

$$\begin{array}{r|l} 6 & -\ 4 \\ 9 & -\ 1 \\ \hline 5\ / & 4 \end{array}$$

Base is 10

Ans. : 54

First Part = (–4)(–1) = 4

Second Part = 6 – 1 = 9 – 4 = 5

ii) 9 × 4

$$\begin{array}{r|l} 9 & -1 \\ 4 & -6 \\ \hline 3\ / & 6 \end{array}$$

Base 10

Ans.: 36

First Part = (–1)(–6) = 6

Second Part = 9 – 6 = 4 – 1 = 3

Proof: $(x - a)(x - b) = x(x - a - b) + ab$ (x is the base) Eq (1)

If the first part of multiplication yields a product consisting of more than n digits then the placement is as per the n – 1 digits where n is the base. The remaining digits are to be carried over to the second part

i) 6 × 5

$$\begin{array}{r|l} 6 & -4 \\ 5 & -5 \\ \hline 1\,/_{2} & 0 \end{array}$$

Base is 10

= 3/0 Ans.: 30

ii) 8 × 5

$$\begin{array}{r|l} 8 & -2 \\ 5 & -5 \\ \hline 3\,/_{1} & 0 \end{array}$$

Base is 10

= 4/0 Ans.: 40

2) Two–digit Numbers Multiplication:

i) 73 × 97

$$\begin{array}{r|l} 73 & -27 \\ 97 & -\ 3 \\ \hline 70\ / & 81 \end{array}$$

Base is 100

Ans.: 7081

ii) 92 × 87

$$\begin{array}{r|l} 92 & -\ 8 \\ 87 & -13 \\ \hline 79\,/_{1} & 04 \end{array}$$

Base is 100

= 80/04 Ans.: 8004

iii) 92 × 99

$$\begin{array}{r|l} 92 & -\ 8 \\ 99 & -\ 1 \\ \hline 91\,/ & 08 \end{array}$$

Base is 100

Ans. = 9108

(iv) 91 x 91

$$\begin{array}{r|l} 91 & -\ 9 \\ 91 & -\ 9 \\ \hline 82\,/ & 81 \end{array}$$

Base is 100

Ans. 8281

3) Three–digit Numbers Multiplication:

i) 996 × 887

$$\begin{array}{r|l} 996 & -\ \ 4 \\ 887 & -113 \\ \hline 883\,/ & 452 \end{array}$$

Base is 1000

Ans.: 883452

ii) 587 × 994

$$\begin{array}{r|l} 587 & -413 \\ 994 & -\ \ 6 \\ \hline 581\,/_{2} & 478 \end{array}$$

Base is 1000

Ans.: 583478

iii) 993 × 995

$$\begin{array}{r|l} 993 & -\ 7 \\ 995 & -\ 5 \\ \hline 988\,/ & 035 \end{array}$$

Base is 1000

Ans.: 988035

iv) 666 x 795 Base is 1000

$$\begin{array}{r|l} 666 & -334 \\ 795 & -205 \\ \hline 461\,/_{68} & 470 \end{array}$$

Ans. 529470

4) Four–digit Numbers Multiplication:

i) 8889 × 9992

$$\begin{array}{r|r} 8889 & -\ 1111 \\ 9992 & -\ \ \ \ 8 \\ \hline 8881\ / & 8888 \end{array}$$

Base is 10000

Ans:88818888

ii) 8587 × 9357

$$\begin{array}{r|r} 8587 & -\ 1413 \\ 9357 & -\ \ 643 \\ \hline 7944\,/_{90} & 8559^{*} \end{array}$$

Base is 10000

Ans:80348559

iii) 9998 x 9992

$$\begin{array}{r|r} 9998 & -\ \ \ 2 \\ 9992 & -\ \ \ 8 \\ \hline 9990\,/ & 0016 \end{array}$$

Base is 10000

Ans.: 99900016

*Here we are using Urdhva Tiryak Sutram for the multiplication of the first part.

$$\begin{array}{cccccc} & & 1 & 4 & 1 & 3 \\ & & 0 & 6 & 4 & 3 \\ \hline 9 & 0 & 8 & 5 & 5 & 9 \\ \hline & 3 & 2 & 3 & 1 & \end{array}$$

5) Nine–digit Numbers Multiplication:

999999996 × 999987962

$$\begin{array}{r|r} 999999996 & -\ \ \ \ \ \ \ 4 \\ 999987962 & -\ \ \ 12038 \\ \hline 999987958\ / & 000048152 \end{array}$$

Base is 1000000000

Ans.: 999987958000048152

(b) Excess from base:

1) Two–digit Numbers Multiplication

i) 13 x 16

$$\begin{array}{r|r} 13 & +\ 3 \\ 16 & +\ 6 \\ \hline 19\,/_{1} & 8 \end{array} = 20 / 8$$

Base is 10

Ans.: 208

ii) 18 x 19

$$\begin{array}{r|r} 18 & +\ 8 \\ 19 & +\ 9 \\ \hline 27\,/_{7} & 2 \end{array} = 34 / 2$$

Base is 10

Ans.: 342

Proof: $(x + a)\ (x + b) = x\ (x + a + b) + ab$ (x is the base) Eq (2)
a, b are deficiencies (–) / excess (+)

(2) Three–digit Numbers Multiplication

i) 126 × 135

$$\begin{array}{r|r} 126 & +\ 26 \\ 135 & +\ 35 \\ \hline 161\,/_{9} & 10 \end{array} = 170 / 10$$

Base is 100

Ans:17010

ii) 113 × 102

$$\begin{array}{r|r} 113 & +\ 13 \\ 102 & +\ \ 2 \\ \hline 115\,/ & 26 \end{array}$$

Base is 100

Ans.: 11526

3) Four–digit Numbers Multiplication:

i) 1013 x 1008

1013 | + 13 Base is 1000
1008 | + 8
1021 / 104 Ans: 1021104

ii) 1021 x 1025

1021 | + 21 Base is 1000
1025 | + 25
1046 / 525 Ans: 1046525

4) Nine–digit Numbers Multiplication:

100000862 × 100005282

100000862 | + 862 Base is 10000000 *multiplication in the 1st part by Urdhva Tiryag
100005282 | + 5282
100006144 / 04553084* Ans.: 10000614404553084

This procedure is extendable to any number multiplication :

c) One of the numbers is above and the other is below the considered base:

In case either the multiplicand or multiplier shows deficit from the base, the other showing excess from the base then the result of the multiplication of the first part will be in vinculum. To get the result, one has to come out from the vinculum

i) 13 × 4
13 | + 3 Base is 10
4 | − 6
$7 / 8_{\overline{1}} = 6 / \overline{8} = 52$ Ans.: 52

ii) 118 × 97
118 | + 18 Base is 100
97 | − 3
$115 / \overline{54}$ Ans.:11446

iii) 1112 × 986
1112 | + 112 Base is 1000
99987 | − 14
$1098 / 568_{1}$ =1099/568 Ans.:1099568

iii) 100031 × 99987
100031 + | 31 Base is 100000
99987 − | 13
$100018 / 00\overline{403}$ Ans.: 10001799597

Proof: $(x + a)(x - b) = x(x + a - b) - ab$ (where x is the base) ⟶ Eq 3 (a)
By substituting the values in equations (1), (2) and (3) one gets the same value as is worked out in parts.(x is the base and a, b are Def/Ex.

(d). Multiplication of 3 or more numbers (V.M.):

1. When the base is common

(A) For example 92 × 93 × 95

Method I.: Making use of the values in the expression derived for three number multiplication one will get the answer.

$(x + a)(x + b)(x + c) = x^2(x + a + b + c) + x(ab + bc + ca) + abc$ ⟶ Eq 3(b)

On substituting x = 100, a =–8, b= –7 c= –5 in the expressions one gets the answer.

$\therefore (100)^2\,[100 - 8 - 7 - 5] + 100\,[(-8)(-7) + (-7)(-5) + (-8)(-5)] + (-8)(-7)(-5) = 810000 + 13100 - 280 = 812820$

II Another method which makes use of writing down the answer in three parts is in the following way :

$$\begin{array}{r|r} 92 & -8 \\ 93 & -7 \\ \underline{95} & \underline{-5} \end{array} \qquad \text{Base is } 100$$

$$80\,/_{1}\quad 31\,/_{\bar{2}}\quad \bar{8}0 \;=\; 81\,/\quad 3\bar{1}\,/\quad \bar{8}0 = 813\bar{1}\bar{8}0 \;= 812820$$

The first part is by mere multiplication of all the three excesses or deficiencies, with reference to the base, $(-8)(-7)(-5) = \bar{2}\bar{8}\bar{0}$. The provision is only for two digits

$\therefore$ I Part = $\genfrac{}{}{0pt}{}{\bar{8}0}{\bar{2}}$

The second part is obtained by adding the product of two by two deficiencies or excesses. i.e., $\Sigma_1^1 \left({}^{3}C_{2} \text{ Products of D/Ex.} \right)$ with reference to the base. The provision is only for two digits.

i.e., $(-8)(-7) + (-8)(-5) + (-7)(-5) = 56 + 35 + 40 = 131$=Placement is. $\genfrac{}{}{0pt}{}{31}{1}$

The last part is obtained by taking any one of the given numbers together with sum of deficiencies or excesses of the other two numbers

i.e., 92 – 7 – 5 = 93 – 8 – 5 = 95 – 8 – 7 = 80.

Answer $80\,/_{1}\quad 31\,/_{\bar{2}}\quad \bar{8}0 = 812820$

The answer obtained by both the methods is same.

(B) Consider another example 89 × 98 × 107

The first method gives by substitution of the values in the general expression x = 100, a= –11, b=–2 c=7.

$\therefore (100)^2\,[100 - 11 - 2 + 7] + 100\,[(-11)(-2) + (-11)(7) + (-2)(7)] + (-11)(-2)(7) = 940000 - 6900 + 154 = 933254.$

II The second method involves the parts of the answers as follows

$$\begin{array}{r|l} 89 & -11 \text{Base is } 100 \\ 98 & -\ 2 \\ 107 & +\ 7 \\ \hline \end{array}$$

$$94 / \ \overline{69} /_1 \ 54 = 94/\ \overline{68} / \ 54 = 933254$$

First Part = (−11) (−2) (7) = 154 placement is $_1{}^{54}$ (Product of D/Ex. w.r.t. base)

Second Part = (−11) (−2) + (−2) (7) + (−11) (7) = 22 − 14 − 77 = $\overline{69} . \Sigma_1^1 \left({}^3C_2 \text{ Products of D/Ex.} \right)$

Third Part = 89 − 2 + 7 = 98 − 11 + 7 = 107 − 2 − 11 = 94.(Cross addition)

A number of examples are given below:

(C) <u>Example: 102 × 109 × 117</u>

Base is 100

$$\begin{array}{r|l} 102 & +\ \ 2 \\ 109 & +\ \ 9 \\ 117 & +\ 17 \\ \hline \end{array}$$

$$128 /_2 \ 05 /_3 \ 06 = 130/08/06 = 1300806$$

First Part = (2) (9) (17) = 306. Placement is $_3{}^{06}$ (Product of D/Ex w.r.t. Base)

Second Part = (2) (9) + (9) (17) + (17) (2) = 205. Placement is $_2{}^{05}$

$\Sigma_1^1 \left({}^3C_2 \text{ Products of D/Ex.} \right)$

Last Part = 102 + 9 + 17 = 109 + 2 + 17 = 117 + 9 + 2 = 128.

Final Answer= $128 /_2 \ 05 /_3 \ 06 = 1300806$

(D) <u>Example: 9999 × 9984 × 9986 × 9998</u>

Base is 10000

$$\begin{array}{r|l} 9999 & -\ \ 1 \\ 9984 & -\ 16 \\ 9986 & -\ 14 \\ 9998 & -\ \ 2 \\ \hline \end{array}$$

$$9967/0316/\overline{0732}/0448 = 9967031592680448$$

First Part = (−1) (−16) (−14) (−2) = 448. (Product of D/Ex) Placement is 0448

Second Par = (−1) (−16) (−14) + (−1) (−16) (−2) + (−1) (−14) (−2) + (−16) (−14) (−2) = $\overline{7}3\overline{2}$.

$\Sigma_1^1 \left[{}^{4}c_{3} \quad \text{Products of D/Ex.} \right]$ Placement is $0 \ \bar{7}\,\bar{3}\,\bar{2}$

Third Part = (−1) (−16) + (−1) (−14) + (−1) (−2) + (−16) (−14) + (−16) (−2) + (−14) (−2) = 316.= $\Sigma_1^1 \left({}^{4}c_{2} \quad \text{Products of D/Ex.} \right)$ Placement is 0136

Last Part = 9999 − 16 − 14 − 2 = 9984 − 1 − 14 − 2 = 9986 − 1 − 16 − 2 = 9998 − 1 − 16 −14 = 9967

Proof: $(x + a)(x + b)(x + c)(x + d) = x^3(x + a + b + c + d) + x^2(ab + ac + ad + bc + bd + cd) + x(abc + abd + acd + bcd) + abcd$ (x is the base) $\longrightarrow$ Eq (3c)

On substituting x = 10000, a = −1, b = −16, c = − 14, d = −2 we get final answer as 9967031592680448.

(E) Example: 11 × 12 × 19 × 17 × 15

Base is 10.

$$\begin{array}{r|rr} 11 & + & 1 \\ 12 & + & 2 \\ 19 & + & 9 \\ 17 & + & 7 \\ 15 & + & 5 \\ \hline \end{array}$$

$${}^{34}/_{20} \quad {}^{8}/_{78} \quad {}^{6}/_{123} \quad {}^{1}/_{63} \quad 0 \quad = \quad 639540$$

First Part = (1) (2) (9) (7) (5) = 630 Placement is $_{63}{}^{0}$ (Products of D/Ex.)

Second Part = (1) (2) (9) (7) + (1) (2) (9) (5) + (1) (2) (7) (5) + (2) (9) (7) (5) + (9) (7) (5) (1) =1231 $\Sigma_1^1 \left({}^{5}c_{4} \quad \text{Products of D/Ex.} \right)$ Placement is $_{123}{}^{1}$

Third Part = (1) (2) (9) + (1) (2) (7) + (1) (2) (5) + (1) (9) (7) + (1) (9) (5) + (1) (7) (5) + (2) (9) (7) + (2) (9) (5) + (2) (7) (5) + (9) (7) (5) = 786 $\Sigma_1^1 \left({}^{5}c_{3} \quad \text{Products of D/Ex.} \right)$ Placement is $_{78}{}^{6}$

Fourth Part = (1) (2) + (1) (9) + (1) (7) + (1) (5) + (2) (9) + (2) (7) + (2) (5) + (9) (7) + (9) (5) + (7) (5) = 208 $\Sigma_1^1 \left({}^{5}c_{2} \quad \text{Products of D/Ex.} \right)$ Placement is $_{20}{}^{8}$

Last Part = 11 + 2 + 9 + 7 + 5 = 12 + 1 + 9 + 7 + 5 = 19 + 1 + 2 + 7 + 5 = 17 + 1 + 2 + 9 + 5 = 15 + 1 + 2 + 9 + 7 = 34. (Cross addition)

Answer : 639540

Proof: $(x + a) (x + b) (x + c) (x + d) (x + e) = x^4 (x + a + b + c + d + e) + x^3 (ab + ac + ad + ae + bc + bd + be + cd + ce + de) + x^2 (abc + abd + abe + acd + ace + ade + bcd + bce + bdc + cde) + x (abcd + abce + abde + acde + debe) + abcde$
(x is the base) $\longrightarrow$ 3 (d)
On substitution in the above formula one can obtain the same multiplication result.

ii) Upasutram – Anurupyena:

1. With Convenient working Base (WB):

Sometimes one can consider multiples or sub–multiples of a theoretical base as a convenient working base depending on the ease with which the deficit or excess can be worked out with the same base for both multiplicand and the multiplier.

But in the answer care is taken for the proper conversion. These are clearly shown in the corresponding examples.

A distinction between theoretical base (TB) and working base (WB) is to be clearly brought out. The excess or the deficiency is to be worked out only with reference to the working base. But placement is with reference to the theoretical base (TB). In the final result, for the conversion one has to necessarily consider either division or multiplication of only the second part of the answer. Any carrying over is to be done only after the conversion depending on the working base being sub–multiple or multiple of the theoretical base. This can also be treated as equivalent to the multiplication of the second part with the ratio of WB to TB.

If the conversion into the theoretical base finally results in fractions, then that fraction of the theoretical base is to be added to the first part of the answer. As a consequence of this, the newly obtained carrying digits, if any in the first part, are to be added to the second part of the answer. [Refer ex. iv (a)]

Finally the application of multiplication by this method makes use of not only the main sutras but also the working details of Vinculum, if necessary.

i) 53 × 47

a) Theoretical base is 100; working base is $\frac{100}{2} = 50$

$$\begin{array}{r|l} 53 & +\ 3 \\ \underline{47} & \underline{-\ 3} \\ \hline 50 & /\ 0\bar{9} \end{array}$$

$$\text{Answer } = \frac{50}{2}\ /0\bar{9}\ = 25\ /0\bar{9} = 2491$$

Proof: $\left(\frac{x}{k}+a\right)\left(\frac{x}{k}+b\right)=\frac{1}{k}\left[x\left(\frac{x}{k}+a+b\right)\right]$ + ab (x is theoretical base, $\frac{x}{k}$ is working base) ⟶Eq (4)

By substituting the values in the expression the same result is obtained

b) Theoretical base is 10; working base is 5 × 10 = 50

$$\begin{array}{l|l} 53 & +3 \\ 47 & -3 \\ \hline 50 & / \; \bar{9} \end{array}$$

$$\text{Answer}\quad \begin{array}{c} 50 \\ \times 5 \end{array} \Big/ \; \bar{9} = 250 \Big/ \bar{9} = 2491.$$

Proof: (kx + a) (kx + b) = k[x (kx + a + b)] + ba (x is theoretical base, kx is working base). Eq(5) (a)

ii) <u>**78 × 61**</u>

Theoretical base is 10; working base is 7 × 10 = 70

$$\begin{array}{l|l} 78 & +8 \\ 61 & -9 \\ \hline 69 & \bar{2} \\ / \; \bar{7} & \end{array}$$

$$\text{Answer}\quad \begin{array}{c} 69 \\ \times 7 \end{array} \Big/ \begin{array}{c} \bar{2} \\ \bar{7} \end{array} = 483 \Big/ \begin{array}{c} \bar{2} \\ \bar{7} \end{array} = 476 \Big/ \bar{2} = 4758$$

iii) <u>**6875 × 5987**</u>

Theoretical base is 1000; working base is 6 × 1000 = 6000

$$\begin{array}{l|l} 6875 & +875 \\ 5987 & -\;\;13 \\ \hline 6862 & \bar{3}\,\bar{7}\,\bar{5} \\ / \; \bar{1}\,\bar{1} & \end{array}$$

$$\text{Answer}\quad \begin{array}{c} 6862 \\ \times 6 \end{array} \Big/ \begin{array}{c} \bar{3}\bar{7}\bar{5} \\ \bar{1}\bar{1} \end{array} = 41172 \Big/ \begin{array}{c} \bar{3}\bar{7}\bar{5} \\ \bar{1}\bar{1} \end{array} = 41161 \Big/ \overline{375} = 41160625$$

By substituting the appropriate values in Equation 5(a) of the expressions, the same result is obtained with different k values in (i) k = 5 (ii) k = 7, (iii) k =6.

iv) 5397256 × 4927425

•

a) Theoretical base is 10000000; working base is $\frac{10000000}{2} = 5000000$

$$\begin{array}{l|l} 5397256 & +397256 \\ 4927425 & -\ 72575 \\ \hline 5324681 \ / & \overline{2}\ \overline{8}\ \overline{8}\ \overline{3}\ \overline{0854200} \end{array}$$

First part by Urdhva Tiryag Multiplication

$$\begin{array}{r} +0\ 3\ \ 9\ 7\ 2\ 5\ 6 \\ -0\ 0\ \ 7\ 2\ 5\ 7\ 5 \\ \hline -0028\ 8\ \ 3\ \ 0\ \ 8\ \ 5\ 4\ 2\ 0\ 0 \\ \hline 7\ \ 9\ \ 11\ \ 16\ \ 16\ \ 9\ \ 8\ \ 7\ \ 3 \end{array}$$

Answer $\frac{5324681}{2} \Big/ \overline{2883}\ \overline{0854200} = 2662340\frac{1}{2} \Big/ \overline{2883}\ \overline{0854200}$

This fraction of 1/2 is with reference to the theoretical base, i.e., 10000000, which is equal to 5000000. This has to be added to the first part, i.e., $\overline{0854200}$. Therefore, the first part becomes $5\overline{854200}$. $(0\ \overline{8}\ \overline{54}\ \overline{00}+5000000)$

$= 2662340 \Big/ \overline{2883}\ 5\overline{854200}$

The second part is $2662340 + \overline{2883} = 2660\ \overline{543}$

$= 2660\overline{543}\ /\ 5\overline{854200}$

$= 2660\overline{543}\ 5\overline{854200}$

$= 26594574145800$

Answer: 26594574145800

Or

b) One can use working base as 5 x TB where TB = Theoretical base is 1000000; working base is 5 × 1000000 = 5000000

$$\begin{array}{l|l} 5397256 & +\ \ 397256 \\ 4927425 & -\ \ \ \ 72575 \\ \hline \end{array}$$

$= 5324681 \Big/ \overline{28830}\ \overline{854200}$

Answer $= \begin{matrix}5324681 \\ \times 5\end{matrix} \Big/ \overline{28830}\ 5\overline{854200}$

$= 26623105 \Big/ \overline{28830}\ \overline{854200}$

$= 26594575 \Big/ \overline{854200}$

$= 26594574145800$

On substitution in the formula of Eq (4) and Eq 5(a) we get the answer as 26594574145800 which is same as that one got by writing down the parts.

(V) Example: 46 × 53 × 49

$$\begin{array}{r|ll} 46 & -4 & \text{Theoretical base 10} \\ 53 & +3 & \text{working base 50} \\ 49 & -1 & \\ \hline \end{array}$$

$$48 \,\Big/_{\bar{1}}\ \bar{1} \,\Big/_{1}\ 2$$

$$\text{Answer} = \underset{\times 25}{48} \Big/ \underset{\times 5}{\begin{pmatrix} & \bar{1} \\ \bar{1} & \end{pmatrix}} \Big/_{1}\ 2$$

$= 1200/_{\bar{5}}\ \bar{5}/_{1}2$

$= 120\bar{5}/\ \bar{4}/\ 2$

$= 119462.$

I Part = $(-4)(3)(-1) = 12$. (Product of D/Ex)

II Part = $(-4)(3) + (-4)(-1) + (3)(-1) = \bar{1}\ \bar{1}$

$$\Sigma_1^1 \begin{pmatrix} 3 & \\ c & \text{Products of D/Ex.} \\ 2 & \end{pmatrix}$$

III Part = $46 + 3 - 1 = 53 - 4 - 1 = 49 - 4 + 3 = 48$. (Cross addition)

(VI) Example: 24 x 15 x 29

$$\begin{array}{r|ll} 24 & +4 & \text{Theoretical base is 10} \\ 15 & -5 & \text{working base is } 2 \times 10 = 20 \\ 29 & +9 & \\ \hline \end{array}$$

$$28 \,\Big/_{\bar{2}}\ \bar{9} \,\Big/_{\overline{18}}\ \bar{0}$$

$$\text{Answer} = \underset{\text{x } 4}{28} \Big/ \underset{\text{x } 2}{\begin{pmatrix} & \bar{9} \\ \bar{2} & \end{pmatrix}} \Big/_{\bar{1}\ \bar{8}}\ \bar{0}$$

$= 11\,\bar{5}\bar{6}\bar{0} = 1\,0440$

I Part = $(4)(-5)(9) = \bar{1}\bar{8}\bar{0}$ (products of D/Ex)

Placement is $_{\overline{18}}{}^{\bar{0}}$

II Part = $(4)(-5) + (4)(9) + (-5)(9) = \bar{2}\ \bar{9}$

$$\Sigma_1^1 \begin{pmatrix} 3 & \\ c & \text{Products of D/Ex.} \\ 2 & \end{pmatrix} \text{Placement is } {}_{\bar{2}}{}^{\bar{9}}$$

III Part = $24 - 5 + 9 = 15 + 4 + 9 = 29 + 4 - 5 = 28$ (cross addition)

The reduced values of second part and Third part are obtained by multiplying by the ratio $\frac{WB}{TB}$ and $\left(\frac{WB}{TB}\right)^2$ respectively

Proof : $(kx + a)(kx + b)(kx + c) = k^2 [x^2 (kx + a + b + c)] + k[x (ab + bc + (a)] + abc$

$\longrightarrow$ Eq 5(b)

On substitution of the values in the expressions the final answers obtained are same for both the methods.

2. Different bases for Multiplicand and Multiplier (V.M.):

So far in the multiplication of numbers the same theoretical base (TB) and working base (WB), or the same T.B. and multiplier or sub–multiplier of T.B. as W.B are used for multiplicand and multiplier. A few problems are worked out and the working details are explained. The multiplication of different numbers can also be carried out by considering different bases for different numbers. This may be applied either to theoretical base or the working base or both.

It is hence thought that a more general method of multiplying many numbers by using different combinations of TBs and WBs and as such a general form of numbers can be written down and multiplications carried out.

The procedure can be explained in the following steps with general expressions. These expressions are worked out to two numbers, three numbers through six numbers and can be extended to any numbers as well.

In case of multiplication of two or more than two numbers, one can allot different theoretical bases (TB) and working bases (WB) for different numbers. This will facilitate to obtain deficiencies / excesses (D/Ex) of one digit value and multiplication can be carried out with the help of these deficiencies / excess. The procedure adopted is to prepare a table consisting of the given numbers, the TB and WB for each number, the corresponding D/Ex with reference to WB, the equalization of all WBs to one of the considered WBs, multiplication factors (MF) which are the ratios of equalized WB to individual WB, modified number and modified D/Ex.

1. For each number a TB and WB are given. Accordingly D/Ex can be obtained.

2. One has to consider the equalization of the WB. Depending on this one can write down (a) the multiplication factors (MF) $= \frac{\text{equalized WB}}{\text{individual WB}}$ for each number.

(b) Modified numbers (given number x MF) and (c) modified D/Ex. (D/Ex. x MF)

<u>**For example consider 201 x 4998**</u> :

Given Number	**Theoretical Base TB**	**Working Base WB**	**Deficiency / Excess w.r.t. WB**	**Multiplication Factor for modifying the working base to 5000 (MF)**	**Modified Numbers**	**Modified Deficiency or Excess**
201	100	200	1	25	5025	25
4998	1000	5000	–2	1	4998	–2

First part = (1) (–2) = $\bar{2}$

Here the placement of the first part is as for the lowest theoretical base i.e., 100. Therefore, first part gives provision for two digits. Now the first part is $0\bar{2}$

Here in order to get the second part, we are equalizing base 200 to 5000 by multiplying it by 25. Therefore, the excess of the first number becomes 1 x 25 = 25 (modified D/Ex.) and first number becomes 201 x 25 = 5025 (modified numbers).

The second part is 5025 – 2 = 5023 or 4998 + 25 = 5023

$$\begin{array}{l} 201 \; + \; 1 \\ 4998 \; - \; 2 \\ \hline 5023 \,/\, 0\,\bar{2} \end{array}$$

The final second part is $5023 \times \frac{200}{100} = 10046$

$$\text{Answer} = \begin{array}{r} 5023 \\ \times 2 \end{array} /0\bar{2} = 10046 \; /0\bar{2} \quad = 1004598$$

It is interesting to note that one can reduce the highest working base to the lower value among the two. Even then the same procedure can be applicable and the result is same

Given Number	Theoretical Base	Working Base	Deficiency / Excess	Multiplication Factor for modifying the working bases to 200 (MF)	Modified Numbers	Modifie Deficien or Exce
201	100	200	1	1	201	1
4998	1000	5000	–2	$\frac{1}{25}$	$\frac{4998}{25}$	$\frac{-2}{25}$

The placement of first part is considered as for the higher base, i.e., 3 places.

In order to get the second part, we are equalizing base 5000 to 200 by multiplying it by $\frac{1}{25}$. Therefore, the number 4998 becomes $\frac{4998}{25}$ and its deficiency becomes $-\frac{2}{25}$.

The second part $201 - \frac{2}{25} = \frac{4998}{25} + 1 = \frac{5023}{25}$

The final second part is $\left(\frac{5023}{25}\right)\left(\frac{5000}{1000}\right) = \frac{5023}{5} = 1004\ \frac{3}{5}$

We take $\frac{3}{5}$ fraction of the theoretical base 1000 i.e., $\frac{3}{5} \times 1000 = 600$ and the result is added to the first part of the answer. The first part becomes $60\bar{2}$. So the answer is $100460\bar{2} = 1004598$

$$\begin{array}{r|l} 201 & +1 \\ 4998 & -2 \\ \hline \frac{5023}{25} \,/ & 0\bar{2} \end{array}$$

$$= \frac{5023}{25} \text{x}\, 5 \Big/ 0\bar{2} = \frac{5023}{5} \Big/ 0\bar{2} = 1004\,\frac{3}{5} \Big/ 0\bar{2} = 1004 \Big/ 60\bar{2} = 1004598$$

Proof: A simple proof can be given for both the above possibilities, which can be substantiated by the following general multiplication.

For the above example, (conversion of lower base to higher base)
$(ax^2 + b)(cx^3 + d)$
Where $a = 2$, $b = 1$, $c = 5$, $d = -2$, and obviously $x = 10$

$$= ax^2(cx^3 + d) + bcx^3 + bd$$

$$= ax^2[cx^3 + d + \frac{bcx}{a}] + bd$$

The first part of the answer is bd. The second part is $cx^3 + d + \frac{bcx}{a}$. Final second part gives $ax^2[cx^3 + d + \frac{bcx}{a}]$ in hundreds.

$\therefore$ Final answer is $ax^2[cx^3 + d + \frac{bcx}{a}] + bd$

And in another way (conversion of higher base to lower base)

$$(cx^3 + d)(ax^2 + b) = cx^3(ax^2 + b) + adx^2 + bd$$

$$= cx^3[ax^2 + b + \frac{ad}{cx}] + bd$$

Then the first part is bd as in the above case. The second part gives $cx^3[ax^2 + b + \frac{ad}{cx}]$ in thousands which is just the same as the previous one in value.

If we look at the two expressions:

1. $ax^2\,[cx^3 + d + \frac{bcx}{a}]$ and
2. $cx^3\,[ax^2 + b + \frac{ad}{cx}]$

Both are identical, but the first one is written in terms of higher working base and the second one is written in terms of lower working base.

While deciding the final answer, care is taken to multiply according as the ratio of working to theoretical base. We can start from the higher working base details and arrive at the result by converting the excess or deficiency of lower working base to higher working base. In such a case the final result is obtained by multiplying the second part by the ratio of lower working base to its theoretical base and vice versa.

In case of multiplication of three or more numbers sometimes it is found easier to consider different theoretical bases or working bases as well. Even then the same procedure is applied but taking care to see that:

1. A proper conversion of all to one common base of any order and accordingly each number and its excess or deficiency also are to be modified to obtain the second part of the answer and to enable cross addition or subtraction.
2. The number of digits that will occupy the first part of the answer is dependent upon the value of the lowest base and the second part of the answer on the next lower base and so on.
3. When the base contains n digits, we have to provide space for n–1 digits for parts of the answer.

For example consider $93 \times 9 \times 1006$:

Given Number	Base (TB=WB)	Deficiency / Excess	Multiplication Factor for modifying the working base (MF) (to 1000)	Modified Numbers	Modified deficiency or excess
93	100	– 7	10	930	–70
9	10	– 1	100	900	–100
1006	1000	+ 6	1	1006	6

First part of the answer is 42 got by multiplying the excess or deficiency as they are. First part of answer is $= (-7)(-1)(6) = 42$

There is a provision shown only for one digit in the first part of the answer, as the minimum base is 10. ∴ Placement is $\,^{2}_{4}$

The middle part of the answer is obtained by multiplying two by two excesses or deficiencies after modification to the common base 1000 and added together.

Second Part $= (-70)(-100) + (-70)(6) + (-100)\,6 = 5980$

Here the second part is 5980. This is to be divided by 10 as is explained in the proof followed. Here the placement of the second part of the answer is considered as 2 digits in accordance with the order of the base, i.e., 100. ∴ Placement is $\,^{98}_{5}$

The third part of the answer is obtained by adding any one of the three numbers suitably modified to a common base to the two other excesses or deficiencies also suitably modified to the same base. Taking each number at a time and checking the result one can verify this.

Third Part $= 1006 - 100 - 70 = 836$

Or $930 - 100 + 6 = 836$

Or $900 - 70 + 6 = 836$

Final Answer $836\,/_{5}\;\; 98\,/_{4}\;\; 2 = 842022$

Proof: For this problem the proof can be worked out in terms of powers of x as the base

For Example $100 = 10^2 = x^2$, $1000 = 10^3 = x^3$, $10 = 10^1 = x$
The numbers considered are 93, 9 and 1006.
93 can be written as $x^2 + a$ Where $a = -7$, $x = 10$
9 can be written as $x + b$ Where $b = -1$
1006 can be written as $x^3 + c$ Where $c = 6$

$$(x^2 + a)(x + b)(x^3 + c)$$
$$= (x^3 + bx^2 + ax + ab)(x^3 + c)$$
$$= x^6 + bx^5 + ax^4 + (ab + c)x^3 + bcx^2 + acx + abc$$
$$= x^3(x^3 + bx^2 + ax + c) + x(ac + bcx + abx^2) + abc \longrightarrow (1)$$

The result of multiplication of the above three numbers can be written as follows:

The first part is abc. On considering lower base placement the first part is placed in units.

In the second part, with the modifications of D/Ex. incorporated to equalize the base for all the three, 'a' becomes 'ax', 'b' becomes 'bx^2' and 'c' is as it is.

The second part is sum of products of two by two–modified excesses or deficiencies. ax, bx^2 and c. As such the value will now be

$(ax)c + (bx^2)c + (ax)(bx^2) = acx + bcx^2 + abx^3$, which has to be placed under tens place by $x(acx + bcx^2 + abx^3)$. $\longrightarrow$ (2)

The comparison with the unmodified value in (1) shows an excess of factor 10, creeping in as a result of modification of the base value in (2). Hence, in writing the final result, one has to divide it by 10. Such a care should be taken in the entire calculations. The placement sorts out 10s, 100s, 1000s and so on.

The last term remains as it is even under modifications. It is $(x^3 + bx^2 + ax + c)$ and is to be shown in 1000's place.

This proof is especially for the case where there is no distinction between theoretical and working base. The numbers can be written in terms of powers of 10 as per the given base and the placement is in terms of units, tens, etc.

In general, one can take care to see that any change in the base has to be incorporated while writing the answer into the parts by the corresponding reduction of the power.

For example, one can write down the reduction factor for modifying the base of every number. The reduction is considered as follows.

a) If the result is obtained by multiplying the modified excesses or deficiencies taking two by two, then the reduction factor is obtained as the product of the two lower (in order) multiplication factors for modification of the bases (i.e., to divide the result by that reduction factor before it is placed in the answer).

b) Similarly when the multiplication is carried out among three deficiencies or excesses, the reduction factor is calculated as the product of three lower multiplication factors (in order) for modification of the bases and so on.

c) The final part is obtained as cross addition and is divided by the lowest MF.

In these problems the first part placement is dependent on the lowest base and we have considered under (a) for the reduction factor, the division of two lower M.F. values. But one can work out with different placements in the first part followed by dividing it by the M.Fs belonging to the other two numbers. This procedure is continued for the second part. Similarly one can work for any equalization of the base. In order to exemplify these, a more general form of multiplication of numbers is considered by introducing a TB and WB where the number is written in terms of WB, instead of TB.

In this case the numbers are generalized as (kx + a), (ly + b) (mz +c). etc, where kx, ly, mz are WBs. This will facilitate working with any choice of equalizations, placements and bases

Hence each number is expressed as (kx + a), (ly + b), (mz + c) .... Where kx, ly, mz are WBs, x, y, z are TBs, a, b, c are D/Ex of each number with reference to its corresponding WBs. The general formulae are worked out for two number, three number through six numbers and they are finally expressed in terms of WB, M.F, D/Exs, modified D/Exs and the modified numbers. The table containing the above details is as follows :

Table I *

Given Numbers	TB	WB	Deficiency / Excess	Multiplication Factor for modifying the working base to kx	Modified Numbers (Given number) (MF)	Modified Deficiency or Excess (D/Ex.) (MF)
kx + a	x	kx	a	$\frac{kx}{kx}=1$	$(kx+a)\left(\frac{kx}{kx}\right)$	$a\left(\frac{kx}{kx}\right)=a'$
ly + b	y	ly	b	$\frac{kx}{ly}$	$(ly+b)\left(\frac{kx}{ly}\right)$	$b\left(\frac{kx}{ly}\right)=b'$
mz + c	z	mz	c	$\frac{kx}{mz}$	$(mz+c)\left(\frac{kx}{mz}\right)$	$c\left(\frac{kx}{mz}\right)=c'$
|	|	|	|	|	|	|
|	|	|	|	|	|	|

a', b', c',......... represent modified D/Ex. w.r.t. specific equalizations of the working base. Similar tables are to be prepared for different equalizations wherein

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* **Explained elaborately under 3 or more number multiplication wherein the deduction of the expressions for the various parts are clearly brought out at the respective places (Page No. 124)**

corresponding M.F., modified numbers and modified D/Ex for given numbers, D/Ex., TBs., and WBs., are to be shown.

The individual formulae for various parts in the final expression for different multiplications ranging from two number through six numbers are given below : -

Method I : (Substitution Method – Derivation of the proof) Making use of the formulae and substituting the values of individual expressions the exact value of different parts are obtained. By adding these parts one gets the final result. Care is taken to use the concerned particular equalization and corresponding table. This is named as Method I.

The formulae : -

(1) For two numbers : -

(kx + a) (ly + b) = (kx) (ly) + (kx) b + (ly) (a) + ab for different equalizations the following expressions can be derived as follows :

$$\text{Equalization of WB to (kx)} = \frac{kx}{\left(\frac{kx}{ly}\right)}\left[(kx+a)+b\left(\frac{kx}{ly}\right)\right]+ab \qquad \text{Eq(6)}$$

$$\text{Where } b\left(\frac{kx}{ly}\right)=b' \text{ modified D/Ex w.r. to } kx$$

$$\text{Equalization of WB to (ly)} = \frac{ly}{\left(\frac{ly}{kx}\right)}\left[(ly+b)+a\left(\frac{ly}{kx}\right)\right]+ab \qquad \text{Eq(7)}$$

$$\text{Where } a\left(\frac{ly}{kx}\right)= a'' \text{ modified D/Ex w.r. to } ly$$

Here ab is the first part and remaining is the second part.

(2) For three numbers : -

(kx + a) (ly + b) (mz + c) = (kx) (ly) (mz) + (ly) (mz)a + (kx) (ly) c + (kx) (mz) (b) + (kx) (bc) + (ly) ac + (mz) ab + abc.

For different equalizations the following expressions can be derived.

Equalization of WB to (mz) :

$$abc+\frac{(mz)}{\left(\frac{mz}{kx}\right)\left(\frac{mz}{ly}\right)}\left[b\left(\frac{mz}{ly}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)b\left(\frac{mz}{ly}\right)\right]$$

$$+\frac{(mz)(ly)}{\left(\frac{mz}{kx}\right)}\left[(mz+c)+b\left(\frac{mz}{ly}\right)+a\left(\frac{mz}{kx}\right)\right]$$

$$=abc+\frac{mz}{\left(\frac{mz}{kx}\right)\left(\frac{mz}{ly}\right)}\left[b'c'+a'c'+a'b'\right]+\frac{(mz)(ly)}{\left(\frac{mz}{kx}\right)}\left[(mz+c)+b'+a'\right] \quad \rightarrow \text{Eq}(8)$$

Where a', b', c', represent modified D/Ex with reference to equalization of the base to (mz)

Equalization of WB to (ly) : -

$$abc+\frac{ly}{\left(\frac{ly}{kx}\right)\left(\frac{ly}{mz}\right)}\left[b''c''+c''a''+a''b''\right]+\frac{(ly)(mz)}{\left(\frac{ly}{kx}\right)}\left[(ly+b)+a''+c''\right]\rightarrow Eq(9)$$

Where a'', b'', c'' are the modified D/Ex with reference to the equalization of the base to (ly)

Here $a''=a\left(\frac{ly}{kx}\right), b''=b\left(\frac{ly}{ly}\right), c''=c\left(\frac{ly}{mz}\right)$

Equalization of WB to (kx) : -

$$=abc+\frac{kx}{\left(\frac{kx}{ly}\right)\left(\frac{kx}{mz}\right)}\left[a'''b''''+b'''c'''+c'''a'''\right]+\frac{(kx)(mz)}{\left(\frac{kx}{ly}\right)}\left[(kx+a)+b'''+c'''\right]\rightarrow Eq(10)$$

Where a''', b''', c''' are modified D/Ex with reference to equalization of the base to (kx)

Here $a'''=a\left(\frac{kx}{kx}\right), b'''=b\left(\frac{kx}{ly}\right), c'''=c\left(\frac{kx}{mz}\right)$

It is seen that the expression consists of three parts. Placement consideration of each part is discussed at the time of specific multiplications as per the principles enumerated in the general statement. (Refer Statements)

(3) For four numbers : -

(kx + a) (ly + b) (mz + c) (nw + d) = (kx) (ly) (mz) (nw) + (ly) (mz) (nw) a + (kx) (mz) (nw) b + (kx) (ly) (nw)c + (kx) (ly) (mz)d + (mz) (nw) ab + (ly) (nw) ac + (kx) (nw) bc + (ly) (mz)ad + (kx) (mz) bd +(kx) (ly) cd + (nw) abc + (mz) abd + (ly) acd + (kx) bcd + abcd.

Equalization of W.B to (kx) :

$$abcd+\frac{kx}{\left(\frac{kx}{nw}\right)\left(\frac{kx}{ly}\right)\left(\frac{kx}{mz}\right)}\left[b'c'd'+a'c'd'+a'b'd'+a'b'c'\right]+\frac{(kx)(nw)}{\left(\frac{kx}{mz}\right)\left(\frac{kx}{ly}\right)}$$

$$\left[a'b'+b'c'+c'd'+a'c'+a'd'+b'd'\right]+\frac{(kx)(mz)(nw)}{\left(\frac{kx}{ly}\right)}\left[(kx+a)+b'+c'+d'\right]\rightarrow Eq(11)$$

Where $a'=a\left(\frac{kx}{kx}\right), b'=b\left(\frac{kx}{ly}\right), c'=c\left(\frac{kx}{mz}\right) d'=d\left(\frac{kx}{nw}\right)$ are modified D / Ex with reference to (kx

Equalization of W.B to (ly) : -

$$abcd+\frac{ly}{\left(\frac{ly}{kx}\right)\left(\frac{ly}{mz}\right)\left(\frac{ly}{nw}\right)}\left[b''c''d''+a''c''d''+a''b''d''+a''b''c''\right]+\frac{(ly)(mz)}{\left(\frac{ly}{kx}\right)\left(\frac{ly}{nw}\right)}$$

$$\left[a''b''+b''c''+c''d''+a''c''+a''d''+b''d''\right]+\frac{(ly)(mz)(nw)}{\left(\frac{ly}{kx}\right)}\left[(ly+b)+a''+c''+d''\right]\rightarrow Eq(12)$$

Where $a''=a\left(\frac{ly}{kx}\right), b''=b\left(\frac{ly}{ly}\right), c''=c\left(\frac{ly}{mz}\right)$ and $d''=d\left(\frac{ly}{nw}\right)$ are the modified D/Ex with reference to equalization of the base to (ly)

Equalization of WB to (mz) : -

$$abcd+\frac{mz}{\left(\frac{mz}{kx}\right)\left(\frac{mz}{ly}\right)\left(\frac{mz}{nw}\right)}\left[b'''c'''d'''+a'''c'''d'''+a'''b'''d'''+a'''b'''c'''\right]$$

$$+\frac{(mz)(nw)}{\left(\frac{mz}{kx}\right)\left(\frac{mz}{ly}\right)}\left[a'''b'''+b'''c'''+c'''d'''+a'''c'''+a'''d'''+b'''d'''\right]$$

$$+\frac{(mz)(nw)(ly)}{\left(\frac{mz}{kx}\right)}\left[(mz+c)+a'''+b'''+d'''\right] \longrightarrow Eq(13)$$

Where $a'''=a\left(\frac{mz}{kx}\right), b'''=b\left(\frac{mz}{ly}\right), c'''=c\left(\frac{mz}{mz}\right), d'''=d\left(\frac{mz}{nw}\right)$ are modified D/Ex with reference to equalization of the base to (mz).

Equalization of WB to (nw) :-

$$abcd+\frac{(nw)}{\left(\frac{nw}{kx}\right)\left(\frac{nw}{ly}\right)\left(\frac{nw}{mz}\right)}\left[b''''c''''d''''+a''''c''''d''''+a''''b''''d''''+a''''b''''c''''\right]+\frac{(nw)(mz)}{\left(\frac{nw}{kx}\right)\left(\frac{nw}{ly}\right)}$$

$$\left[a''''b''''+b''''c''''+c''''d''''+a''''c''''+a''''d''''+b''''d''''\right]+\frac{(nw)(mz)(ly)}{\left(\frac{nw}{kx}\right)}$$

$$\left[(nw+c)+a''''+b''''+d''''\right] \longrightarrow Eq(14)$$

Where $a''''=a\left(\frac{nw}{kx}\right), b''''=b\left(\frac{nw}{ly}\right), c''''=c\left(\frac{nw}{mz}\right) d''''=d\left(\frac{nw}{nw}\right)$ are modified D/Ex with reference to equalization of the base (nw).

By substituting the value in the expressions from (1) to (14) for the concerned multiplication one can get the result of the multiplication.

There is another way of writing down the answer by evaluating the individual parts of the answer as explained below. This is called Method II.

METHOD II (Simplified method):

The value of multiplication can also be simply written down in its parts by making use of the table consisting of the numbers, TB, WB, D/Ex, M.F, modified numbers and modified D/Ex already prepared for the I Method. This method is explained as follows. The following points are to be considered.

1. The answer consists of as many parts as the numbers in the multiplication. For example two parts for two number multiplication, three parts for three number multiplication and so on. Consider the multiplication of n numbers. Write down (a) the first part as the product of D/Ex with reference to W.B (b) The second part is evaluated by $\Sigma_1^1 \left({}^{n}c_{n-1} \text{ Products of D/Ex.} \right)$ values of the given numbers. This holds for n > 2. If n = 2, the II part is the final part and is as explained later. (c) For n > 3 the third part of the answer is deduced by evaluating the value of $\Sigma_1^1 \left({}^{n}c_{n-2} \text{ Products of D/Ex.} \right)$ of the given numbers.

   But if n = 3 the third part is the final part. This can be followed for other parts also depending on 'n'.
2. The final part for multiplication of any numbers is obtained by taking any one of the modified numbers and adding the modified D/Ex of the remaining numbers. This is called cross addition.
3. Now one has to sort out for every part the placement of the answer on the basis of following procedure

(a) For the placement of the I part, which is obtained using 1(a) one, can consider any one of the TBs of the problem. For example if the base considered is 10 then only one digit (i.e. in units) can be accommodated, the remaining, if any are to be shown below and to be carried to the next higher part accordingly. If the base considered is 100 then a provision for two digits (i.e. units and tens) can be shown and the remaining answer of the I part are to be shown below and to be carried out to the next higher part. Similarly for the other bases say for example 1000, three digits provision is shown with digits – in units, tens and hundreds and remaining answer of the I part are to be carried out. (Refer page 89 for the accommodation of the digits).

IV Part	III Part	II Part	I Part
(a) allowed digits	(a) allowed digits	(a) allowed digits	(a) allowed digits
________	(b) Digits to be carried out to (a) of IV Part	(b) Digits to be carried out to (a) of III Part	(b) Digits to be carried out to (a) of II Part

(b). For the placement of answer in the II part, one should reduce the value of the II part obtained from 1(b) in the following way, which is again based on the placement of the I part.

For the reduction of the II part one has to consider (i) multiplication of the II part value by the ratio of WB to TB (TB being that considered for the placement in the I part and WB being its corresponding value) (ii) a division by the M.F. values of all the numbers other than that considered for multiplication in b(i).

The value so obtained is called the reduced II part. The placement of this reduced II part is to be then considered. The placement can be with reference to any one of the other TBs of the given numbers and is other than that which is already considered for the I part

(c). For the placement of the answer in the III part one should reduce the value of the III part obtained from 1(c) in the following way which is again based on the placement of the I and II parts.

For the reduction of the III part, the value of III part is to be (i) multiplied by the two $\frac{WB}{TB}$ ratios – the one which is considered in the II part and the other is the ratio of $\frac{WB}{TB}$ where the TB value is that which is used for the placement in the II part and WB being its corresponding value. (ii) This is followed by the division of MF values of the remaining numbers other than those considered for the multiplication in c(i). The placement of the III part can be with reference to any one of the bases other than those, which are considered for I and II parts. If there is a recurrence of the base in the given problem then one can consider the repetition. If $n > 3$ then reduction of all the parts can be similarly carried out by considering the necessary multiplications and divisions.

In all these cases the final part is to be reduced in the same manner but has to be placed as it is.

The value can be read out as the simplification of various parts wherein the actual placement and excess are shown clearly. The excesses are to be carried over to the next higher part. Various possibilities of placements and equalizations can be worked out but it is noticed that

(i) For any equalization of the WB the placement of the parts according as lower and next lower and so on is found to be easier.

(ii) Consideration of highest WB equalization and together with placement of the above order in various parts is considered to be workable with more ease.

As otherwise one is likely to come across fractional values in M.F, modified D/Ex and modified numbers which make the evaluation of the parts more complicated. While a number of possibilities are worked out it is for the reader to attempt all the other combinations as well, to experience the ease or difficulty.

If one gets a fractional value in any part then the fraction has to be multiplied by the T.B considered for the placement in the immediate lower part and the value thus obtained is to be carried to the accommodated digits of this lower part [Refer the structure shown in 3(a)].

Value of different number multiplications could be considered by both the substitution method I as well as the simplified method II as is shown in the worked examples

2 (a) Multiplication of two numbers with different theoretical bases but each one having the same Theoretical base and working base:

Base is not the same for multiplicand and multiplier. (But Theoretical base = working base for each number) In such case one has to equalize the base to any one of the considered bases:

A table consisting of details of the given numbers, base, D/Ex, multiplication factors (MF) for equalization of the bases and accordingly the modified numbers and modified D/Ex is prepared. $MF = \frac{\text{equalized WB}}{\text{individual WB}}$.

Tables are prepared keeping in view the different equalizations. In this case of two number multiplication we get two tables T_1 and T_2. :

Let us consider an example: 101 x 8

METHOD I: We get the value of the multiplication by substitution of the corresponding values in the concerned equations 6(a) (making use of higher base equalization) or 7 (a) (making use of lower base equalization). The method gives the final result on substitution. The method doesn't require any placement details. In this method the different equalizations are worked out. Refer tables T1 and T2.

By making use of 6(a) and substituting x = 100, y =10, a = 1, b = –2 one can

deduce the answer from T_1 as $\frac{(100)}{\frac{(100)}{(10)}} \quad 100+1+(-2)\left(\frac{100}{10}\right)\rfloor + (1)(-2).$

= 10[101 – 20] – 2 = 810 – 2 = 808.

Similarly by making use of 7(a) and T_2 which is concerned with lower base equalization, one gets the same value.

<u>METHOD II</u> :

We can also simply write down the answer, which consists of different parts, two parts in two numbers multiplication and so on. This method makes use of a specific placement order and details are worked out for different equalizations and placements as follows:

In this method we have four possibilities, two each under same equalization. Each referring to different placements of the I part of the answer with reference to any one of the two TBs. These are exemplified for same multiplication as follows:

Equalization to higher base (100): -

Table I (T1)

Given Numbers	Base TB=WB	Deficiency / Excess	Multiplication Factor for modifying the working base to 100 (MF)	Modified Numbers	Modified D/Ex.
101	100	1	1	101	1
8	10	–2	10	80	–20

One can prepare a similar table, with the equalization of the working base to 10, which is T2.

In both the cases one can try placement of the first part of the answer on the basis of either the higher or the lower base. The following are the working details of the above considerations

(1) Equalization to higher base 100 and placement on the basis of lower base (10)

From the table prepared containing the theoretical base, working base for the numbers, the D/Ex, multiplication factor for modification to a base and accordingly the modified numbers, modified D/Ex. the two parts of the answer can be written as follows:

I Part = (1) (–2) = –2, the Placement is $\bar{2}$

II Part = 101 – 20 = 80 + 1 = 81

This has to be divided by the lowest M.F. value and multiplied by the ratio of $\frac{\text{WB}}{\text{TB}}$ of the other number.

$$\text{Reduced II Part} = \frac{81}{1}\left(\frac{10\,\text{WB}}{10\,\text{TB}}\right) = 81$$

$$\text{Final Answer} = 81 \,/\, \bar{2} = 81\bar{2} = 808$$

(2) <u>Equalization to higher base (100) and placement on the basis of higher base (100)</u>:

I Part $= (1)(-2) = \bar{2}$ Placement is $0\bar{2}$
II Part $= 101 - 20 = 80 + 1 = 81$

This has to be divided by the higher M.F. value 10 and multiplied by the ratio of $\frac{\text{WB}}{\text{TB}}$ of the other number.

$$\text{Reduced II Part} = \frac{81}{10}\left(\frac{100(\text{WB})}{100(\text{TB})}\right) = \frac{81}{10} = 8\frac{1}{10}$$

$\frac{1^{th}}{10}$ part of $100 = \frac{100}{10} = 10$. This is to be added to the I Part i.e. $10 + 0\bar{2} = 1\bar{2}$

$$\therefore \text{Final Answer} = 8 \,/\, 1\bar{2} = 81\bar{2} = 808$$

<u>Equalization to lower base(10)</u> : -

The corresponding table prepared is given below. By making use of it the answer can be worked out.

Table II (T2)

Given numbers	Base TB = WB	D/Ex	Multiplication factor for modifying the working base to 10 (MF)	Modified numbers	Modified D/Ex
101	100	+1	1/10	$\frac{101}{10}$	$\frac{1}{10}$
8	10	– 2	1	8	–2

(3) Equalization to lower base (10) and placement considered on the basis of lower base (10):

I Part $= (1)(-2) = \bar{2}$ Placement is $\bar{2}$

II Part $= \frac{101}{10} - 2 = 8 + \frac{1}{10} = \frac{81}{10}$

This is to be divided by the lower M.F. value i.e., $\frac{1}{10}$ and multiplied by the ratio of $\frac{WB}{TB}$ of the other number.

$$\text{Reduced II Part} = \frac{\frac{81}{10}}{\frac{1}{10}}\left(\frac{10}{10}\right) = 81$$

Final Answer is $81 / \bar{2} = 81\bar{2} = 808$

(4) Equalization to lower base (10) and placement is considered on the basis of higher base (100):

I Part $= (1)(-2) = \bar{2}$ Placement is $0\bar{2}$

II Part $= \frac{101}{10} - 2 = 8 + \frac{1}{10} = \frac{81}{10}$

This is to be divided by the higher M.F. value 1 and multiplied by the ratio of $\frac{WB}{TB}$ of the other number.

$$\text{Reduced II Part} = \frac{\frac{81}{10}}{1}\left(\frac{100}{100}\right) = \frac{81}{10} = 8\frac{1}{10}$$

$\frac{1}{10}$th of 100 = 10 is added to I Part i.e., $10 + \bar{2} = 1\bar{2}$

$\therefore$ Final Answer $= 8 / 1\bar{2} = 81\bar{2} = 808$

It is noticed that out of the four types of evaluation of the answer the consideration of equalization to higher base followed by placement on the basis of lower base is easier and it doesn't deal with fractions in general.

The equations (6) and (7) can be reduced to

Equalization of the base to x :

$$(x + a)(y + b) = \left(\frac{x}{\frac{x}{y}}\right)\left[(x+a)+b\left(\frac{x}{y}\right)\right] + ab$$ (x is the base for multiplicand, y is the base for multiplier) → 6 (a)

and equalization of the base to y

$$\left(\frac{y}{\frac{y}{x}}\right)\left[y+b+a\left(\frac{y}{x}\right)\right]+ab \qquad \rightarrow 7(a)$$

Where k = l =1; as TB = WB for each number.

2. (b) Multiplication of two numbers each having different Theoretical Base and different Working Base (ratio of Working base/Theoretical Base is same for both multiplicand and multiplier) : placement with reference to lower base. (10)

For example 503 × 53

Given Number	Theoretical Base	Working Base	D/Ex	Multiplication Factor for modifying the working base to 500 (MF)	Modified Numbers	Modified Deficiency or Excess
503	100	500	3	1	503	3
53	10	50	3	10	530	30

METHOD I:

By making use of Eq(6) one can deduce the answer from the above table. Here higher base equalization is considered.

If the ratio of WB to TB is same for both the numbers then k = 1 in (6).

∴ Eq (6) Becomes Eq 6(b)

In this case k = 5, x = 100, y = 10, a = 3, b =3.

$$\text{Answer} = \frac{(5)(100)}{\left(\frac{500}{50}\right)}\left[500+3+3\left(\frac{100}{10}\right)\right]+(3)(3)=50[533]+9=26659$$

For the other equalization Eq (7) becomes Eq 7(b).

METHOD II:

The different parts can be written as follows. Here the equalization is done for higher base and the placement is with reference to lower base.

For the same equalization the placement can also be with reference to higher base with different equalization to lower base and the placement with reference to lower or higher bases can also be similarly worked out. In all these cases the answer is same.

I Part = (3) (3) = 9 Placement is 9
II Part = 503 + 30 = 530 + 3 = 533

$$\text{Reduced II part} = \frac{533}{1}\left(\frac{50}{10}\right) = 2665$$

$$\text{Final Answer} = 2665 \,/\, 9 = 26659$$

Proof: $$(kx + a)(ky + b) = \left(\frac{kx}{\frac{kx}{ky}}\right)\left(kx + a + b\left(\frac{x}{y}\right)\right) + ab \qquad \rightarrow 6(b)$$

$$\text{Or } \frac{ky}{\left(\frac{ky}{kx}\right)}\left(ky + b + a\left(\frac{y}{x}\right)\right) + ab \qquad \rightarrow 7\,(b)$$

(x, y are theoretical bases for multiplicand and multiplier respectively and kx, ky are working bases for multiplicand and multiplier respectively)

2. (c) Multiplication of two numbers having same theoretical base and different working base (ratio of Working Base / Theoretical Base is not same for both multiplicand and multiplier) : Placement with reference to lower base (10)

For example 28 × 46

Given Number	Theoretical Base	Working Base	Deficiency/ Excess	Multiplication Factor for modifying the working base to 40 (MF)	Modified Numbers	Modified Deficiency or Excess
28	10	30	–2	$\frac{4}{3}$	$\frac{112}{3}$	$-\frac{8}{3}$
46	10	40	+6	1	46	+6

Method I : By making use of Eq(6) one can deduce Eq6(c) where x = y (as TB is same for both numbers).

Here the higher base equalization is considered

In Eq6(c) x = y = 10, k = 3, l = 4, a = –2, b = 6

$$\therefore \text{Answer} = \frac{30}{\frac{30}{40}}\left[30 - 2 + 6\left(\frac{3}{4}\right)\right] + (-2)(6) = 40\left[28 + \frac{9}{2}\right] - 12 = 1288$$

Method II : The different parts are obtained as follows:

First part is obtained by multiplying the deficiency or excess of multiplicand and multiplier (original values).

First Part = (–2) (6) = $\bar{1}\,\bar{2}$. Placement is $_{\bar{1}}{}^{\bar{2}}$

Consider conversion of lower working base into the higher as the common working base in order to arrive at the second part of the answer, i.e., the conversion of 30 to 40. Then one should convert accordingly the given number and the deficiencies/excess. So we have to multiply 28 and its deficiency by 4/3. Therefore, 28 becomes $28\times\frac{4}{3}=\frac{112}{3}$ and the deficiency, -2, becomes $-2\times\frac{4}{3}=-\frac{8}{3}$.

$$\text{Second Part}=\frac{112}{3}+6=46-\frac{8}{3}=\frac{130}{3}$$

Finally one has to take into consideration the multiplication of the second part of the result by the ratio of the working base to theoretical base of the converted one i.e 30/10 = 3, and divided by lower M.F. value = 1.

$$\text{Thus the reduced II Part}=\frac{130}{(1)(3)}\left(\frac{30}{10}\right)=130$$

$$\text{Final Answer}=\ 130\underset{\bar{1}}{/}\ \bar{2}\ =\ 13\bar{1}\ /\ \bar{2}\ =1288$$

This answer can also be arrived at, by making the lower working base common.

Proof: $(kx+a)\,(lx+b)=\dfrac{kx}{\left(\dfrac{kx}{lx}\right)}\left[kx+a+b\dfrac{k}{1}\right]+ab \qquad \rightarrow \text{Eq}6(c)$

$$\text{or }\ \frac{lx}{\left(\dfrac{lx}{kx}\right)}\left[\left(lx+b+\frac{al}{k}\right)\right]+ab \qquad \rightarrow \text{Eq}\,7(c)$$

(x is theoretical base, kx, lx are working bases for multiplicand and multiplier respectively)

2 (d) Multiplication of two numbers where both theoretical and working bases are different for multiplicand and multiplier (ratio of Working Base/Theoretical Base is not the same for multiplicand and multiplier):

The table is prepared showing the details of the problem as is done in the previous cases.

For example consider 201 × 4998

Equalization to higher W.B (5000)

Given Number	Theoretical Base	Working Base	Deficiency /Excess	Multiplication Factor for modifying the working base to 5000 (MF)	Modified Numbers	Modified Deficiency or Excess
201	100	200	1	25	5025	25
4998	1000	5000	− 2	1	4998	−2

Method I : In Eq(6) the values to be substituted are k = 2, l = 5, x = 100, y = 1000 a = 1, b =–2.

$$\dfrac{\dfrac{200}{200}}{5000}\left[200+1+(-2)\left(\frac{2000}{5000}\right)\right]+(1)(1)-2=5000\left[201-\frac{2}{25}\right]-2=1004598$$

Even from Eq(7) the same result is obtained.(Refer Equalization to lower WB)

Method II : In The different equalizations and placements are as follows :

The first part is the multiplication of the deficiencies or excesses of multiplicand and multiplier, as the case may be, and the placement is as per the least theoretical base.

For the second part of the answer one needs to consider the equalization of one working base on par with the other working base. For example, consider the equalization to the highest WB. While doing so care should be taken to multiply the number and deficiency or excess accordingly. The second part of the answer is obtained by considering the cross addition of one of the modified numbers and modified D/Ex. of the other and this to be further reduced. In order to get the final result; the second part is to be multiplied by the ratio of the least working base to its theoretical base and is to be divided by the lower M.F. value

(1) Equalization to higher base (5000) placement with reference to lower base (100):

First Part = $(1)(\bar{2})=\bar{2}$

Here the placement of the first part is as for the lowest theoretical base, i.e., 100. Therefore, first part gives provision for two digits. Now the first part is $0\bar{2}$.

The second part is 5025 – 2 = 4998 + 25 = 5023

The reduced second part is $\left(\frac{5023}{1}\right)\left(\frac{200}{100}\right)$ = 10046

Final Answer = 10046 / $0\bar{2}$ = 1004598

(2) Equalization to higher base (5000) and placement with reference to higher base (1000) :

Instead of considering the lower theoretical base for placement, one can consider the higher theoretical base.

I Part = $\bar{2}$ Placement $00\bar{2}$ (i.e., 1000)

II Part = 5025–2 = 5023

Reduced II Part = $\frac{5023}{25}\left(\frac{5000}{1000}\right)=\frac{5023}{5}=1004\frac{3}{5}$

The $\frac{3}{5}$th part of 1000 = 600 is to be added to I part. i.e., 600 + $00\bar{2}$ = $60\bar{2}$

Final Answer = 1004 / $60\bar{2}$ = 1004598

It is interesting to note that one can reduce the highest working base to have the lower value among the two. Even then the same procedure can be applicable, and the result is same.

Equalization to lower W.B (200) :

Given Number	Theoretical Base	Working Base	Deficiency /Excess	Multiplication Factor for modifying the working base to 200 (MF)	Modified Numbers	Modified Deficiency or Excess
201	100	200	1	1	201	1
4998	1000	5000	− 2	$\frac{1}{25}$	$\frac{4998}{25}$	$\frac{-2}{25}$

(3) <u>Equalization to lower base (200) and placement with reference to higher base (1000)</u>.

The placement of first part is considered as for the higher base, i.e., 3 places.

In order to get the second part, we are equalizing base 5000 to 200 by multiplying it by $\frac{1}{25}$. Therefore, the number 4998 becomes $\frac{4998}{25}$ and its deficiency becomes $-\frac{2}{25}$.

I Part = $(1)(-2)=\bar{2}$ Placement is $00\bar{2}$

II Part = $201-\frac{2}{25}=\frac{4998}{25}+1=\frac{5023}{25}$

Reduced II Part = $\frac{\frac{5023}{25}}{1}\left(\frac{5000}{1000}\right)=\frac{5023}{5}=1004\frac{3}{5}$

We take $\frac{3}{5}$ fraction of the theoretical base, i.e., 1000, and the result is added to the first part of the answer. Then first part becomes $60\bar{2}$.

So the Final Answer = 1004 / $60\bar{2}$ = 1004598

(4) Equalization to lower base (200) and placement with reference to lower base (100) :

I part = (1) (–2) = $\bar{2}$

The placement is as per the lowest theoretical base i.e., 2 **places = $0\bar{2}$**

II part is similarity calculated as $201-\frac{2}{25}=\frac{4998}{25}+1=\frac{5023}{25}$

This is to be modified by dividing it by the lowest multiplication factor which is $\frac{1}{25}$ and is then raised to the ratio of the minimum W.B. to its TB which is $\frac{200}{100}$.

$$\text{Reduced II Part} = \frac{\left(\frac{5023}{25}\right)}{\left(\frac{1}{25}\right)}\left(\frac{200}{100}\right)=10046$$

Final Answer $= 10046 \,/\, 0\bar{2} = 1004598$

On a comparison of all the possibilities it is clear that one can obtain the answer in a much simpler way by considering the higher base concept for the equalization together with the placement on the basis of lower base.

Consider another example 99 × 43:

Given Number	Theoretical Base	Working Base	Deficiency /Excess	Multiplication Factor for modifying the working base to 100 (MF)	Modified Numbers	Modified Deficiency or Excess
99	100	100	–1	1	99	–1
43	10	40	3	$\frac{5}{2}$	$\frac{215}{2}$	$\frac{15}{2}$

I Part = $(-1)(3)=\bar{3}$ Placement is $\bar{3}$ w.r. to (10)

II Part = $99+\frac{15}{2}=\frac{215}{2}-1=\frac{213}{2}$

$$\text{Reduced II Part}=\frac{\frac{213}{2}}{1}\left(\frac{40}{10}\right)=426$$

Final Answer = $426 \,/\, \bar{3} = 4257$

Three number multiplication :

3(a) Different theoretical bases for different numbers (Theoretical and working bases being the same for each number):

For example consider, 93 × 9 × 1006. We have worked out the details of multiplications by method II and different base equalization but with lowest base placement for the I part and next lower for the II part and so on.

Equalization to higher base (1000)

Given Number	Base TB=WB	Deficiency / Excess	Multiplication Factor for modifying the working base to 1000 (MF)	Modified Numbers	Modified deficiency or excess
93	100	− 7	10	930	−70
9	10	− 1	100	900	−100
1006	1000	+ 6	1	1006	6

$\text{I Part} = (-7)(-1)(6) = 42$ Placement is $\,_{4}^{\;2}$

$\text{II Part} = (-70)(-100) + (-70)(6) + (-100)(6) = 5980$

$\text{Reduced II part} = \frac{5980}{(1)(10)}\left(\frac{10}{10}\right) = 598$ Placement is $\,_{5}^{\;98}$

$\text{III Part} = 930 - 100 + 6 = 836$

$\text{Reduced III Part} = \frac{836}{(1)}\left(\frac{10}{10}\right)\left(\frac{100}{100}\right) = 836$

Final Answer = $836\,/_{5}\;\;98\,/_{4}\;\;2 = 842022$

The above problem can be worked out by considering the equalization on any other two bases also i.e., 100 or 10 :

The detail for consideration of equalization to 100 (Medium) is as follows.

Equalization to medium base (100) :

Given Number	Base	D/Ex	Multiplication Factor for modifying the working base to 100 (MF)	Modified Numbers	Modified D/Ex
93	100	− 7	1	93	−7
9	10	− 1	10	90	−10
1006	1000	+ 6	1/10	$\frac{1006}{10}$	$\frac{6}{10}$

$\text{I Part} = (-7)\,(-1)\,(6) = 42$ Placement $\,_{4}^{\;2}$

$$\text{II Part} = (-7)\,(-10) + (-7)\left(\frac{6}{10}\right)+(-10)\left(\frac{6}{10}\right)=\frac{598}{10}$$

$$\text{Reduced II Part} = \frac{\dfrac{598}{10}}{(1)\left(\dfrac{1}{10}\right)}\left(\frac{10}{10}\right)= 598 \text{ Placement is } {}_{5}{}^{98}$$

$$\text{III Part} = 93-10+\frac{6}{10}=\frac{836}{10}$$

$$\text{Reduced III Part} = \frac{\dfrac{836}{10}}{\dfrac{1}{10}} = \left(\frac{10}{10}\right)\left(\frac{100}{100}\right) = 836$$

$$\text{Final Answer} = \; 836\,/_{5}\;\; 98\,/_{4}\;\; 2 \; = 842022$$

The details for consideration of equalization to 10 (lower) is as follows

Equalization to lower base (10):

Given Number	Base	D/Ex	Multiplication Factors for modifying the working base to 10 (MF)	Modified Numbers	Modified D / Ex
93	100	− 7	$\frac{1}{10}$	$\frac{93}{10}$	$\frac{-7}{10}$
9	10	− 1	1	9	−1
1006	1000	6	$\frac{1}{100}$	$\frac{1006}{100}$	$\frac{6}{100}$

$$\text{I Part} = (-7)\,(-1)\,(6) = 42 \quad \text{Placement is } {}_{4}{}^{2}$$

$$\text{II Part} = \left(\frac{-7}{10}\right)(-1)+(-1)\left(\frac{6}{100}\right)+\left(\frac{-7}{10}\right)\left(\frac{6}{100}\right)=\frac{598}{1000}$$

$$\text{Reduced II Part} = \frac{\dfrac{}{1000}}{\left(\dfrac{1}{}\right)\left(\dfrac{1}{100}\right)} = \qquad \text{Placement is } {}_{5}$$

$$\text{III Part} = \frac{93}{10}-1+\frac{6}{100}=\frac{836}{100}$$

$$\text{Reduced III Part} = \frac{\frac{836}{100}}{\frac{1}{100}} = 836$$

$$\text{Final Answer} = 836 \,/_{5}\; 98 \,/_{4}\; 2 = 842022$$

3(b) Three number Multiplication with different TB and WB(for different numbers : (different equalization and placement is in the order of lower to higher)

A more general deduction of the multiplication can be explained as follows :

This is extendable to multiplication of four numbers, five numbers etc.,

The multiplication of three numbers can be carried out as follows :

Let each number be given a theoretical base (TB) and also a working base (WB). (in general different from TB). In terms of these, the numbers are written as

(kx + a), (ly +b), (mz + c) where x,y,z, are TBs; kx, ly, mz are WBs a,b,c are deficiencies (D) / excesses (Ex) from the considered WB.

Let us consider product of the numbers as (kx+a) (ly+b) (mz+c).

$$=[(kx)(ly)+(kx)b+(ly)a+ab]\,(mz+c)$$

$$=(kx)(ly)(mz+c)+(kx)b\,(mz+c)+(ly)a\,(mz+c)+ab(mz+c)$$

The answer can be written in three parts.

$$= \overset{\text{III Part}}{\overline{[(kx)(ly)(mz+c)+(kx)(mz)b+(ly)(mz)a]}} + \overset{\text{II Part}}{\overline{[(kx)bc+(ly)ac+(mz)ab]}} + \overset{\text{I Part}}{\overline{abc}}$$

One has to consider an equalization of the W.B. to any one of the working bases of the three numbers.

This gives rise to multiplication factors (M.F) for each number of the magnitude $= \dfrac{\text{Equalized WB}}{\text{The WB of each number}}$

One has to prepare a table of all such modifications of (1) the given numbers (2) D/Ex with respect to common WB, which make use of corresponding multiplication factors

The final table is given as follows for the Equalization of WB to mz :

Table I(a)

Given numbers	TB	WB	D/Ex w.r.t to WB	Multiplication factor for modifying the working base to (mz)	Modified numbers	Modified D/Ex
kx + a	x	kx	a	$\left(\frac{mz}{kx}\right)$	$(kx+a)\left(\frac{mz}{kx}\right)$	$a\left(\frac{mz}{kx}\right)=a'$
ly + b	y	ly	b	$\left(\frac{mz}{ly}\right)$	$(ly+b)\left(\frac{mz}{ly}\right)$	$b\left(\frac{mz}{ly}\right)=b'$
mz + c	z	mz	c	$\left(\frac{mz}{mz}\right)=1$	$(mz+c)\left(\frac{mz}{mz}\right)$	$c\left(\frac{mz}{mz}\right)=c'$

Similar tables can be prepared for equalization to the other two WBs kx or ly.

The answer consists of three parts.

The first part = abc.

The second part = (kx) bc + (ly) ac + (mz) ab.

When we consider mz as the common working base. The second part of the expression is modified as follows:

$$(kx)(ly)(mz)\left[\frac{bc}{(mz)(ly)}+\frac{ac}{(mz)(kx)}+\frac{ab}{(kx)(ly)}\right]$$

$$=\frac{(kx)(ly)(mz)}{(mz)^2}\left[bc\left(\frac{mz}{mz}\right)\left(\frac{mz}{ly}\right)+ac\left(\frac{mz}{mz}\right)\left(\frac{mz}{kx}\right)+ab\left(\frac{mz}{kx}\right)\left(\frac{mz}{ly}\right)\right]$$

$$=\frac{(kx)}{\left(\frac{mz}{mz}\right)\left(\frac{mz}{ly}\right)}\left[b\left(\frac{mz}{ly}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)b\left(\frac{mz}{ly}\right)\right]$$

$$=\frac{(kx)}{\left(\frac{mz}{mz}\right)\left(\frac{mz}{ly}\right)}\left[b'c'+a'c'+a'b'\right]$$ Which gives the value of the II part, by substituting for the expressions.

<u>The third part</u> = (kx) (ly) (mz + c) + (kx) (mz) b + (ly) (mz) a.

$$= (kx)(ly)\left[(mz+c)+b\left(\frac{mz}{ly}\right)+a\left(\frac{mz}{kx}\right)\right]$$

$$= \frac{(kx)(ly)}{\left(\frac{mz}{mz}\right)}\left[(mz+c)+b'+a'\right]$$ which gives the value of the III part, by substituting for the expressions

If one considers the equalization of the WB to either kx, or ly, the first part remains same i.e. unchanged

If the equalization is to the working base (kx), then the second part is

$$= \frac{ly}{\left(\frac{kx}{kx}\right)\left(\frac{kx}{mz}\right)}\left[b''\ \ c''+a''\ c''+a''b''\right]$$

Where a'', b'', c'' are the modifications of D/Ex with reference to (kx)

On substitution for the expressions, the value of the II part can be obtained.

The third part is $\frac{(ly)(mz)}{\left(\frac{kx}{kx}\right)}\left[(kx+a)+b''+c''\right]$

The value of the third part can be obtained from the above expression.

Similarly if the equalization is carried out to the working base (ly) then the second part is $\frac{(kx)}{\left(\frac{ly}{ly}\right)\left(\frac{ly}{mz}\right)}\left[b'''\ \ c'''+a'''\ c'''+a'''\ b'''\right]$

Where a''', b''', c''' are the modifications of D/Ex with reference to (ly)

The third part is $\frac{(kx)(mz)}{\left(\frac{ly}{ly}\right)}\left[(ly+b)+a'''+c'''\right]$

From the expressions of all the parts one can get the values of the various different parts and by combining the values of these parts the final answer is obtained.

This procedure is workable to multiplication of any numbers i.e., a four number, five number etc. However the answer consists of as many parts as the numbers. For four number multiplication four parts and for five number multiplication five parts. Etc.,

An example of three number multiplication can be illustrated as follows (where TB and WB are different for different bases).

Consider multiplication 41 x 48 x 199:

The table is prepared as follows:

Equalization to WB 200 (i.e., mz) : -

Given numbers	**TB**	**WB**	**D/Ex**	**Multiplication factor for modifying the working base to 200 (MF)**	**Modified numbers**	**Modified D/Ex**
41	10 (x)	40(kx)	+1(a)	$5\left(\frac{mz}{kx}\right)$	205 $(kx+a)\left(\frac{mz}{kx}\right)$	5(a′)
48	10 (y)	50(ly)	−2 (b)	$4\left(\frac{mz}{ly}\right)$	192 $(ly+b)\left(\frac{mz}{ly}\right)$	−8(b′)
199	100 (z)	200(mz)	−1(c)	$1\left(\frac{mz}{mz}\right)$	199 $(mz+c)\left(\frac{mz}{mz}\right)$	−1(c′)

METHOD I

The first part of answer = abc

$$= (1)(-2)(-1) = 2.$$

On substitution of the various values in the second part

$$= \frac{kx}{\left(\frac{mz}{mz}\right)\left(\frac{mz}{ly}\right)}\left[a'b' + b'c' + c'a'\right] = \frac{40}{(1)(4)}[(5)(-8) + (-8)(-1) + (5)(-1)] = \overline{370}$$

The value of the third part is obtained by substituting the corresponding values in the expression $\frac{(kx)(ly)}{\left(\frac{mz}{mz}\right)}\left[(mz+c)+b'+a'\right] = \frac{(40)\,(50)}{1}\,[199-8+5] = 392000$

By adding all the three parts, the final answer is obtained as $2+\overline{370}+392000$

$$= 392002 - 370 = 391632.$$

METHOD II (Equalization along with part placements* ranging from lower to higher)

The multiplication can be simplified (directly evaluated) by using the table in the following way. This result is arrived by setting apart the individual parts and placement of the digits.

Step (1) : Form the table by equalization to one of the WBs. (say mz)

Step (2) : The answer consists of three parts. Calculate I, II and III parts of the answer in the following way. :

I Part : abc, product of D/Ex with reference to WB. The placement is with reference to the minimum* of the three TBs

II Part :
(1) Sum of products of two by two modified D/Ex. i.e., $a' b' + b' c' + c' a'$
(2) Divide this sum by the product of two minimum* multiplication factors. i.e., $\frac{mz}{mz}, \frac{mz}{ly}$
(3) This is to be multiplied by the ratio of minimum* WB to its TB.
(4) The placement is as per the next lower TB

III Part: (1) Cross Addition: One modified number is added to the other two modified D/Ex. i.e., $(mz + c) + a' + b'$

(2) Divide this by the least multiplication factor. i.e., $\frac{mz}{mz}$

(3) This is to be multiplied by the ratio of two minimum* WBs to the respective TBs. i.e., $\frac{kx}{x}, \frac{ly}{y}$

The values obtained by this procedure is as follows :

I Part = (1) (–2) (–1) = 2 and the placement is as per the lowest TB (10) i.e. 2

II Part = $(5)(-8) + (5)(-1) + (-8)(-1) = \overline{37}$

Reduced II Part = $\frac{\overline{37}}{(1)(4)}\left(\frac{40}{10}\right) = \overline{37} = {}_{\bar{3}}\,{}^{\bar{7}}$ (Placement as per the next lower TB i.e., 10) where $\bar{3}$ is to be carried to the III Part

III Part = 199 – 8 + 5 = 192 – 1 + 5 = 205 – 8 – 1 = 196

Reduced III Part = $\left(\frac{196}{1}\right)\left(\frac{40}{10}\right)\left(\frac{50}{10}\right) = 3920$

Final Answer = $3920 \;/_{\bar{3}}\;\; \bar{7} \;/\; 2 = 392\overline{37}2 = 391632$

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* One can also use placement pertaining to any base and consequently the M.F. Values.

Equalization to WB 50 (i.e., ly)

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 50 (MF)	Modified numbers	Modified D/Ex
41	10	40	+1	5/4	$\frac{205}{4}$	5/4 a''
48	10	50	–2	1	48	–2 b''
199	100	200	–1	1/4	$\frac{199}{4}$	–1/4 c''

METHOD II

I Part $= (1)(-2)(-1) = 2$ (Placement with reference to 10)

$$\text{II Part} = \left(\frac{5}{4}\right)(-2) + \left(\frac{5}{4}\right)\left(-\frac{1}{4}\right) + (-2)\left(-\frac{1}{4}\right) = \frac{-10}{4} - \frac{5}{16} + \frac{2}{4} = \frac{-40-5+8}{16} = \frac{-37}{16}$$

$$\text{Reduced II Part} = \frac{\frac{\overline{37}}{16}}{(1)\left(\frac{1}{4}\right)}\left(\frac{40}{10}\right) = \overline{37} = {}_{\bar{3}}\ {}^{\bar{7}}$$ (Placement with reference to 10).

$$\text{III Part} = \frac{205}{4} - 2 - \frac{1}{4} = \frac{205-8-1}{4} = \frac{196}{4}$$

$$\text{Reduced III Part} = \frac{\frac{196}{4}}{\frac{1}{4}}\left(\frac{40}{10}\right)\left(\frac{50}{10}\right) = 3920$$

$$\text{Final Answer} = 3920 \Big/ {}_{\bar{3}}\ \bar{7} \Big/ 2 = 392\overline{37}2 = 391632$$

Equalization to WB 40 (i.e., kx) : -

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 40 (MF)	Modified numbers	Modified D/Ex
41	10	40	+1	1	41	+1 a'''
48	10	50	–2	$\frac{4}{5}$	$\frac{192}{5}$	$\frac{-8}{5}$ b'''
199	100	200	–1	$\frac{1}{5}$	$\frac{199}{5}$	$\frac{-1}{5}$ c'''

I Part = (1) (–2) (–1) = 2 (Placement with reference to 10)

$$\text{II Part} = (1)\left(\frac{-8}{5}\right)+(1)\left(\frac{-1}{5}\right)+\left(\frac{-8}{5}\right)\left(\frac{-1}{5}\right)=\frac{-8}{5}-\frac{1}{5}+\frac{8}{25}=\frac{-40-5+8}{25}=\frac{-37}{25}$$

$$\text{Reduced II Part} = \frac{\frac{\overline{37}}{25}}{\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)}\left(\frac{40}{10}\right)=\overline{37}=\ _{\bar{3}}\quad ^{\bar{7}}$$ (Placement with reference to 10)

$$\text{III Part} = 41-\frac{8}{5}-\frac{1}{5}=\frac{205-8-1}{5}=\frac{196}{5}$$

$$\text{Reduced III Part} = \frac{\frac{196}{5}}{\frac{1}{5}}\left(\frac{40}{10}\right)\left(\frac{50}{10}\right)=3920$$

$$\text{Final Answer} = 3920\Big/_{\bar{3}}\quad \bar{7}\ \Big/2=392\overline{37}2\ =391632$$

Consider another three number multiplication : -

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 600 (MF)	Modified numbers	Modified D/Ex
18	10	20	–2	30	540	–60
402	100	400	+2	3/2	603	3
597	100	600	–3	1	597	–3

<u>Method I</u>: - Here kx = 20, ly = 400, mz = 600, a = –2, b = 2, c = –3

I Part = abc = (– 2) (2) (– 3) = 12

$$\text{II Part} = \frac{kx}{\left(\frac{mz}{mz}\right)\left(\frac{mz}{ly}\right)}\left[b\left(\frac{mz}{ly}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)c\left(\frac{mz}{mz}\right)+a\left(\frac{mz}{kx}\right)b\left(\frac{mz}{ly}\right)\right]$$ (Refer Table I)

$$=\frac{20}{(1)\left(\frac{3}{2}\right)}[(3)\,(-3)+(-60)(-3)+(-60)\,(3)]=\frac{40}{3}[-9+180-180]=-120$$

$$\text{III Part} = \frac{(kx)(ly)}{\left(\frac{mz}{mz}\right)}\left[(mz+c)+b\left(\frac{mz}{ly}\right)+a\left(\frac{mz}{kx}\right)\right]$$

$$=\frac{(20)(400)}{1}[597+3-60]=(20)\,(400)\,(540)=4320000$$

Adding all these values the final answer is = 12 – 120 + 4320000 = 4319892.

<u>Method II</u> : -

I Part $= (-2)\,(2)\,(-3) = 12$ the placement is $= \begin{matrix} & 2 \\ 1 & \end{matrix}$

II Part $= (3)\,(-3) + (-60)\,(-3) + (-60)\,(3) = -9 + 180 - 180 = \bar{9}$

Reduced II part $= \dfrac{\bar{9}}{(1)\,(3/2)}\left(\dfrac{20}{10}\right) = \overline{120}$ the placement is $\begin{matrix} & \bar{2}\,\bar{0} \\ \bar{1} & \end{matrix}$

III Part $= 597 + 3 - 60 = 600 - 60 = 540$

Reduced III Part $= \left(\dfrac{540}{1}\right)\left(\dfrac{20}{10}\right)\left(\dfrac{400}{100}\right) = 4320$

Final Answer $= 4320 \,\Big/\, \begin{matrix} & \overline{20} \\ \bar{1} & \end{matrix} \,\Big/\, \begin{matrix} & 2 \\ 1 & \end{matrix} = 4319892$

3(c) Different TB and WB for the different numbers (TB   WB). Different equalization and different part placements:

<u>Equalization to higher base (8000)</u>

Given numbers	TB	WB	D/Ex	Multiplication factor for modifying the working base to 8000 (MF)	Modified Numbers	Modified D/Ex
41	10	40	+1	200	8200	200
196	100	200	–4	40	7840	–160
8002	1000	8000	+2	1	8002	2

II Method is applied :

(a) <u>Equalization to higher base and placement of the I Part on the basis of lower base</u> : –

I Part $= (1)\,(-4)\,(2) = \bar{8}$ Placement is $\bar{8}$ (i.e., w.r.to 10)

II Part $= (200)\,(-160) + (200)\,(2) + (-160)\,(2) = -32000 + 400 - 320 = \overline{31920}$

Reduced II Part $= \dfrac{\overline{31920}}{(1)(40)}\left(\dfrac{40}{10}\right) = \overline{3192}$ Placement is $\begin{matrix} & \bar{9}\,\bar{2} \\ \overline{31} & \end{matrix}$ (i.e., w.r.to 100)

III Part $= 8200 - 160 + 2 = 7840 + 200 + 2 = 8002 + 200 - 160 = 8042.$

Reduced III Part $= \dfrac{8042}{1}\left(\dfrac{40}{10}\right)\left(\dfrac{200}{100}\right) = 64336$

Final Answer $= 64336 \,\Big/\, \begin{matrix} & \overline{92} \\ \overline{31} & \end{matrix} \,\Big/\, \bar{8} = 64305 \,\Big/\, \overline{92} \,\Big/\, \bar{8} = 64304072$

(b) **Equalization to higher base and placement of the I Part on the basis of higher base** : -

I Part = (1) (–4) (2) = $\bar{8}$ Placement is $00\bar{8}$ (i.e., w.r.to 1000)

II Part = (200) (–160) + (200) (2) + (–160) (2) = – 32000 + 400 – 320 = $\overline{31920}$

$$\text{Reduced II Part} = \frac{\overline{31920}}{200 \times 40}\left(\frac{8000}{1000}\right) = \frac{\overline{31920}}{1000} = \frac{\overline{3192}}{100} = \bar{3}\bar{1}\,\frac{\bar{9}\bar{2}}{100}$$ Placement is $\bar{3}\bar{1}$ (w.r.to 100)

The fractional Part $\frac{\overline{92}}{100}$ of 1000 = $\overline{920}$ is to be added to $00\bar{8} = \overline{92}0 + 00\bar{8} = \overline{928}$

III Part = 8200 – 160 + 2 = 8042

$$\text{Reduced III Part} = \frac{8042}{200}\left(\frac{8000}{1000}\right)\left(\frac{200}{100}\right) = \frac{16084}{25} = 643\frac{9}{25}$$

$\frac{9}{25}$th part of 100 = $\left(\frac{9}{25}\right)100 = 36$ This is to be added to II Part i.e., $\bar{3}\ \bar{1} + 36 = 05$

$$\text{Final Answer} = 643\,\frac{9}{25} \Big/ \overline{31}\ \frac{\overline{92}}{100} \Big/ 00\bar{8} = 643 \Big/ 05 \Big/ \bar{9}\bar{2}\bar{8} = 64304072$$

(c)(i) **Equalization to higher base and placement of the I Part on the basis of medium base and II Part on the basis of lower base (10) than the medium**:-

I Part = $\bar{8}$ placement is $0\bar{8}$ (i.e., w.r. to 100)

II Part = $\overline{31920}$

$$\text{Reduced II Part} = \frac{\overline{31920}}{(1)(200)}\left(\frac{200}{100}\right) = \frac{\overline{3192}}{10} = \overline{319}\frac{\bar{2}}{10}$$

= $\overline{319}$ Placement is ${}_{\overline{31}}\ {}^{\bar{9}}$ (i.e., w.r.to 10)

$\frac{\bar{2}}{10}$th part of 100 = $\bar{2}0$ is added to I part = $\bar{2}0 + 0\bar{8} = \overline{28}$

III Part = 8200 – 160 + 2 = 8042

$$\text{Reduced III Part} = \frac{8042}{1}\left(\frac{200}{100}\right)\left(\frac{40}{10}\right) = 64336.$$

$$\text{Final Answer :}\quad 64336 \Big/ {}_{\overline{31}}\ {}^{\bar{9}} \Big/ \bar{2}\bar{8} = 64304072$$

(c)(ii) <u>Equalization to higher base and placement of the I Part on the basis of medium base and II part on the basis of higher base 1000 them the medium:</u>

I Part = $\bar{8}$ Placement is $0\bar{8}$ (i.e., w.r.to 100)

II Part = $\overline{31920}$

Reduced II Part = $\dfrac{\overline{31920}}{(1)(200)}\left(\dfrac{200}{100}\right)=\overline{319}\dfrac{\bar{2}}{10}$ placement is $\overline{319}$ (i.e., w.r.to 1000) and

$\dfrac{\bar{2}}{10}$ th part of 100 $=\bar{2}0$ is added to I part $=\bar{2}0+0\bar{8}=\bar{2}\bar{8}$

III Part = 8200 – 160 + 2 = 8042

Reduced III Part = $\dfrac{8042}{200}\left(\dfrac{200}{100}\right)\left(\dfrac{8000}{1000}\right)=643\dfrac{9}{25}$

$\dfrac{9}{25}$ th part of $1000=360$ is to be added to II part $=360+\overline{319}=05\bar{9}$

Final Answer $= 643\dfrac{9}{25}\Big/\ \overline{31}\,\bar{9}\ \dfrac{\bar{2}}{10}\ \Big/0\bar{8}\ =\ 643\ \Big/\ 05\bar{9}\ \Big/\ \overline{28}=64304072$

<u>Highest base equalization (Placement)</u> :

		I	II
(a) Lower base	→	10	100
(b) Higher base	→	1000	100
(c) Medium base	→	100	10 1000

Equalization to lower(WB = 40) : -

Given numbers	TB	WB	D/Ex	Multiplication factor for modifying the working base to 40 (MF)	Modified numbers	Modified D/Ex.
41	10	40	+1	1	41	1
196	100	200	–4	$\frac{1}{5}$	$\frac{196}{5}$	$\frac{-4}{5}$
8002	1000	8000	+2	$\frac{1}{200}$	$\frac{8002}{200}$	$\frac{1}{100}$

(a) <u>Placement with reference to lower base (10) for the I Part</u>:

I Part = (1) (–4) (2) = $\bar{8}$ placement is $\bar{8}$ (w.r. to 10)

II Part = (1) $\left(\dfrac{-4}{5}\right)+\left(\dfrac{-4}{5}\right)\left(\dfrac{1}{100}\right)+(1)\left(\dfrac{1}{100}\right)=\dfrac{-4}{5}-\dfrac{4}{500}+\dfrac{1}{100}=\dfrac{-400-4+5}{500}=\dfrac{\overline{399}}{500}$

Reduced II Part = $\dfrac{\dfrac{\overline{399}}{500}}{\left(\dfrac{1}{5}\right)\left(\dfrac{1}{200}\right)}\left(\dfrac{40}{10}\right)=\overline{3192}$ Placement is ${}_{\overline{31}}\ {}^{\overline{92}}$ (w.r.to 100)

III Part = $41-\dfrac{4}{5}+\dfrac{1}{100}=\dfrac{20500-400+5}{500}=\dfrac{20105}{500}$

Reduced III Part = $\dfrac{\dfrac{20105}{500}}{\dfrac{1}{200}}\left(\dfrac{40}{10}\right)\left(\dfrac{200}{100}\right)=64336$

Final Answer = $64336 \Big/ {}_{\overline{31}}\ \overline{92} \Big/ \ \bar{8} = 64304072$

(b) Placement with reference to higher base 1000 for the I Part :-

I Part = (1) (–4) (2) = $\bar{8}$ Placement is $00\bar{8}$ (w.r. to 1000)

II Part = $\dfrac{\dfrac{\overline{399}}{500}}{(1)\left(\dfrac{1}{5}\right)}\left(\dfrac{8000}{1000}\right)=\dfrac{\overline{798}}{25}=\overline{31}\,\dfrac{\overline{23}}{25}$ Placement is $\bar{3}\ \bar{1}$ (w.r. to. 100)

$\dfrac{\overline{23}}{25}$th Part of 1000=$\overline{920}$ is to be added to I Part i.e., $00\bar{8}+\overline{920}=\overline{928}$ → (1)

III Part = $\dfrac{20105}{500}$

Reduced III Part = $\dfrac{\dfrac{20105}{500}}{1}\left(\dfrac{8000}{1000}\right)\left(\dfrac{200}{100}\right)=\dfrac{1684}{25}=643\dfrac{9}{25}$

$\dfrac{9}{25}$th part of 100 = 36 is to be added to the II part i.e., $\overline{31}+36=05$ → (2)

Final Answer = $643\dfrac{9}{25} \Big/ \overline{31}\dfrac{\overline{23}}{25} \Big/ \ 00\bar{8}$

$= 643\dfrac{9}{25} \Big/ \bar{3}\,\bar{1} \Big/ \overline{928}$ [by (1)]

$= 643 \Big/ 05 \Big/ \overline{928}$ [by (2)]

$= 64304072$

(c) (i) Placement of the I Part w.r. to medium base, 100 and the reduced II Part on the basis of lower base (10) than the medium :–

I Part : $(1)(-4)(2) = \bar{8}$ Placement is $0\bar{8}$ (w.r. to 100)

II Part $= \dfrac{\overline{399}}{500}$

Reduced II Part: $\dfrac{\dfrac{\overline{399}}{500}}{(1)\left(\dfrac{1}{200}\right)}\left(\dfrac{200}{100}\right) = \dfrac{\overline{1596}}{5} = \overline{319}\,\dfrac{\bar{1}}{5}$ Placement is $\overset{\bar{9}}{\overline{31}}$ (i.e., w.r. to 10)

The $\dfrac{\bar{1}}{5}$th part of 100 is to be added to I Part i.e., $0\bar{8} + \overline{20} = \overline{28}$ → (3)

III Part $= \dfrac{20105}{500}$

Reduced III Part $= \dfrac{\dfrac{20105}{500}}{\dfrac{1}{200}}\left(\dfrac{200}{100}\right)\left(\dfrac{40}{10}\right) = 64336$

Final Answer $= 64336 \;/\; \overline{319}\,\dfrac{\bar{1}}{5} \;/\; 0\bar{8}$

$= 64336 \;/\; \overset{\bar{9}}{\overline{31}} \;/\; \overline{28}$ → [from (3)]

$= 64304072$

(c)(ii) Placement of I part w.r. to medium base 100 and the reduced II Part on the basis of higher base 1000 than the medium: -

I Part $= (1)(-4)(2) = \bar{8}$ Placementis $0\bar{8}$ (w.r. to 100)

II Part $= \dfrac{\overline{399}}{500}$

Reduced II Part $= \dfrac{\dfrac{\overline{399}}{500}}{(1)\left(\dfrac{1}{200}\right)}\left(\dfrac{200}{100}\right) = \dfrac{\overline{1596}}{5} = \overline{319}\,\dfrac{\bar{1}}{5}$ Placement is $= \bar{3}\bar{1}\bar{9}$ (w.r. to 1000)

$\dfrac{\bar{1}}{5}$th part of 100 $= \overline{20}$ is to be added to I part i.e., $0\bar{8} + \overline{20} = \overline{28}$ → (4)

III Part $= \dfrac{20105}{500}$

$$\text{Reduced III Part} = \frac{\frac{20105}{500}}{(1)}\left(\frac{200}{100}\right)\left(\frac{8000}{1000}\right) = \frac{16084}{25} = 643\,\frac{9}{25}$$

$\frac{9}{25}$th part of 1000 = 360 is to be added to II part i.e., $360 + \overline{319} = 05\bar{9}$ → (5)

$$\text{Final Answer} = 643\ \frac{9}{25} \Big/ \overline{319}\ \frac{\bar{1}}{5} \Big/ 0\bar{8}$$

$$= 643\ \frac{9}{25} \Big/ \overline{31}\,\bar{9} \Big/ \overline{28} \qquad \rightarrow \text{(from 4)}$$

$$= 643 \Big/ 05\bar{9} \Big/ \overline{28} \qquad \rightarrow \text{(from 5)}$$

= 64304072

Equalization to medium (WB = 200) base :

Given numbers	TB	WB	D/Ex	Multiplication factor for modifying working base to 200 (MF)	Modified numbers	Modified D/Ex.
41	10	40	+1	5	205	5
196	100	200	–4	1	196	–4
8002	1000	8000	+2	$\frac{1}{40}$	$\frac{8002}{40}$	$\frac{1}{20}$

(a) Placement of the I Part w.r. to lower base, 10: –

I Part = (1) (–4) (2) = $\bar{8}$ Placement is $\bar{8}$

$$\text{II Part} = (5)(-4) + (5)\left(\frac{1}{20}\right) + (-4)\left(\frac{1}{20}\right) = \frac{-400+5-4}{20} = \frac{\overline{399}}{20}$$

Reduced II Part $= \dfrac{\dfrac{\overline{399}}{20}}{(1)\left(\dfrac{1}{40}\right)}\left(\dfrac{40}{10}\right) = \overline{3192}$ Placement is $\underset{\overline{31}}{}\ \overset{\overline{92}}{}$ (w.r. to 100)

III Part $= 205 - 4 + \dfrac{1}{20} = \dfrac{4021}{20}$

Reduced III Part $= \dfrac{\dfrac{4021}{20}}{\left(\dfrac{1}{40}\right)}\left(\dfrac{40}{10}\right)\left(\dfrac{200}{100}\right) = 64336$

Final Answer $= \quad 64336 \Big/ \underset{\overline{31}}{} \ \overline{92} \Big/ \ \bar{8} = 64304072$

(b) Placement of the I Part with reference to higher base, 1000 : -

I Part $= \bar{8}$ Placement is $00\bar{8}$

II Part $= \dfrac{\overline{399}}{20}$

Reduced II Part $\dfrac{\dfrac{\overline{399}}{20}}{(1)(5)}\left(\dfrac{8000}{1000}\right) = \dfrac{\overline{798}}{25} = \bar{3}\bar{1}\,\dfrac{\overline{23}}{25}$ Placement is $\bar{3}\ \bar{1}$ (w.r. to 100)

$\dfrac{\overline{23}}{25}$th part of $1000 = \overline{920}$ is to be added to I Part i.e., $00\bar{8} + \overline{920} = \overline{928}$ → (6)

III Part $= \dfrac{4021}{20}$

Reduced III Part $= \dfrac{\dfrac{4021}{20}}{(5)}\left(\dfrac{8000}{1000}\right)\left(\dfrac{200}{100}\right) = \dfrac{16084}{25} = 643\dfrac{9}{25}$

$\dfrac{9}{25}$th part of $100 = 36$ is to be added to II Part i.e., $\overline{31} + 36 = 05$ → (7)

Final Answer = $643\,\frac{9}{25} \Big/ \overline{31}\,\frac{\overline{23}}{25} \Big/ 00\bar{8}$

$= 643\,\frac{9}{25} \Big/ \overline{31} \Big/ \overline{928}$ →(from 6)

$= 643 \Big/ 05 \Big/ \overline{928}$ →(from 7)

= 64304072

(c)(i) Placement of the I Part with reference to medium base 100 and reduced II part on the basis of lower base 10 than the medium :-

I Part = $\bar{8}$ placement is $0\bar{8}$

II Part = $\frac{\overline{399}}{20}$

Reduced II Part $= \dfrac{\frac{\overline{399}}{20}}{(5)\left(\frac{1}{40}\right)}\left(\frac{200}{100}\right) = \frac{\overline{7596}}{5} = \overline{319}\,\frac{\bar{1}}{5}$ Placementis = $\begin{matrix} & \bar{9} \\ \bar{3}\bar{1} & \end{matrix}$ (w.r. to 10)

$\frac{\bar{1}}{5}$th part of 100 = $\overline{20}$ is to be added to I part i.e., $\overline{20} + 0\bar{8} = \overline{28}$ →(8)

III Part = $\frac{4021}{20}$

Reduced III Part $= \dfrac{\frac{4021}{20}}{\left(\frac{1}{40}\right)}\left(\frac{200}{100}\right)\left(\frac{40}{10}\right) = 64336$

Final Answer = $64336 \Big/ \overline{319}\,\frac{\bar{1}}{5} \Big/ 0\bar{8}$

$= 64336 \Big/ \begin{matrix} & \bar{9} \\ \overline{31} & \end{matrix} \Big/ \overline{28}$ →[from(8)]

$$= \quad 64304072$$

(c)(ii) Placement of the I part on the basis of medium base and the reduced II Part on the basis of higher base (1000) than medium :

I Part = $\bar{8}$ placement is $0\bar{8}$

II Part = $\dfrac{\overline{399}}{20}$

Reduced II Part = $\dfrac{\frac{\overline{399}}{20}}{(5)\left(\frac{1}{40}\right)}\left(\dfrac{200}{100}\right) = \dfrac{\overline{1596}}{5} = \overline{319}\dfrac{\bar{1}}{5}$ Placeme is $\bar{3}\bar{1}\bar{9}$ (w. r. to 1000)

$\dfrac{\bar{1}}{5}$ th part of 100 = $\overline{20}$ is to be added to I part i.e., $\overline{20} + 0\bar{8} = \overline{28}$ → (9)

III Part = $\dfrac{4021}{20}$

Reduced III Part = $\dfrac{\frac{4021}{20}}{(5)}\left(\dfrac{200}{100}\right)\left(\dfrac{8000}{1000}\right) = \dfrac{16084}{25} = 643\,\dfrac{9}{25}$

$\dfrac{\bar{9}}{25}$ th part of 1000 = 360 is to be added to II part i.e., $\overline{319} + 360 = 05\bar{9}$ →` (10)

Final Answer = $643\,\dfrac{9}{25} \;\Big/\; \overline{319}\,\dfrac{\bar{1}}{5} \;\Big/\; 0\bar{8}$

$= 643\,\dfrac{9}{25} \;\Big/\; \overline{319} \;\Big/\; \overline{28}$ →[From(9)]

$= 643 \;\Big/\; 05\bar{9} \;\Big/\; \overline{28}$ →[From(10)]

$= \quad 64304072$

II Equalization to lower base (10) :

	Placement	I Part	II Part
(a)	Lower base	10	100
(b)	Higher base	1000	100
(c)	Medium base	100	10 1000

III Equalization to medium base (100) :

Placement		I Part	II Part
(a)	Lower base	10	100
(b)	Higher base	1000	100
(c)	Medium base	100	10 1000

(3d) Another example with equal TBs : - $\left(\frac{WB}{TB} \text{ are different}\right)$

Equalization to higher base 6000

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 6000 (MF)	Modified numbers	Modified D/Ex
1999	1000	2000	–1	3	5997	–3
4012	1000	4000	+12	3/2	6018	+18
6008	1000	6000	+8	1	6008	+8

Method II Direct Evaluation :

I Part $= (-1)(12)(8) = \overline{96}$ Placement is $\overline{096}$ (w.r. to 1000)

II Part $= (-3)(18) + (-3)(8) + (18)(8) = -54 - 24 + 144 = 66$

Reduced II Part $= \dfrac{66}{(1)\left(\dfrac{3}{2}\right)}\left(\dfrac{2000}{1000}\right) = 88$ Placement is 088 (w.r. to 1000)

III Part $= 5997 + 18 + 8 = 6018 - 3 + 8 = 6008 - 3 + 18 = 6023$

Reduced III Part $= \dfrac{6023}{1}\left(\dfrac{4000}{1000}\right)\left(\dfrac{2000}{1000}\right) = 48184$

Final Answer $= 48184 \,/\, 088 \,/\, \overline{096} = 48184088\,\overline{096} = 48184087904$

(3e) Another example: Consider 19 x 201 x 1998 $\left(\frac{\text{WB}}{\text{TB}}\text{ is same for all numbers}\right)$

Equalization to higher WB base 2000

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 2000 (MF)	Modified numbers	Modified D/Ex
19	10	20	−1	100	1900	−100
201	100	200	+1	10	2010	10
1998	1000	2000	−2	1	1998	− 2

I Part = (−1) (1) (−2) = 2 Placement is 2 (w.r. to 10)
II Part = (−100) (10) + (−100) (−2) + (10) (−2)

$$= -1000 + 200 - 20 = \overline{8}\,\overline{2}\,0$$

Reduced II Part = $\frac{\overline{820}}{(1)(10)}\left(\frac{20}{10}\right)=\overline{164}$ Placement is $\begin{matrix} & \overline{64} \\ \overline{1} & \end{matrix}$ (w.r. to 10)

III Part = 1900 + 10 − 2 = 2010 − 100 − 2 = 1998 − 100 + 10 = 1908

Reduced III Part = $\left(\frac{1908}{1}\right)\left(\frac{20}{10}\right)\left(\frac{200}{100}\right) = 7632$

Final Answer = $7632 \,/\, \begin{matrix} & \overline{64} \\ \overline{1} & \end{matrix} \,/\, 2 = 7631\overline{64}2 = 7630362$

The same procedure is extendable to four number multiplication.

This procedure can be extended to multiplication of any number by any numbers.

(4) Four number multiplication :

4(a) Consider 999 × 995 × 98 × 96 (TB = WB)

Given Number	TB=WB Base	Deficiency / Excess	Multiplication Factor for Modifying the working base to 1000 (MF)	Modified Numbers	Modified excess or deficiency
999	1000	− 1	1	999	−1
995	1000	− 5	1	995	−5
98	100	− 2	10	980	−20
96	100	− 4	10	960	−40

Method II

I Part $= (-1)(-5)(-2)(-4) = 40$, Placement is 40 (w.r. to 100)

II Part $= (-1)(-5)(-20) + (-1)(-5)(-40) + (-1)(-20)(-40) + (-5)(-20)(-40)$

$= \bar{5}\ \bar{1}\ \bar{0}\ \bar{0}$

Reduced, II Part $= \dfrac{-\bar{5}\bar{1}\bar{0}\bar{0}}{(1)(1)(10)} = \bar{5}\bar{1}\bar{0}$ Placement is ${}_{\bar{5}}{}^{\overline{10}}$ (w.r.to100)

III Part $= (-1)(-5) + (-1)(-20) + (-1)(-40) + (-5)(-20) + (-5)(-40) + (-20)(-40)$
$= 1165$

Reduced III part $\dfrac{1165}{(1)(1)} = 1165$ Placement is ${}_{1}{}^{165}$ (w.r. to 1000)

IV Part $= 999 - 5 - 20 - 40 = 934$

Reduced IV part $\dfrac{934}{1} = 934$

Final Answer =

$= 934 \,/_{1}\; 165 \,/_{\bar{5}}\; \bar{1}\bar{0} \,/\, 40 = 935 \,/\, 160 \,/\, \bar{1}\ \bar{0} \,/\, 40 = 9351599040$

4 (b) Consider 18 x 290 x 401 x 5999 (TB ≠ WB)

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 6000 (MF)	Modified numbers	Modified D/Ex
18	10	20	–2	300	5400	– 600
290	100	300	–10	20	5800	– 200
401	100	400	+1	15	6015	+ 15
5999	1000	6000	–1	1	5999	– 1

Method II

I Part: $= (-2)(-10)(+1)(-1) = \bar{2}0$ Placement is ${}_{\bar{2}}{}^{0}$ (w.r. to 10)

II Part: $(-600)(-200)(15) + (-600)(-200)(-1) + (-600)(15)(-1) + (-200)(15)(-1)$
$= 1800000 - 120000 + 9000 + 3000.$
$= 1692000$

Reduced II Part $= \dfrac{1692000}{(1)(15)(20)}\left(\dfrac{20}{10}\right) = 11280$ Placement is ${}_{112}{}^{80}$ (w.r. to 100)

III Part $= (-600)(-200) + (-600)(15) + (-600)(-1) + (-200)(15) + (-200)(-1)$
$+ (15)(-1) = 120000 - 9000 + 600 - 3000 + 200 - 15$
$= 108785$

Reduced III Part = $\frac{108785}{(1)\,(15)}\left(\frac{20}{10}\right)\left(\frac{300}{100}\right)$ = 43514 Placement is $_{435}\;^{14}$ (w.r.to 100)

IV Part = 5400 – 200 + 15 – 1 = 5214

Reduced IV Part = $\left(\frac{5214}{1}\right)\left(\frac{400}{100}\right)\left(\frac{300}{100}\right)\left(\frac{20}{10}\right)$=125136

Final Answer = $125136\;\big/_{435}\;^{14}\big/_{112}\;^{80}\big/_{\bar{2}}\;0 = 12557226780$

Proof :

Consider the four numbers as (kx + a) (ly + b) (mz + c) (nw +d) in the above problem

kx = 20 a = – 2

Ly = 300 b = –10

mz = 400 c = + 1

nw = 6000 d = – 1

The actual multiplication gives

(kx) (ly) (mz) (nw) + (ly) (mz) (nw)a + (kx) (mz) (nw)b + (kx) (ly) (nw)c + (kx) (ly) (mz)d + (mz) (nw)ab + (ly) (nw)ac + (kx) (nw) (bc) + (ly) (mz)ad + (kx) (mz) bd + (kx) (ly) cd + (nw) abc + (mz) abd + (ly) acd + (kx) bcd + abcd

When equalized to kx, This can also be visualized as :

I Part = abcd

II Part = $\frac{kx}{\left(\frac{kx}{nw}\right)\left(\frac{kx}{ly}\right)\left(\frac{kx}{mz}\right)}$ [$b'\ c'\ d' + a'\ c'\ d' + a'\ b'\ d' + a'\ b'\ c'$]

$$\Sigma\left[\;{}^{4}c_{3}\ \text{products of modified D/Ex}\right]$$

III Part = $\frac{(kx)(nw)}{\left(\frac{kx}{mz}\right)\left(\frac{kx}{ly}\right)}\left[a'\ b' + b'\ c' + c'\ d' + a'\ c' + a'\ d' + b'\ d'\right]$

$$\Sigma\left[\ {}^{4}c_{2}\ \text{products of modified D/Ex}\right]$$

$$\text{IV Part} = \frac{(kx)(mz)(nw)}{\left(\frac{kx}{kx}\right)}\left[(kx+a)+b' + c' + d'\right] \text{ (cross addition)}$$

Where $a' = a\left(\frac{kx}{kx}\right), b' = b\left(\frac{kx}{ly}\right), c' = c\left(\frac{kx}{mz}\right), d' = d\left(\frac{kx}{nw}\right)$ are modified D/Ex. w.r.t. kx

4 (c) Consider another example : 17 x 304 x 392 x 6022 : TB, WB are different for different numbers

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 6000 (MF)	Modified numbers	Modified D/Ex
17	10	20	–3	300	5100	–900
304	100	300	+4	20	6080	80
392	100	400	–8	15	5880	–120
6022	1000	6000	+22	1	6022	22

METHOD I

From the expressions the values are :

I Part = abcd = (–3) (4) (– 8) (22) = 2112

$$\text{II Part} = \frac{(nw)}{\left(\frac{nw}{kx}\right)\left(\frac{nw}{ly}\right)\left(\frac{nw}{mz}\right)}\left[b'c'd' + a'c'd' + a'b'd' + a'b'c'\right] \text{When modified to nw}$$

$$= \frac{6000}{(300)(15)(20)}[(-900)(80)(-120)+(-900)(80)(22)+(-900)(-120)(22)+(80)(-120)(22)$$

$$= \frac{1}{15}\ [8640000 - 1584000 + 2376000 - 211200] = \frac{1}{15}[9220800] = 614720$$

$$\text{III Part} = \frac{(nw)(mz)}{\left(\frac{nw}{kx}\right)\left(\frac{nw}{ly}\right)}\left[a'b' + b'c' + c'd' + a'c' + a'd' + b'd'\right] \text{when modified to nw}$$

$$=\frac{(6000)(400)}{(300)(20)}[(-900)\ (80)+(-900)\ (-120)+(-900)\ (22)+(80)\ (-120)+(80)\ (22)+(-120)\ (2$$

$$= 400\ [-72000 + 108000 - 19800 - 9600 + 1760 - 2640]$$

$$= (400)\ [5720] = 2288000$$

$$\text{IV Part} = \frac{(nw)(ly)(mz)}{(nw)(kx)}\left[(nw+d) + a' + b' + c'\right] \text{ When equalised to nw}$$

$$=\frac{(6000)\ (300)\ (400)}{300}[6022-120+80-900] = (2400000)\ (5082)$$

$$= 12196800000$$

Final answer = 12196800000 + 2288000 + 614720 + 2112 = 12199704832

METHOD II

I Part $= (-3)\ (4)\ (-8)\ (22) = 2112$ Placement is $\frac{}{211}\,2$ (w.r. to 10)

$$\text{II Part} = (-900)\ (80)\ (-120) + (-900)\ (80)\ (22) + (-900)\ (-120)\ (22) + (80)\ (-120)\ (22)$$

$$= 8640000 - 1584000 + 2376000 - 211200$$

$$= 9220800$$

Reduced II Part $=\frac{9220800}{(1)\ (15)\ (20)}\left(\frac{20}{10}\right) = 61472$ Placement is $\frac{}{614}\,72$ (w.r. to 100)

$$\text{III Part} = (-900)\ (80) + (-900)\ (-120) + (-900)\ (22) + (80)\ (-120) + (80)\ (22)$$
$$+ (-120)\ (22) = -72000 + 108000 - 19800 - 9600 + 1760 - 2640 = 5720$$

Reduced III Part $=\frac{5720}{(1)(15)}\left(\frac{20}{10}\right)\left(\frac{300}{100}\right) = 2288$ and the placement is $\frac{}{22}\,88$ (w.r.to 100)

IV Part = 6022 − 120 + 80 − 900 = 5082

Reduced IV Part $= \frac{5082}{1}\left(\frac{20}{10}\right)\left(\frac{300}{100}\right)\left(\frac{400}{100}\right) = 121968.$

Final Answer $= 121968 \,/\, \frac{88}{22} \,/\, \frac{72}{614} \,/\, \frac{2}{211} = 12199704832$

Consider another example : 199 x 999 x 55 x 4004

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 1000 (MF)	Modified numbers	Modified D/Ex
199	100	200	−1	5	995	−5
999	1000	1000	−1	1	999	−1
55	10	50	+5	20	1100	+100
4004	1000	4000	+4	$\frac{1}{4}$	1001	1

METHOD II

I Part $= (-1)(-1)(5)(4) = 20$ Placement is $_{2}\,{}^{0}$ (w.r. to 10)

II Part $= (-5)(-1)(100) + (-5)(-1)(1) + (-1)(100)(1) + (-5)(100)(1)$
$= 500 + 5 - 100 - 500 = \overline{95}$

Reduced II Part $= \dfrac{\overline{95}}{(1)\left(\frac{1}{4}\right)(5)}\left(\dfrac{50}{10}\right) = \overline{380}$ Placement is $_{\bar{3}}\,{}^{\overline{80}}$ (w.r. to 100)

III Part $= (-5)(-1) + (-5)(100) + (-5)(1) + (-1)(100) + (-1)(1) + (100)(1)$
$= 5 - 500 - 5 - 100 - 1 + 100 = \overline{501}$

Reduced III Part $= \dfrac{\overline{501}}{(1)(1/4)}\left(\dfrac{50}{10}\right)\left(\dfrac{200}{100}\right) = \overline{20040}$ Placement is $_{\overline{20}}\,{}^{\overline{040}}$ (w.r. to 1000)

IV Part $= 995 - 1 + 100 + 1 = 1095$

Reduced IV Part $= \left(\dfrac{1095}{1/4}\right)\left(\dfrac{50}{10}\right)\left(\dfrac{200}{100}\right)\left(\dfrac{1000}{1000}\right) = 43800$

Final Answer $= 43800 \,\Big/_{\overline{20}}\; {}^{\overline{040}} \Big/_{\bar{3}}\; {}^{\overline{80}} \Big/_{2}\; 0 = 438\bar{2}00\overline{438}20 = 43779956220$

*Consider another four number multiplication: 51 x 78 x 196 x 403**

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 400 (MF)	Modified numbers	Modified D/Ex
51	10	50	+1	8	408	+8
78	10	80	–2	5	390	–10
196	100	200	–4	2	392	–8
403	100	400	+3	1	403	+3

METHOD II

I Part: $(+1)(-2)(-4)(+3) = 24$ Placement is $_{2}\,{}^{4}$ (w.r. to 10)

II Part: $(8)(-10)(-8) + (8)(-10)(3) + (8)(-8)(3) + (-10)(-8)(3) = 640 - 240 - 192 + 240 + 448$

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* The different equalizations together with different Placements and other possibilities can be also tried as exercise for any numbers.

Reduced II Part : $\frac{448}{(1)(2)(5)}\ \frac{50}{10} = 224$ Placement is ${}_{22}\ {}^{4}$ (w.r. to 10)

III Part : (8) (–10) + (8) (–8) + (8) (3) + (–10) (–8) + (–10) (3) + (–8) (3)
$= -80 - 64 + 24 + 80 - 30 - 24 = -\ \overline{9}\ \overline{4}$

$$\text{Reduced III Part}\left(\frac{\overline{94}}{(1)(2)}\right)\left(\frac{80}{10}\right)\left(\frac{50}{10}\right) = \overline{1}\,\overline{880} \text{ Placement is } {}_{\overline{18}}\ {}^{\overline{80}}$$

IV Part = 408 –10 –8 + 3 = 393

$$\text{Reduced IV Part: } = \frac{393}{1}\left(\frac{200}{100}\right)\left(\frac{80}{10}\right)\left(\frac{50}{10}\right) = 31440$$

$$\text{Final Answer } = 31440 \,/_{\overline{18}}\ {}^{\overline{80}}/_{22}\ {}^{4}/_{2}\ {}^{4}$$

$= 3143\ \overline{86}\ 264$
$= 314214264$

*** 5. Five number multiplication:**

5(a) Consider five number multiplication with TB = WB: 1011 × 1009 × 11 × 108 × 98

Given number	Base	D/Ex	Multiplication factor for modifying the working base to 1000 (MF)	Modified numbers	Modified D/Ex
1011	1000	+ 11	1	1011	11
1009	1000	+ 9	1	1009	9
11	10	+ 1	100	1100	100
108	100	+ 8	10	1080	80
98	100	– 2	10	980	–20

Method II

First part = (11) (9) (1) (8) (– 2) = $\overline{1584}$ Placement is ${}_{\overline{158}}\ {}^{\overline{4}}$ (w.r. to 10)

Second Part = (11) (9) (80) (100) + (11) (9) (100) (–20) + (11) (9) (80) (–20) + (11) (100) (80) (–20) + (9) (80) (100) (–20) = 792000 – 198000 – 158400 – 1760000 – 1440000 = $\overline{2764400}$

Reduced Second Part = $\dfrac{\overline{2764400}}{(1)(1)(10)(10)} = \bar{2}\,\bar{7}\bar{6}\bar{4}\bar{4}$ Placement is $_{\overline{276}}\ {}^{\overline{44}}$ (w.r. to 100)

Third Part = (11) (9) (100) + (11) (9) (80) + (11) (9) (–20) + (11) (80) (100)
+ (11) (–20) (100) + (11) (–20) (80) + (9) (80) (100) + (9) (–20) (100)
+ (9) (80) (–20) + (80) (–20) (100) = 9900 + 7920 – 1980 + 88000
– 22000 – 17600 + 72000 – 18000 – 14400 –160000 = $\overline{56160}$

The different equalizations together with different placements and other possibilities can be also tried as exercise for any numbers.

Reduced Third Part = $\dfrac{\overline{56160}}{(1)(1)(10)} = \overline{5616}$ Placement is $_{\overline{56}}\ {}^{\overline{16}}$ (w.r.to 100)

Fourth Part = (11) (9) + (11) (100) + (11) (80) + (11) (–20) + (9) (100) + (9) (80)
+ (9) (–20) + (100) (80) + (100) (–20) + (80) (–20) = 99 + 1100
+ 880 – 220 + 900 + 720 – 180 + 8000 – 2000 – 1600 = 7699

Reduced Fourth Part = $\dfrac{7699}{(1)(1)} = +\,7699$ Placement is $_{7}\ {}^{699}$ (w.r. to 1000)

Fifth part = 1011 + 9 + 100 + 80 – 20 = 1180

Reduced fifth part = $\dfrac{1180}{1} = 1180$.

$$\text{Final Answer} = 1180\ \Big/_{7}\ 699\ \Big/_{\overline{56}}\ \overline{16}\ \Big/_{\overline{276}}\ \overline{44}\ \Big/_{\overline{158}}\ \bar{4}$$

$$= 1187 \Big/ 641 \Big/ \overline{94} \Big/ \overline{02} \Big/ \bar{4} = 1187\ 641\ \bar{9}\bar{4}\bar{0}\bar{2}\bar{4} = 118764005976$$

(b) Consider five number multiplication as 98 x 16 x 43 x x601 x 4997 (TB WB)

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 20 (MF	Modified numbers	Modified D/Ex
98	100	100	–2	1/5	$\frac{98}{5}$	–2/5
16	10	20	–4	1	16	–4
43	10	40	+3	1/2	43/2	3/2
601	100	600	1	1/30	601/30	1/30
4997	1000	5000	–3	1/250	$\frac{4997}{250}$	–3/250

METHOD II

I part = (– 2) (–4) (3) (1) (–3) = $\overline{72}$ Placement is $\bar{7}\;^{\bar{2}}$ (w.r. to 10)

$$\text{II Part} = \left(\frac{-2}{5}\right)(-4)\left(\frac{3}{2}\right)\left(\frac{1}{30}\right)+\left(\frac{-2}{5}\right)(-4)\left(\frac{3}{2}\right)\left(\frac{-3}{250}\right)+\left(\frac{-2}{5}\right)\left(\frac{3}{2}\right)\left(\frac{1}{30}\right)\left(\frac{-3}{250}\right)+(-4)\left(\frac{3}{2}\right)\left(\frac{1}{30}\right)\left(\frac{-3}{250}\right)+\left(\frac{-2}{5}\right)(-4)\left(\frac{1}{30}\right)\left(\frac{-3}{250}\right)=\frac{3990}{75000}$$

$$\text{Reduced II Part} = \frac{\frac{3990}{75000}}{\left(\frac{1}{250}\right)\left(\frac{1}{30}\right)\left(\frac{1}{5}\right)\left(\frac{1}{2}\right)}\left(\frac{20}{10}\right)=7980 \text{ Placement is } 798\;^{0} \text{ (w.r. to 10)}$$

In order to evaluate II Part if one uses working base 20 as common base then it is noticed that multiplication factors, modified numbers and almost all modified D/Ex will be in fractions or mixed fractions. As such the working will be little complicated. Hence we tried a different working base i.e., 5000 as common base. The working details with this is found simpler. At the same time it is interesting to note that the values that are obtained for each part will be same as the one's that are obtained for any other given W.B. as common working base of the problem. We may here point out that even in the middle of the problem we can choose any given common W.B for obtaining the values of the parts. For example: in the above problem with common WB, 20 we obtained the value of II Part as 7980. as we have noticed that we had to work with number of fractions we changed the common WB to 5000 and the corresponding modifications were carried out . Even then it is found that the value of the II Part is 7980 with the modification to 5000 as common WB the remaining values were evaluated using the new modified table.

<u>Change of Table modified to base 5000</u> : -

Given number	TB	WB	D/Ex	Multiplication factor for modifying the working base to 5000 (MF)	Modified numbers	Modified D/Ex
98	100	100	–2	50	4900	–100
16	10	20	–4	250	4000	–1000
43	10	40	3	125	5375	375
601	100	600	1	$\frac{25}{3}$	$\frac{15025}{3}$	$\frac{25}{3}$
4997	1000	5000	–3	1	4997	–3

METHOD II

I Part$=(-2)(-4)(3)(1)(-3)=\overline{72}$ Placement is ${}_{\bar{7}}{}^{\bar{2}}$ (w.r. to 10)

$$\text{II Part}=(-100)\ (-1000)\ (375)\left(\frac{25}{3}\right)+(-100)\ (-1000)\ (375)\ (-3)+(-100)\ (-1000)\left(\frac{25}{3}\right)$$

$$+(-100)\ (375)\left(\frac{25}{3}\right)(-3)\ +(-1000)\ (375)\left(\frac{25}{3}\right)(-3)$$

$$=312500000-112500000-2500000+937500+93751000=207812500$$

Reduced II Part$=\dfrac{207812500}{(1)\left(\frac{25}{3}\right)(50)\ (125)}\left(\dfrac{20}{10}\right)=$ 7980 Placment is ${}_{798}{}^{0}$ (w.r. to 10)

$$\text{III Part}=(-100)(-1000)(375)+(-100)(-1000)\left(\frac{25}{3}\right)+(-100)(-1000)(-3)+(-100)(375)\Bigg($$

$$+(-100)(375)(-3)+(-100)\left(\frac{25}{3}\right)(-3)+(-1000)(375)\left(\frac{25}{3}\right)+(-1000)\left(\frac{25}{3}\right)(-3)$$

$$+(375)\left(\frac{25}{3}\right)(-3)+(-1000)(375)(-3)=\frac{107554375}{3}$$

Reduced III Part$=\dfrac{\frac{107554375}{3}}{(1)\left(\frac{25}{3}\right)(50)}\left(\dfrac{20}{10}\right)\left(\dfrac{40}{10}\right)=688348$ Placement is ${}_{6883}{}^{48}$ (w.r. to 100)

$$\textit{IV Part}\ (-100)(-1000)+(-100)(375)+(-100)\left(\frac{25}{3}\right)+(-100)(-3)+(-1000)(375)+$$

$$(-1000)\left(\frac{25}{3}\right)+(-1000)(-3)+(375)\left(\frac{25}{3}\right)+(375)(-3)+\frac{25}{3}(-3)=\frac{\overline{949175}}{3}$$

$$\text{Re}\textit{duced IV Part}=\frac{\frac{\overline{949175}}{3}}{(1)\left(\frac{25}{3}\right)}\left(\frac{20}{10}\right)\left(\frac{40}{10}\right)\left(\frac{100}{100}\right)=\overline{303736}\ \textit{Placement is}\ {}_{\overline{3037}}{}^{\overline{36}}\ (\textit{w.r.to}\ 100)$$

$$\text{V Part} = 4900 - 1000 + 375 + \frac{25}{3} - 3 = \frac{12841}{3}$$

$$\text{Reduced V part} = \frac{\left(\frac{12841}{3}\right)}{1}\left(\frac{20}{10}\right)\left(\frac{40}{10}\right)\left(\frac{100}{100}\right)\left(\frac{600}{100}\right) = 205456$$

$$\text{Final Answer} = 205456 \Big/ {}_{\bar{3}0\overline{37}}\ {}^{\overline{36}} \Big/ {}_{6883}\ {}^{48} \Big/ {}_{798}\ {}^{0} \Big/ {}_{\bar{7}}\ \bar{2} = 202487554528$$

5 (c) Consider another five number multiplication 5999 x 5986 x 4012 x 208 x 19 :

Given numbers	Th.B	WB	D/Ex	M.F. Equalized to 6000	Modified Numbers	Modified D/Ex.
5999	1000	6000	– 1	1	5999	– 1
5986	1000	6000	– 14	1	5986	– 14
4012	1000	4000	+ 12	3/2	6018	18
208	100	200	+ 8	30	6240	240
19	10	20	– 1	300	5700	–300

$$\text{I Part} = (-1)\,(-14)\,(12)\,(18)\,(-1) = \bar{1}\,\bar{3}\,\bar{4}\,\bar{4} = \overset{\bar{4}}{\bar{1}\bar{3}\,\bar{4}} \quad (\text{Placement is } 10)$$

$$\text{II Part} = (-1)\,(-14)\,(18)\,(240) + (-1)\,(-14)\,(18)\,(-300) + (-1)\,(-14)\,(240)\,(-300)$$
$$+ (-1)\,(18)\,(240)\,(-300) + (-14)\,(18)\,(240)\,(-300) = 60480 - 75600$$
$$- 1008000 + 1296000 + 1814000 = 18416880$$

$$\text{Reduced II Part}=\frac{18416880}{(1)\,(1)\left(\frac{3}{2}\right)(50)}\left(\frac{20}{10}\right)818528 = {}_{8185}\ {}^{28}\ (\text{Placement is } 100)$$

III Part = (–1) (–14) (18) + (–1) (–14) (240) + (–1) (–14) (–300) + (–1) (18) (240) + (–1) (18) (–300) + (–1) (240) (–300) + (14) (18) (240) + (–14) (18) (–300) + (–14) (240) (–300) + (18) (240) (–300) = 252 + 3360 – 4200 – 4320 + 5400 + 72000 – 60480 + 75600 = $\bar{2}\,\bar{0}\,\bar{0}\,\bar{3}\,\bar{8}\,\bar{8}$

$$\text{Reduced III Part} = \frac{\bar{2}\,\bar{0}\,\bar{0}\,\bar{3}\,\bar{8}\,\bar{8}}{(1)(1)\left(\frac{3}{2}\right)}\left(\frac{20}{10}\right)\left(\frac{200}{100}\right) = \bar{5}\,\bar{3}\,\bar{4}\,\bar{3}\;\bar{6}\,\bar{8} = \bar{5}\,\bar{3}\,\bar{4}^{\,\bar{3}\bar{6}\bar{8}} \qquad (\text{Placement is } 1000)$$

IV Part = (–1) (–14) + (–1) (18) + (–1) (240) + (–1) (–300) + (–14) (18) + (–14) (240) + (–14) (–300) + (18) (240) + (18) (–300) + (240) (–300) = 14 – 18 – 240 + 300 – 252 – 3360 + 4200 + 4320 – 5400 – 72000 = $\bar{7}\,\bar{2}\,\bar{4}\,\bar{3}\,\bar{6}$

$$\text{Reduced II Part} = \frac{\bar{7}\bar{2}\bar{4}\bar{3}\bar{6}}{(1)\,(1)}\left(\frac{20}{10}\right)\left(\frac{200}{100}\right)\left(\frac{4000}{1000}\right) = \bar{1}\bar{1}\bar{5}\bar{8}\bar{9}\bar{4}\bar{4} = \bar{1}\,\bar{1}\,\bar{5}\,\bar{8}^{\,\bar{9}\bar{7}\bar{6}} \qquad (\text{Placement is } 1000)$$

V Part = 5999 – 14 + 18 + 240 – 300 = 5943

$$\text{R.F} = \frac{5943}{1}\left(\frac{20}{10}\right)\left(\frac{200}{100}\right)\left(\frac{4000}{1000}\right)\left(\frac{6000}{1000}\right) = 570528$$

$$\text{Final Answer} = 570528 \Big/_{\bar{1}\bar{1}\bar{5}\bar{8}}\ \bar{9}\bar{7}\bar{6} \Big/_{\bar{5}\bar{3}\bar{4}}\ \bar{3}\bar{6}\bar{8} \Big/_{8185}\ 28 \Big/_{\bar{1}3\bar{4}}\ \bar{4}$$

$$= 57\bar{1}\,4\,\bar{3}\,\bar{1}\,\bar{5}\,0\,\bar{2}\,\bar{2}2\bar{4}\,\bar{1}4\bar{4} = 569368497815936$$

(6)Six number multiplication TB WB :

Consider six number multiplication as 110 x 204 x 38 x 46 x 2005 x 3999.

Given number	TB	WB	D/Ex	Multiplication factor for modified the working base to 4000 (MF)	Modified numbers	Modified D/Ex
110	100	100	+10	40	4400	400
204	100	200	+4	20	4080	80
38	10	40	–2	100	3800	–200
46	10	50	–4	80	3680	–320
2005	1000	2000	+5	2	4010	10
3999	1000	4000	–1	1	3999	–1

METHOD II

I Part $= (10)\,(4)\,(-2)\,(-4)\,(5)\,(-1) = \overline{1600}$ Placement is $\overline{160}\;{}^{\bar{0}}$ (w.r. to 10)

II Part $= (400)\,(80)\,(-200)\,(-320)\,(10) + (400)\,(80)\,(-200)\,(-320)\,(-1) + (400)\,(80)\,(-200)\,(10)\,(-1) + (400)\,(80)\,(-320)\,(10)\,(-1) + (80)\,(-200)\,(-320)\,(10)\,(-1) + (400)\,(-200)\,(-320)\,(10)\,(-1) = 18291200000$

Reduced II Part: $\dfrac{18291200000}{(1)\,(2)\,(20)\,(40)\,(80)}\left(\dfrac{40}{10}\right) = 571600.$ Placement is $57160\;{}^{0}$ (w.r. to 10)

III Part $= (400)\,(80)\,(-200)\,(-320) + (400)\,(80)\,(-200)\,(10) + (400)\,(80)\,(-200)\,(-1) + (400)\,(-200)\,(-320)\,(10) + (400)\,(-200)\,(10)\,(-1) + (400)\,(80)\,(-320)\,(10) + (400)\,(80)\,(-320)\,(-1) + (400)\,(-200)\,(-320)\,(-1) + (400)\,(-320)\,(10)\,(-1) + (400)\,(80)\,(10)\,(-1) + (80)\,(-200)\,(-320)\,(10) + (80)\,(-200)\,(-320)\,(-1) + (80)\,(-320)\,(10)\,(-1) + (80)\,(-200)\,(10)\,(-1) + (-200)\,(-320)\,(10)\,(-1) = 2176256000$

Reduced III Part $= \dfrac{2176256000}{(1)\,(2)\,(20)\,(40)}\left(\dfrac{40}{10}\right)\left(\dfrac{50}{10}\right)$ 27203200 Placement is $272032\;{}^{00}$ (w.r.to 100)

IV Part $= (400)\,(80)\,(-200) + (400)\,(80)\,(-320) + (400)\,(80)\,(10) + (400)\,(80)\,(-1) + (400)\,(-200)\,(-320) + (400)\,(-200)\,(10) + (400)\,(-200)\,(-1) + (400)\,(-320)\,(10) + (400)\,(-320)\,(-1) + (400)\,(10)\,(-1) + (80)\,(-200)\,(-320) + (80)\,(-200)\,(10) + (80)\,(-200)\,(-1) + (-200)\,(-320)\,(10) + (-200)\,(-320)\,(-1) + (--320)\,(10)\,(-1) + (-200)\,(10)\,(-1) + (80)\,(10)\,(-1) + (80)\,(-320)\,(10) + (80)\,(-320)\,(-1) = 12698000.$

Reduced IV Part $= \dfrac{12698000}{(1)\,(2)(20)}\left(\dfrac{40}{10}\right)\left(\dfrac{50}{10}\right)\left(\dfrac{100}{100}\right) = 6349000.$ Placement is $63490\;{}^{00}$ (w.r.to 100)

V Part $= (400)\,(80) + (400)\,(-200) + (400)\,(-320) + (400)\,(10) + (400)\,(-1) + (80)\,(-200) + (80)\,(-320) + (80)\,(10) + (80)\,(-1) + (-200)\,(-320) + (-200)\,(10) + (-200)\,(-1) + (-320)\,(10) + (-320)\,(-1) + (10)\,(-1) = \overline{153970}$

Reduced V Part $= \dfrac{\overline{153970}}{(1)(2)}\left(\dfrac{40}{10}\right)\left(\dfrac{50}{10}\right)\left(\dfrac{100}{100}\right)\left(\dfrac{200}{100}\right) = \overline{3079400}$ Placement is $\overline{3079}\;{}^{\overline{400}}$ (w.r.to 1000)

VI Part $= 3999 + 10 - 320 - 200 + 80 + 400 = 3969$

Reduced VI Part $= \left(\dfrac{3969}{1}\right)\left(\dfrac{40}{10}\right)\left(\dfrac{50}{10}\right)\left(\dfrac{100}{100}\right)\left(\dfrac{200}{100}\right)\left(\dfrac{2000}{1000}\right) = 317520$

$$\text{Final Answer} = \begin{array}{ccccccccccc} 317520 & / & & \overline{400} & / & & 00 & / & & 00 & / & & 0 & / & 0 \\ & / & \overline{3079} & & / & 63490 & & / & 272032 & & / & 57160 & & / & \overline{160} \end{array}$$

$$= \begin{array}{r} 317520 \\ \overline{3013} \\ \hline 31451\bar{3} \end{array} \Big/ \begin{array}{r} \overline{3079} \\ 66 \\ \hline \overline{3013} \end{array} \quad \begin{array}{r} \overline{400} \\ 216 \\ \hline \bar{2}16 \end{array} \Big/ \begin{array}{r} 63490 \\ 2726 \\ \hline 66216 \end{array} \quad \begin{array}{r} 00 \\ 03 \\ \hline 03 \end{array} \Big/ \begin{array}{r} 272032 \\ 571 \\ \hline 272603 \end{array} \quad \begin{array}{r} 00 \\ 5\bar{6} \\ \hline 5\bar{6} \end{array} \Big/ \begin{array}{r} 57160 \\ \overline{16} \\ \hline 5715\bar{6} \end{array}$$

$$= \quad 3\ 1\ 4\ 5\ 1\ \bar{3} \quad \bar{2}\ 1\ 6\ 0\ 3\ 5\ \bar{6}\ 0\ 0$$

$$= \quad 3\ 1\ 4\ 5\ 0\ 6 \quad 8\ 1\ 6\ 0\ 3\ 4\ 4\ 0\ 0$$

iii) A further simplified method I - ALPHA METHOD

It is also attempted to explore the possibility to obtain some common factor in the process of application of general formula with regards to working bases and placement. In our trial to arrive at the simplification, it is suggested that
(i) Slight reorientation of the formula results in simplification, as can been seen in the following way:

Let us consider a six number multiplication (kx + a) (ly + b) (mz +c) (nw +d) (ρu + e) qv + f). There is no change in the first part of the expression, abcdef.

The second part of the expression

$$\frac{kx}{\left(\frac{kx}{ly}\right)\left(\frac{kx}{mz}\right)\left(\frac{kx}{nw}\right)\left(\frac{kx}{\rho u}\right)\left(\frac{kx}{qv}\right)}[a'b'c'd'e' + a'b'c'd'f' + a'b'c'e'f'$$
$$+ a'b'd'e'f' + a'c'd'e'f' + b'c'd'e'f']$$

In the reduction factor of the II part if one cancels out one (kx) in the numerator and denominator, also by taking out the fractional parts (ly) (mz) (nw) (ρu) (qv) to the numerator and keeping all the other (kx) as it is the expression is further reduced to $\frac{(ly)(mz)(nw)(\rho u)(qv)}{(kx)^4}$.

Similarly the reduction factor of III part can be further reduced as $\frac{(ly)(mz)(nw)(\rho u)(qv)}{(kx)^3}$.

The reduction factor of IV part can be further reduced to $\frac{(ly)(mz)(nw)(\rho u)(qv)}{(kx)^2}$

The reduction factor of V part can be further reduced to $\frac{(ly)(mz)(nw)(\rho u)(qv)}{(kx)^1}$

The reduction factor of VI part i.e., the last part can be reduced to $\frac{(ly)(mz)(nw)(\rho u)(qv)}{(kx)^0 = 1}$

This shows that the numerator is a constant α divided by $(kx)^{n-m}$ where n is the number of multiplications i.e., 6 in this case and m is number of the part considered. and hence the revised reduction with this concept to II, III, IV, V and VI parts are $\frac{\alpha}{(kx)^4}, \frac{\alpha}{(kx)^3}, \frac{\alpha}{(kx)^2}, \frac{\alpha}{(kx)^1}, \frac{\alpha}{(kx)^0 = 1}$ respectively. Thus the calculations can be simplified. But these are to be multiplied by the corresponding $\Sigma\begin{pmatrix} n \\ c \\ n-1 \end{pmatrix}$, $\Sigma\begin{pmatrix} n \\ c \\ n-2 \end{pmatrix}$, $\Sigma\begin{pmatrix} n \\ c \\ n-3 \end{pmatrix}$, $\Sigma\begin{pmatrix} n \\ c \\ n-4 \end{pmatrix}$, products of modified D/Ex respectively and the last part is the multiplication of $\frac{\alpha}{(kx)^0 = 1}$ with the cross addition. The values of each part so obtained can be simply added to get the final result. One ca try with any equalization base in such case, α is accordingly evaluated and the problem worked out accordingly.

For example considering the specific six number multiplication: 110 x 204 x 38 x 46 x 2005 x 3999. equalizing to (kx) : α (ly) (mz) (nw) (kx) (qv)

Adding all the reduced parts we get

$$\begin{array}{r}
\bar{1}\bar{6}\bar{0}\bar{0} \\
5716000 \\
2720320000 \\
63490000000 \\
\bar{3}\bar{0}\bar{7}\bar{9}\bar{4}00000000 \\
\underline{317520000000000} \\
31451\bar{4}816035\bar{6}00 \\
\hline
\end{array} = 314506816034400$$

Where kx = 4000, ly = 2000, mz = 50

nw = 40, pu = 200, qv = 100.

Using the reduced parts one obtained

METHOD - II

The result can be obtained by an additional simplification carried out in the values of the parts obtained as above.

Each part value is considered for the same placement. Say for example 10 as per the same base the different parts can be read as follows:

I Part = $\overline{1}\,\overline{6}\,\overline{0}\,\overline{0}$ placement as per base 10 it is to be read as $\overline{1}\,\overline{6}\,\overline{0}^{\,0}$.

II Part = 5716000. If we divide this by 10 and place the result as per base 10 then it is. $571060^{\,0}$. Thus we obtain the result in tens.

III Part = 2720320000. If we divide this by 100 and place the result as per base 10 then it is $2720320^{\,0}$. Thus we obtain the result in hundreds

IV part = 63490000000. If we divide this by 1000 and place the result as per base 10 then it is $6349000^{\,0}$. The value is in thousands

V Part = $\overline{3}\,\overline{0}\,\overline{7}\,\overline{9}\,\overline{4}00000000$. If we divide this by 10000 and place the result as per base 10 then it is $\overline{3}\,\overline{0}\,\overline{7}\,\overline{9}\,\overline{4}000^{\,0}$. Thus it is in ten thousands

VI Part = 317520000000000 If we divide this by 100000

Then we get 3175200000. The values in the different parts can be bifurcated as per the placements as follows.

$$\begin{array}{ccccccccccc}
6^{th}\text{ part} & & 5^{th}\text{ part} & & 4^{th}\text{ part} & & 3^{rd}\text{ part} & & 2^{nd}\text{ part} & & 1^{st}\text{ part}\\
3175200000 & / & 0 & / & 0 & / & 0 & / & 0 & / & 0\\
 & / & \overline{3}\,\overline{0}\,\overline{7}\,\overline{9}\,\overline{4}000 & / & 6349000 & / & 2720320 & / & 57160 & / & \overline{1}\,\overline{6}0
\end{array}$$

$$\begin{array}{cccccccccccc}
 & & / & 0 & / & 0 & / & 0 & / & 0 & / & \overline{6}\\
 & 3175200000 & / & 3 & / & 5 & / & \overline{6} & / & 0 & / & \\
 & \overline{3}\,\overline{0}\,\overline{1}\,\overline{3}2160 & / & \underline{3} & / & \underline{5} & / & \underline{\overline{6}} & / & \underline{0} & / & \\
= & & / & \overline{3}\,\overline{0}\,\overline{7}\,\overline{9}\,\overline{4}\;000 & / & 6349000 & / & 2720320 & / & 57160 & / & \\
 & & / & 662\,160 & / & 272603 & / & 5715 & / & \overline{1}\,\overline{6} & / & \\
\hline
 & 31\overline{4}51\overline{3}\,\overline{2}160 & / & \overline{3}\,\overline{0}\,\overline{1}\,\overline{3}\,\overline{2}160 & / & 6621603 & / & 2726035 & / & 5715\overline{6} & / &
\end{array}$$

$$=31451\bar{3}\bar{2}160 \quad /3 \quad /5 \quad /\bar{6} \quad /0 \quad /0$$

$$= 314506816034400$$

Let us consider an example of five number multiplication as 5999 x 5986 x 4012 x 208 x 19 :

In this problem let kx = 6000, ly = 6000, mz = 4000, nw = 200 pu = 20

Equalizing to (kx) 6000 ; $\alpha = (ly)\,(mz)\,(nw)\,(pu) = 96 \times 10^{9}$

I Part = $\bar{1}\,\bar{3}\,\bar{4}\,\bar{4}$

$$\text{II Part} = \left(\frac{\alpha}{(kx)^3}\right)(18416880) = 8185280$$

$$\text{III Part} = \left(\frac{\alpha}{(kx)^2}\right)(\bar{2}\,\bar{0}\,\bar{0}\,\bar{3}\,\bar{8}\bar{8}) = \bar{5}\,\bar{3}\,\bar{4}\,\bar{3}\,\bar{6}\,\bar{8}\ \ \bar{0}\,\bar{0}\,\bar{0}$$

$$\text{IV Part} = \left(\frac{\alpha}{(kx)^1}\right)(\bar{7}\,\bar{2}\,\bar{4}\,\bar{3}\,\bar{6}) = \bar{1}\,\bar{1}\,\bar{5}\,\bar{8}\,\bar{9}\,\bar{7}\,\bar{6}\,\bar{0}\,\bar{0}\,\bar{0}\,\bar{0}\,\bar{0}\,\bar{0}$$

$$\text{V Part} = \left(\frac{\alpha}{(kx)^0}\right)(5943) = 570528000000000$$

Adding all these parts we get the final answer as : 569368497815936

(iv) SUMMARY:

The proofs are developed in different ways, in case TB = WB or TB ≠ WB, base being represented as x, x^2, x^3 etc depending on the value (where x = 10) :

(1) The numbers are written in terms of working bases as ax or ax^2 or ax^3, the D/Ex with reference to WB. The multiplication is simplified in having specific parts, two for two number multiplication, three for three number multiplication etc.

(2) A second model proof is more general than the above in considering each number as kx + D/Ex, ly + D/Ex, mz + D/Ex, where x, y, z are theoretical bases and kx, ly, mz are working bases, the D/Ex being with reference to working base.

(3) The above method is further simplified using a common multiplication factor for all the parts excepting the first part. The common multiplier is obtained by multiplying all the working bases excepting the equalized base. This method is called Alpha method.

In all these cases one can make an attempt to obtain the result by the following procedures.

(a) A master table is to be prepared with different columns (i) the numbers to be multiplied (ii) theoretical base of each number (iii) working base for each number (iv) D/Ex, from working base of the number (v) multiplication factor $\left(MF = \frac{\text{Equalized WB}}{\text{Individual WB}} \right)$ (vi) modified numbers (vii) modified D/Ex.

(b) Equalization to any working base can be considered.

(c) The answer can be obtained by the corresponding substitution of the values in the corresponding expressions from the algebraic expansion in the proof. This gives the direct value of the answer (Method I).

(d) The answer can be written in terms of different parts as many as the number in the multiplication. For example two parts in two number multiplication and three parts in three number multiplication and so on. Here the placement with reference to the theoretical base is also considered, (Method II).

(e) Through Alpha method (Method III).

(f) A combination of groups, base and series multiplication by reducing the problem to a two column series multiplication method (Method IV).

A good choice is left to the reader for the placement of the allowed digits in the parts and the digits which are to be carried to the next part. This can be done by choice of the placement as per the choice of the theoretical base as such one can consider any of the theoretical bases as a matter of fact, to obtain any stage of the part.

However if the base considered is 1000 for any part then the placement in that part is allowed for three digits (i.e., one less the number of digits in the base). The remaining digits if any are to be carried over to the next higher part. For the last part the entire value is placed as it is. The placement in any part is considered only after reduction, which is explained in each problem.

Thus the multiplication can be carried out using also any of the options for equalization and placement.

V Extension of base method:

There is still another method, for multiplication of series of numbers, which makes use of a combination of group concept as a unit, a base generally different for each number followed by the method of series multiplication. This can also be written in a polynomial form. This can be exemplified with the following examples. The base may be powers of ten or multiplies of powers of ten.

For example:

(a) 99 x 43

The base for 99 is 100 and for 43, 4 x 10 = 40.

The D/Ex w.r.to the bases are –1 and +3 respectively, for the two numbers.

For the two numbers x=10

2nd Col.	1st Col.
Base	D/Ex
(100)	(– 1)
(40)	(+3)

In terms of polynomials x = 10

$$\begin{array}{l} x^2 + 0x - 1 \\ 0 + 4x + 3 \\ \hline 4x^3 + 3x^2 - 4x - 3 = 4000 + 300 - 40 - 3 = 4257 \end{array}$$

Apply series multiplication method. Applying the group concept in the first column where the groups indicated in the brackets are in units and in the second column in 10's. The series multiplication gives values as follows.

Step (1)

(100) – (1)

(40) (3) = –3

Step (2)

(100) (–1)

(40) (3) = 300 – 40 = 260

Step (3)

(100) – (1)

(40) (3) = 4000

The final answer is obtained by adding the values in the steps as –3 + 260 + 4000 = 4257.

The procedure is same for many number multiplications. By writing down the numbers into two columns, the first column being the D/Ex and the second column, representing the bases of each number in the multiplication.

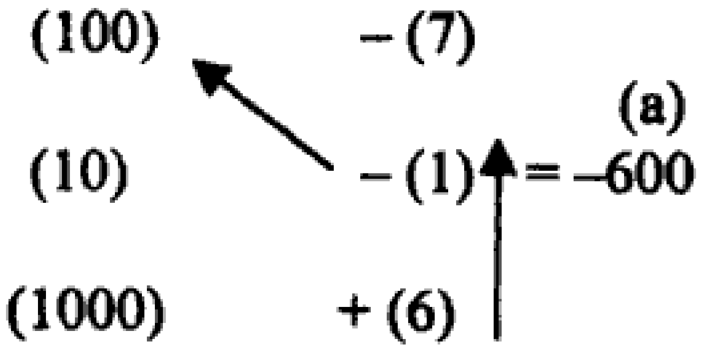
(b) Let us consider 93 x 9 x 1006 (100, 10, 1000 being the bases correspondingly)

$$\begin{aligned} 93 &= 100 - 7 \\ 9 &= 10 - 1 \\ 1006 &= 1000 + 6 \end{aligned}$$

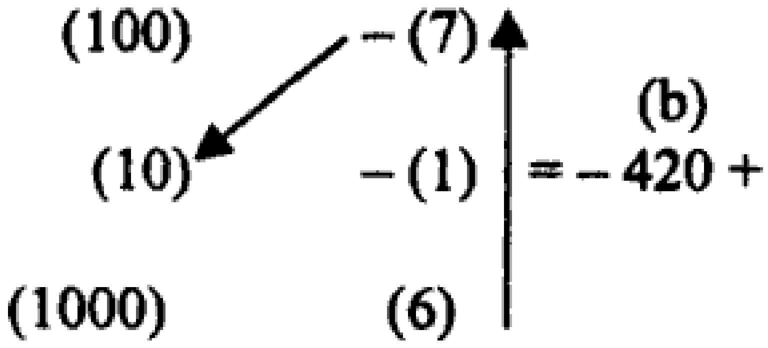
Step (i)

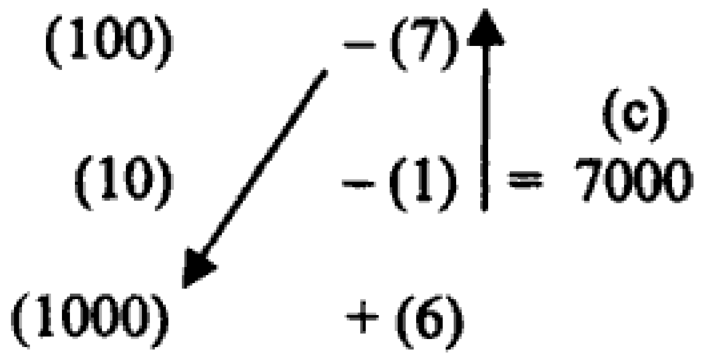
Base	D/Ex		Converting into polynomial
(100)	–(7)		$x^2 + 0x - 7$
(10)	–(1)	= 42	$0x^2 + x - 1$
(1000)	+(6)		$x^3 + 0x^2 + 0x + 6$

Step (ii)



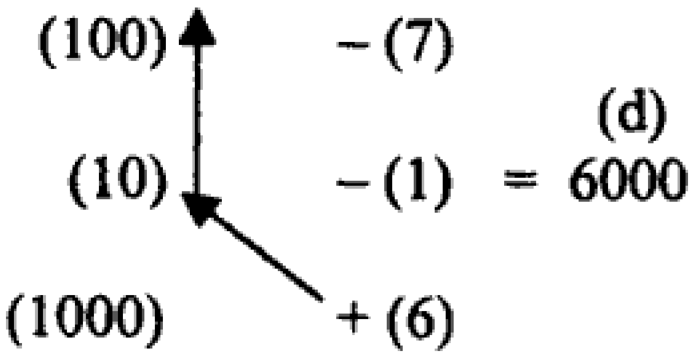
+



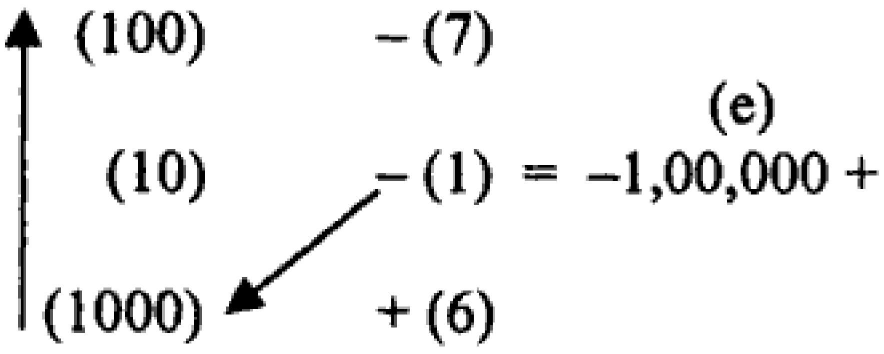


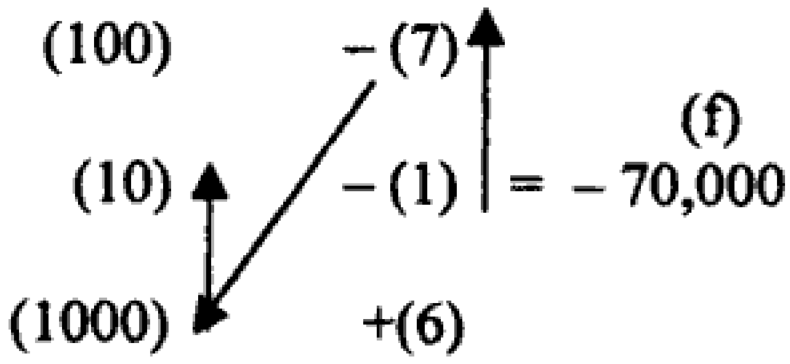
a + b + c = – 600 – 420 + 7000 = 5980

Step (iii)



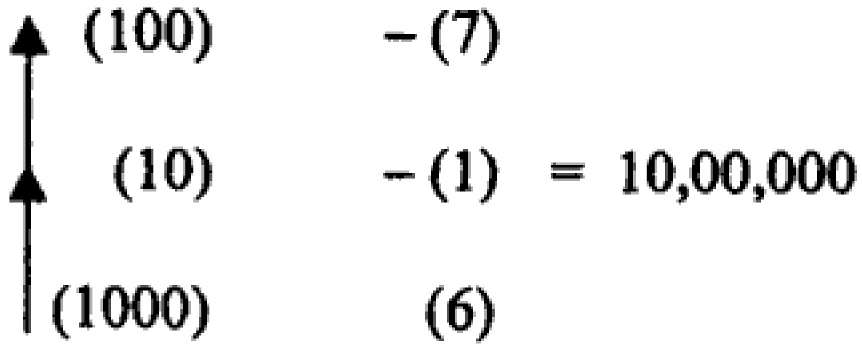
+





d + c + f = 6000 – 1,00,000 – 70,000 = – 1,64,000

Step (iv)



Final Answer = 42
+ 5980
+1000000
1006022
– 164000
842022

(c) Consider a six number multiplication: 110 x 204 x 38 x 46 x 2005 x 3999.

The bases are 100, 200, 40, 50, 2000 and 4000 respectively.

The D/Ex are + 10, + 4, –2, –4, +5 and –1 respectively. Writing down the problem into two columns, base and D/Ex.

Base	**D/Ex.**	
(100)	+(10)	$x^2 + 0x + 10$
(200)	+ (4)	$2x^2 + 0x + 4$
(40)	– (2)	$0x^2 + 4x - 2$
(50)	– (4)	$0x^2 + 5x - 4$
(2000)	+ (5)	$2x^3 + 0x^2 + 0x + 5$
(4000)	– (1)	$4x^3 + 0x^2 + 0x - 1$

The problem now is a series multiplication consisting of two columns. According to Vedic method, the series multiplication for two column is already explained. (Ref. Page No: 37). Here the number is shown in two columns as groups. The multiplication is to be carried out considering the group which is explained under the topic "Multiplication with groups".

Step 1 Urdhva of first column groups: i.e., (10) (+4) (–2) (–4) (+5) (-1) = -16×10^2.

Step 2 To Multiply the $\begin{pmatrix} 6 & \\ c & \text{Products of D/Ex} \\ 5 & \end{pmatrix}$ with the second column base which is not lying in the horizontal row of the considered D/Exs. This gives rise to six combinations. The sum of all such combinations = 5716 x 103.

Step 3: To consider products of $\begin{pmatrix} 6 & \\ c & \text{D/Ex} \\ 4 & \end{pmatrix}$ (15 combinations) with the allowed two groups of the second column which do not lie in the horizontal rows of the considered D/Exs. The value is 272032×10^4

Step 4: Similarly to work out all Products of $\begin{pmatrix} 6 & \\ c & \text{D/Ex} \\ 3 & \end{pmatrix}$ 20 combinations of the first column with allowed three groups of the second column, which do not lie in the horizontal rows of the D/Ex considered in the first column. This gives the value = 6349×10^7.

Step 5: Extending this to the products of $\begin{pmatrix} 6 & \\ c & \text{D/Ex} \\ 2 & \end{pmatrix}$ combinations (15) with the corresponding allowed four groups in the second column, one gets the value = -30794×10^8.

Step 6: The products of $\begin{pmatrix} 6 & \\ c & D/Ex \\ 1 & \end{pmatrix}$ combinations (6) with the corresponding allowed five groups in the second column, gives the value = -248×10^{10}.

Step 7: Urdhva of the second column groups gives the value 320×10^{12}.

Finally adding all steps, the answer = 314506816034400

The values of various combinations D/Ex after multiplying with the corresponding allowed bases in the different steps are as follows:

Step (1) = (10) (+4) (–2) (–4) (5) (–1) = –1600 = -16×10^2 (product of all D/Ex)

Step (2) = obtained from 6 combinations.
= (10) (+4) (–2) (–4) (5) (4000) + (10) (+4) (–2) (–4) (–1) (2000) + (10) (+4) (–2) (+5) (–1) (50) + (10) (+4) (–4) (5) (–1) (40) + (10) (–2) (–4) (5) (–1) (200) + (4) (–2) (–4) (5) (–1) (100)
= 5716000 = 5716×10^{3x}

Step (3) Obtained from 15 combinations are (10) (4) (–2) (–4) (2000) (40) + (10) (4) (–2) (5) (50) (4000) + (10) (4) (–2) (–1) (50) (2000) + (10) (4) (–4) (5) (40) (2000) + (10) (4) (–4) (–1) (40) (4000) + (10) (4) (5) (–1) (40) (50) + (10) (–2) (–4) (5) (200) (4000) + (10) (–2) (–4) (–1) (200) (2000) + (10) (–2) (5) (–1) (200) (50) + (10) (–4) (5) (–1) (200) (40) + (4) (–2) (–4) (5) (100) (4000) + (4) (–2) (–4) (–1) (100) (2000) + (4) (–2) (5) (–1) (100) (50) + (4) (–4) (5) (–1) (100) (40) + (–2) (–4) (5) (–1) (100) (2000)
= 272032×10^4

Step (4): Obtained from 20 combinations.
(100) (200) (40) (–4) (5) (–1) + (100) (200) (50) (–2) (5) (–1) + (100) (200) (2000) (–2) (–4) (–1) + (100) (200) (4000) (–2) (–4) (5) + (100) (40) (50) (4) (+5) (–1) + (100) (40) (2000) (4) (–4) (–1) + (100) (40) (4000) (4) (–4) (5) + (100) (50) (2000) (4) (–2) (–1) + (100) (50) (4000) (4) (–2) (+5) + (100) (2000) (4000) (4) (–2) (–4) + (200) (40) (50) (10) (5) (–1) + (200) (40) (2000) (10) (–4) (–1) + (200) (40) (4000) (10) (–4) (5) + (200) (50) (2000) (10) (–4) (–1) + (200) (50) (4000) (10) (–2) (5) + (200) (2000) (4000) (10) (–2) (–4) + (40) (50) (2000) (10) (4) (–1) + (40) (50) (4000) (10) (4) (5) + (40) (2000) (4000) (10) (4) (–4) + (50) (2000) (4000) (10) (4) (–2)
= 6349×10^7

Step (5): Obtained from 15 combinations;

(10) (4) (40) (50) (2000) (4000) + (10) (−2) (200) (50) (2000) (4000)
+ (10) (−4) (200) (40) (2000) (4000) + (10) (5) (200) (40) (50) (4000)
+ (10) (−1) (200) (40) (50) (2000) + (4) (−2) (100) (50) (2000) (4000)
+ (4) (−4) (100) (40) (2000) (4000) + (4) (5) (100) (40) (50) (4000)
+ (4) (−1) (100) (40) (50) (2000) + (−2) (−4) (100) (200) (2000) (4000)
+ (−2) (+5) (100) (200) (50) (4000) + (−2) (−1) (100) (200) (50) (2000)
+ (−4) (5) (100) (200) (40) (4000) + (−4) (−1) (100) (200) (40) (2000)
+ (5) (−1) (100) (200) (40) (50)
$= -30794 \times 10^{8}$

Step (6): Obtained from 6 combinations.

(10) (200) (40) (50) (2000) (4000) + (4) (100) (40) (50) (2000) (4000)
+ (−2) (100) (200) (50) (2000) (4000) + (−4) (100) (200) (40) (2000) (4000)
+ (5) (100) (200) (40) (50) (4000) + (−1) (100) (200) (40) (50) (2000)
$= -248 \times 10^{10}$

Step (7): Urdhva of the second column (100) (200) (40) (50) (2000) (4000) $= 32 \times 10^{13}$

Final Answer $= -16 \times 10^{2} + 5716 \times 10^{3} + 272032 \times 10^{4} + 6349 \times 10^{7} - 30794 \times 10^{8}$
$- 248 \times 10^{10} + 32 \times 10^{13} = 314506816034400$

vi) Upasutram Antayor Dasakepi, together with Ekadhikena Purvena (V.M):

Special Case:

Multiplication of two numbers whose last digits[*] together total to 10 and powers of 10 and whose previous part is exactly the same:

The modus operandi of this Upasutram Antyayor Dasakepi is as follows

One has to identify from left side the common part in both the numbers and the remaining part of the numbers when added together giving rise to 10 or powers of 10. This part is called Antyayor Dasakepi.

In such cases the operation of the sutram consists of two parts. The first part is the mere multiplication of the two numbers whose total is 10 or powers of 10. The second part is the multiplication of the common number by its Ekadhika. In the first part of the answer the provision is shown as per the multiplication of the actual number of digits taking zeros also into consideration.

1) 38×32

38 First part $= 8 \times 2 = 16$

32 second part$=3 \times 4 = 12$

12/16 Ans.: 1216

2) 67×63

67 First part $= 7 \times 3 = 21$

63 second part$=6 \times 7 =42$

42/21 Ans.: 4221

Proof: The given numbers are $(kx + a)(kx + b)$ where x is the base, $a + b = x$. Here $k = 3$, $x = 10$, $a = 8$, $b = 2$. $(kx + a)(kx + b) = k^2x^2 + akx + bkx + ab = k^2x^2 + kx(a + b) + ab = k(k + 1)x^2 + ab$

3) 281×219

$81 + 19 = 100$

281 First part. $= 81 \times 19 = 1539$

219 Second part $= 2 \times 3 = 6$

6/ 1539 Ans.: 61539

4) 396×304

$96 + 04 = 100$

396 First Part $= 96 \times 04 = 0384$

304 Second Part$=3 \times 4 = 12$

12 / 0384 Ans.: 120384

(5) 3086 x 3914

086 + 914 = 1000

3086 First part = 086 x 914 = 078604

3914 Second part = 3 x 4 = 12

12/ 078604 Ans: 12078604

6) 4008 x 4992

088 + 992 = 1000

4008 First part = 008 x 992 = 007936

4992 Second part = 4 x 5 = 20

20/ 007936 Ans: 20007936

---

[*] Digits in Units place

vii) Ekanyunena Purvena as corollary to Nikhilam Sutram (V.M.):

Special Case:

Multiplication wherein the multiplier digits consist entirely of nines:

Another special type of multiplication, which employs the sub-sutram Ekanyunena Purvena and the modus operandi, is as follows:

1. The multiplication is being represented as: multiplicand × multiplier.
2. The multiplicand is Purva and multiplier signifies the Apara.

Case a) Multiplicand and multiplier in 9's consist of same number of digits.

This multiplication can be done by applying the sutram Ekanyunena Purvena. The modus operandi is as follows.

The result consists of two parts. The second part is derived by applying Ekanyunena Sutram meaning one less than the multiplicand. First part is its complement of the multiplier. Examples are as follows:

i) 6 × 9 = 5 / 4 Ans.: = 54
Second part = 6 – 1 = 5 (Ekanyunena Purvena)
First part = 9 – 5 = 4 (Complement of the modified purva from 9)

ii) 32 × 99 = 31 / 68 Ans.: = 3168
Second part = 32 – 1 = 31
First part. = 99 – 31 = 68

iii) 763 × 999 = 762 / 237 Ans.: = 762237
Second part = 763 – 1 = 762
First part = 999 – 762 = 237

iv) 2345678 × 9999999
= 2345677 / 7654322
= 23456777654322

Caseb) Multiplicand consists of a smaller number of digits than the multiplier (in 9's): Even when the multiplicand consists of smaller number of digits than the multiplier, the same procedure can be applied. Examples are given below:

i) 5 × 99 = 4 /95
Ans. = 495
ii) 65 × 999 = 64 / 935
Ans. = 64935
iii) 57 × 99999999 = 56 / 99999943
Ans. = 5699999943

Case c) Multiplier (in 9's) contains a smaller number of digits than the multiplicand:

First one has to partition the multiplicand into two portions as Left hand and Right hand such that right hand side consists of as many digits as the multiplier. Apply Ekanyunena for the multiplicand, i.e., subtracting one from it and further subtract the left – hand portion of the partition. Now the left – hand side of the result is completed. For the Right hand side of the product apply Nikhilam for the Right Hand portion of the partition. Specific examples are shown in the following steps.

i) 16 × 9 = 14 / 4 Ans. = 144
Divide 16 into two parts
L.H.P → 1/6 ← R.H.P
L.H.S. = (16 – 1) – 1 = 14
R.H.S. = value obtained by applying Nikhilam Sutram to 6 = 4

ii) 24 × 9 = 21 / 6 Ans. = 216
partition of the multiplicand is 2/4
LHS = (24 – 1) – 2 = 21
RHS = value obtained by applying Nikhilam Sutram to 4 = 6

iii) 84 × 9 = 75 / 6 Ans. = 756
partition of the multiplicand is 8 / 4
LHS = (84 – 1) – 8 = 75
RHS = value obtained by applying Nikhilam Sutram to 4 = 6

iv) 36789 × 999 = 36752 / 211 Ans. = 36752211
partition of the multiplicand is 36 /789
LHS = (36789 – 1) – 36 = 36752
RHS = value obtained by applying Nikhilam Sutram to 789 = 211

v) 9862453217 × 9999999
partition of the multiplicand is 986 / 2453217
LHS = (9862453217 – 1) – 986 = 9862452230
RHS = value obtained by applying Nikhilam Sutram to 2453217 = 7546783
Ans. = 98624522307546783.

CHAPTER – IV*

A preliminary attempt is made to program many of the methods described in this lecture notes, using C and C++ language, It is quite interesting to note that successful results could be arrived at all the programs within limitation of the S/W and H/W. The programming is carried out for the following works of the lecture notes.

I. **RIGHT TO LEFT MULTIPLICATION**
II. **LEFT TO RIGHT MULTIPLICATION METHOD 2**
III. **STEP WISE RIGHT TO LEFT MULTIPLICATION**
IV. **STEP WISE LEFT TO RIGHT MULTIPLICATION METHOD 1**
V. **SERIES MULTIPLICATION OF TWO DIGITS**
VI. **MOVING MULTIPLICATION**
VII. **SUM OF PRODUCTS**
VIII. **VINCULUM RIGHT TO LEFT MULTIPLICATION**
IX. **VINCULUM LEFT TO RIGHT MULTIPLICATION**
X. **MULTIPLICATION OF POLYNOMIAL IN X**
XI. **MULTIPLICATION OF GENERAL PLACEMENT & BASE**
XII. **APPLYING DIRECT FORMULA**
XIII. **ALPHA METHOD**
XIV. **NUMBER IN TERMS OF BASE & D/EX — SERIES MULTIPLICATION**

The programming was carried out by Mr. Pillala Ashok Babu, and the details were verified by Prof. P.S. Avadhani–Computer Science Department, Andhra University. The authors are extremely grateful to Prof. P.S. Avadhani for his valuable suggestions made in the final outputs.

A suggestion by way of improvement or addition to the programs are welcome and will be received with due attention. We can say, definitely that this is the first time the Programmings are included in the lecturer notes of this nature.

****AS PER THE WISH OF THE PROGRAMMER MR. P. ASHOK BABU, THIS CHAPTER ON PROGRAMMING IS DEDICATED TO HIS PARENTS (SMT. P. RENU AND SRI. P. RAMBABU).***

I. RIGHT TO LEFT MULTIPLICATION.

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
class calc{                          //Class calc to include the method as single object
  int a[100];                        //array to store first number
  int b[100];                        //array to store second number
  int c[10000],p[100];               //arrays to store temporary & final result
  int ai,bi,ci,nc;                   //Variables used as index of arrays
public:
  calc(){ ai=bi=ci=nc=0; }           //constructor to assign 0 to all indeices
  void readnum1();                   //To accept multiplicand
  void readnum2();                   //To accept multiplier
  void eqliz();                      //Method used to equalize the no. of digits of both nos.
  void mult();                       //Method to multiply
  void disp();                       //Method to display the final results
};
void calc::eqliz(){                  //equlizing the size of both the numbers
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readnum1(){               //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else ai++;
   readnum1();
  }
  if(ch!='-')  a[i++]=(int)ch-48;
}
void calc::readnum2(){               //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else
    bi++;
   readnum2();
  }
  if(ch!='-')   b[i++]=(int)ch-48;
}
void calc::mult(){                   //multiplication of two number stored in array a & b, following the method.
  int i=-1,j=0,t=0,s;
  int c1,c2;
  while(j<bi){
    i++;
    s=0;
    if (i==ai){
      j++;i--;
    }
```

```
 for(c1=i,c2=j;c2<=c1;c1--,c2++)
   if(c1!=c2)
        s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
   else
         s+=a[c1]*b[c2];
    s+=t;
    c[ci++]=s%10;
    t=s/10;
    p[ci]=t;
 }
}
void calc::disp(){
 clrscr();
 cout<<"OUTPUT:\n\nMultiplication of Two Numbers (Right to Left)\n\n\n";
 for(int l=0;l<79;l++) cout<<"-";
 cout<<"\n First Number is  : "; for(int k=ai-1;k>=0;k--) cout<<a[k];
 cout<<"\n Second Number is : "; for(k=bi-1;k>=0;k--) cout<<b[k];
 cout<<endl;
 for(l=0;l<79;l++) cout<<"-";
 cout<<"\n\n The Final Result is \n";for(int j=ci-1,cno=1;c[j]==0;j--);
 if (!(j>=0)) {cout<<"0"; return;}
 for(int i=j;i>=0;i--){
   gotoxy(cno,13);cout<<c[i];gotoxy(cno+4,14);
   if(i>1)
      cout<<p[i-1];
   cno+=3;
 }
 gotoxy(1,16);  if(nc%2!=0) cout<<"-";
 for(i=j;i>=0;i--) cout<<c[i];
   cout<<endl;
}void main(){                                    //main method calling all the operations of the calc class
 clrscr();
 calc obj;
 cout<<"\nPlease Enter Only Digits\n";
 cout<<"\nEnter 1st Number \t"; obj.readnum1();
 cout<<"\nEnter 2nd Number \t"; obj.readnum2();
 obj.eqliz();  cout<<"\n";
 obj.mult();
 cout<<"\nThe Product is\t";  obj.disp();
 getch();
 return;
```

OUTPUT

```
UTPUT:
ultiplication of Two Numbers (Right to Left)


irst Number is     : 8721532
econd Number is : 5642791


he Final Result is

  9  2  1  3  7  8  2  2  7  5  8  1  2
  9  9  7  12 18 14 7  6  7  4  2  0

9213782275812
```

II. LEFT TO RIGHT MULTIPLICATION METHOD 2

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
class calc{                           //Class calc to include the method as single object
  int a[100];                         //array to store first number
  int b[100];                         //array to store second number
  int c[1000],d[1000];                //arrays to store temporary & final result
  int ai,bi,ci,di,nc;                 //Variables used as index of arrays
public:
  calc(){  ai=bi=di=ci=nc=0;  }       //constructor to assign 0 to all indeices
  void readnum1();                    //To accept multiplicand
  void readnum2();                    //To accept multiplier
  void eqliz();                       //Method used to equalize the no. of digits of both nos.
  void mult();                        //Method to multiply
  void disp();                        //Method to display the final results
};
void calc::eqliz(){
 {                                    //verifying the size of first number and equlizing
 for(int m=0,o=0,n=ai-1;n>=0;n--)
   if(a[n]==-40) o++;
 for(n=ai-1;n>0;n--)
   if (a[n]==-40)
     for(m=n;m>0;m--)
          a[m+1]=a[m-1];
 for(m=0,n=ai-(2*o)+1;n<=ai;n++,m++)
       a[m]=a[n];
 ai=ai-(2*o);
 }
 {                                    //verifying the size of second number and equlizing
 for(int m=0,o=0,n=bi-1;n>=0;n--)
   if(b[n]==-40) o++;
 for(n=bi-1;n>0;n--)
   if (b[n]==-40)
     for(m=n;m>0;m--)
          b[m+1]=b[m-1];
 for(m=0,n=bi-(2*o)+1;n<=bi;n++,m++)
       b[m]=b[n];
 bi=bi-(2*o);
 }
 if(ai>bi){                           //equlizing the size of both the numbers
   for(int i=bi;i<ai;i++) b[i]=0;
   bi=ai;
 }
 else if (bi>ai){
   for(int i=ai;i<bi;i++) a[i]=0;
   ai=bi;
 }
}
void calc::readnum1(){                //recursive function to read the number as chars and store it in reverse
 char ch;
 static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   ai++;
  readnum1();
 }
 if(ch!='-')
  a[i++]=(int)ch-48;
}
```

```
void calc::readnum2(){                    //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
    bi++;
   readnum2();
 }
 if(ch!='-')
 b[i++]=(int)ch-48;
}
void calc::mult(){                        //multiplication of two number stored in array a & b, following the method.
  int i=ai,j=bi-1,s;
  int c1,c2;
  d[di-1]=0;
 while(j>=0){
   i--;
   s=0;
   if (i==-1){
     j--;i++;
   }                                      // end of if (i==-1)
   for(c1=i,c2=j;c2>=c1;c1++,c2--)
     if(c1!=c2)
          s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
     else
          s+=a[c1]*b[c2];
    s+=(d[di-1]*10);
     c[ci++]=s/10;
     d[di++]=s%10;
 }                                        // end of while(j>=0)
 d[di]=0;
}
void calc::disp(){
 clrscr();
 cout<<"OUTPUT:\n\nMultiplication of Two Numbers (Left to Right)\n\n\n";
 for(int l=0;l<79;l++) cout<<"-";
  cout<<"\n";
 for(int n=bi-1;n>=0;n--) printf("%5d",/*cout<<*/a[n]);
  cout<<"\n";
 for(n=bi-1;n>=0;n--)  printf("%5d",/*cout<<*/b[n]);
 cout<<endl;
 for(l=0;l<79;l++) cout<<"-";
 cout<<endl;
 {for(int i=ci-1;i>=0;i--)
  if(c[i]>9){
    c[i-1]+=c[i]/10;
    c[i]%=10;
  }
 }
 int cno=0;
 for(int i=0,j=0;i<ci||j<di-2;i++,j++)
 { gotoxy(cno+=5,10);
   cout<<c[i];
  if(j<di-2){
   gotoxy(cno+2,12);
   cout<<d[j];
  }
 }
gotoxy(cno+3,10); cout<<"->Answer";
gotoxy(cno,12); cout<<"--->Carry";
cout<<endl;
for(l=0;l<79;l++) cout<<"-";
```

```
  cout<<endl;
  for(l=0;c[l]==0 && l<ci ;l++);
  if (l==ci) cout<<"0";
  else
  {if(nc%2!=0) cout<<"-";
  for(;l<ci;l++) cout<<c[l];}
}
void main(){                                    //main method calling all the operations of the calc class
  clrscr();
  calc obj;
  cout<<"\nPlease Enter Only Digits\n";
  cout<<"\nEnter 1st Number \t";obj.readnum1();
  cout<<"\nEnter 2nd Number \t";obj.readnum2();
  obj.eqliz();cout<<"\n";
  obj.mult();
  obj.disp();
  getch();
  return;
}
```

OUTPUT

```
Multiplication of Two Numbers (Left to Right)

--------------------------------------------------------
  8  7  2  1  5  3  2
  5  6  4  2  7  9  1
--------------------------------------------------------
  4  9  2  1  3  7  8  2  2  7  5  8  1  2 ->Answer

   0  3  4  1  9  4  5  6  0  1  6  1    --->Carry
--------------------------------------------------------
49213782275812
```

III. STEP WISE RIGHT TO LEFT MULTIPLICATION

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
//Class calc to include the method as single object
class calc{
  int a[100];                       //array to store first number
  int b[100];                       //array to store second number
  int c[10000];                     //arrays to store temporary & final result
  int ai,bi,ci,nc;                  //Variables used as index of arrays
public:
  calc(){  ai=bi=ci=nc=0;  }        //constructor to assign 0 to all indeices
  void readnum1();                  //To accept multiplicand
  void readnum2();                  //To accept multiplier
  void eqliz();                     //Method used to equalize the no. of digits of both nos.
  void mult();                      //Method to multiply
  void disp();                      //Method to display the final results
};
void calc::eqliz(){                 //equlizing the size of both the numbers
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readnum1(){              //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   ai++;
  readnum1();
 }
 if (ch!='-')
 a[i++]=(int)ch-48;
}
void calc::readnum2(){              //recursive function to read the number as chars and store it in reverse
 char ch;
 static int i=-1;
 if ((ch=getche())!=13){
  if (ch=='-') nc++;
  else
  bi++;
 readnum2();
 }
 if (ch!='-')
 b[i++]=(int)ch-48;
}
void calc::mult(){                  //multiplication of two number stored in array a & b, following the method.
 int i=-1,j=0,s;
 int c1,c2;
 while(j<bi){

  s=0;
```

```
   if (i==al){
     j++;i--;
   }
   for(c1=i,c2=j;c2<=c1;c1--,c2++)
     if(c1!=c2)
          s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
     else
          s+=a[c1]*b[c2];
      c[ci++]=s;
  }
  cout<<"The Series of Products Right To Left\n";
  for(j=ci-1;c[j]==0;j--);
  gotoxy(20,6);
   cout<<"^";
  int m=7;
  for(i=0;i<=j;i++){
   gotoxy(1,m++);
    printf("%5d",c[i]);//cout<<c[i]<<" ----- ";
  }  cout<<"\n\n";
  for(j=ci-1;c[j]==0;j--);
  m=7;
  for(i=0;i<=j;i++){
    gotoxy(7,m);
     printf("%5d",c[i]/10);
     //cout<<c[i]/10;
    gotoxy(12,m);
     printf("%5d",c[i]%10);
     //cout<<c[i]%10<<"\n";
    gotoxy(20,m++);
     cout<<"|";
     c[i+1]+=c[i]/10;
     c[i]=c[i]%10;
   }
  gotoxy(10,m);
  cout<<"----------";
}
void calc::disp(){
  for(int j=ci-1;c[j]==0;j--);
  if(!(j>=0)){cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
  for(int i=j;i>=0;i--)
    cout<<c[i];
    cout<<endl;
}
void main(){                                    //main method calling all the operations of the calc class
  clrscr();
  cout<<"OUTPUT:\tPlease enter only digits";
  cout<<"\nMultiplication Using Step Wise Right to Left\n";
  calc obj;
  cout<<"Enter 1st Number(Only Digits) \t";  obj.readnum1();
  cout<<"\nEnter 2nd Number(Only Digits) \t";  obj.readnum2();
  obj.eqliz();  cout<<"\n";
  obj.mult();
  cout<<"\nThe Product is\t";  obj.disp();
  getch();
  return;
}
```

OUTPUT

```
OUTPUT:  Please enter only digits
Multiplication Using Step Wise Right to Left
Enter 1st Number(Only Digits)  8721532
Enter 2nd Number(Only Digits)  5642791
The Series of Products Right To Left
                                ^
     2    0    2   |
    21    2    1   |
    46    4    8   |
    71    7    5   |
    60    6    7   |
    66    7    2   |
   135   14    2   |
   174   18    8   |
   109   12    7   |
    61    7    3   |
    84    9    1   |
    83    9    2   |
    40    4    9   |
          ---------
The Product is  49213782275812
```

IV. STEP WISE LEFT TO RIGHT MULTIPLICATION METHOD 1

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
class calc{                                   //Class calc to include the method as single object
  int a[100];                                 //array to store first number
  int b[100];                                 //array to store second number
  int c[1000];                                //arrays to store temporary & final result
  int ai,bi,ci,nc;                            //Variables used as index of arrays
public:
  calc(){ai=bi=ci=nc=0;c[0]=0;}               //constructor to assign 0 to all indeices
  void readnum1();                            //To accept multiplicand
  void readnum2();                            //To accept multiplier
  void eqliz();                               //Method used to equalize the no. of digits of both nos.
  void mult();                                //Method to multiply
  void disp();                                //Method to display the final results
};
void calc::eqliz(){                           //constructor to assign 0 to all indeices
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readnum1(){                        //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   ai++;
  readnum1();
 }
 if (ch!='-')
 a[i++]=(int)ch-48;
}
void calc::readnum2(){                        //recursive function to read the number as chars and store it in reverse
 char ch;
 static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   bi++;
  readnum2();
 }
 if (ch!='-')
 b[i++]=(int)ch-48;
}
void calc::mult(){                            //multiplication of two number stored in array a & b, following the method.
 {for(int i=0;i<ai;i++)
   if(a[i]!=0) break;
  for(int j=0;j<ai;j++)
   if(b[j]!=0) break;
 if(i>=ai || j>=bi)
```

```
   return;
   }
   int i=ai,j=bi-1,s;
   int c1,c2;
  while(j>0){
    i--;
    s=0;
    if (i==-1){
      j--;i++;
    }
    for(c1=i,c2=j;c2>=c1;c1++,c2--)
      if(c1!=c2)
           s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
      else
           s+=a[c1]*b[c2];
      c[ci++]=s;
  }
  cout<<"The Series of Products Right To Left\n";
  int rn=wherey();
  gotoxy(4,rn);
   cout<<"^";
  gotoxy(14,rn);
  cout<<"----------->";
  for(j=ci-1;c[j]==0;j--);
  for(i=0;i<=j;i++){
  gotoxy(4,++rn);
  printf("|%5d",c[i]);
  }
  cout<<endl;
   gotoxy(25,rn);
    cout<<"v";
 for(i=ci-1;i>0;i--){
   gotoxy(10,rn--);
    printf("%5d  %5d",c[i]/10,c[i]%10);
   gotoxy(25,rn);
    cout<<"|";
    c[i-1]+=c[i]/10;
    c[i]%=10;
 } gotoxy(10,rn--);
   printf("%5d  %5d",c[i]/10,c[i]%10);
}
void calc::disp(){
   gotoxy(1,25);
  for(int j=0;c[j]==0 && j<ci;j++);
 if(!(j<ci)){cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
  for(;j<ci;j++)     cout<<c[j];
}
void main(){                                    //main method calling all the operations of the calc class
 clrscr();
 cout<<"OUTPUT:\tStep wise Left To Right Multiplication-Method1";
 calc obj;
 cout<<"\nPlease Enter only digits";
 cout<<"\nEnter 1st Number \t"; obj.readnum1();
 cout<<"\nEnter 2nd Number \t";  obj.readnum2();
 obj.eqliz();  cout<<"\n";
 obj.mult();
 gotoxy(1,23);
```

```
 cout<<"\nThe Product is\t";  obj.disp();
 getch();
 return;
}
```

OUTPUT

```
OUTPUT: Step wise Left To Right Multiplication-Method1
Please Enter only digits
Enter 1st Number          8721532
Enter 2nd Number          5642791
The Series of Products Right To Left
     ^             ------->
     |  40    4   9     |
     |  83    9   2     |
     |  84    9   1     |
     |  61    7   3     |
     | 109   12   7     |
     | 174   18   8     |
     | 135   14   2     |
     |  66    7   2     |
     |  60    6   7     |
     |  71    7   5     |
     |  46    4   8     |
     |  21    2   1     |
     |   2    0   2     v

The Product Is
49213782275812
```

V. SERIES MULTIPLICATION OF TWO DIGITS

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
#include <string.h>
#include <math.h>
class calc{                        //Class calc to include the method as single object
  int a[100][100];                 //a two dim array to store all the two digited values
  long res[1000];                  // a single dim array to store results
  int b[100][100],bi[100];         //arrays to store the temporary results
public:
  int tns,tds;                     //variables to store no. of values
  int tp,nc;                       //variables to store no. of +ve values and -ve values
  calc(){                          //constructor to assign 0 to all variables and arrays
    tns=tds=nc=0;
    int i,j;
    for(i=0;i<1000;res[i++]=0);
    for(i=0;i<100;i++)
    { bi[i]=0;
     for(j=0;j<100;j++)
          b[i][j]=0;
    }
  }
  void readnums();                 //function to read the numbers
  void mult();                     //function for performing multiplication
  void disp();                     //function to display final results
};
void calc::readnums(){             //function to read the numbers
  char *str;
  cout<<"OUTPUT:"<<endl<<"Type \"#\" if no numbers are to be entered\n";
  do{
    cout<<endl<<"Enter any Number :";
    gets(str);
    if(str[0]=='-') {
     nc++;
     int i=0;
     while(str[i]!='\0')
        str[i++]=str[i+1];
    }
    if (stricmp(str,"#")==0) break;
    if (tds<strlen(str)) tds=strlen(str);
    for(int i=strlen(str),j;i<100;i++) a[tns][i]=0;
   for(i=strlen(str)-1,j=0;i>=0;i--)
     if (str[i]>=48 && str[i]<=57)
          a[tns][j++]=(int)str[i]-48;
     else{
          cout<<endl<<"Illeagle Value ReEnter";
          tns--;
          break;
     }
     tns++;
  }while(stricmp(str,"#")!=0);     //ends when entered '#' char
  return;
}
void calc::mult(){                 //function to multiple n numbers of two digits
  int n,i,j,k,r,l;
 n=pow(2,tns);
 for(i=n-1;i>0;i--){
   r=1;l=0;j=i;
  for(k=tns-1;k>=0;k--){
    if(j&1)
         r*=a[k][0];
    else{
         r*=a[k][1];
         l++;
    }
```

```
      j>>=1;
     }
     res[l]+=r;
      b[l][bi[l]++]=r;
   }
   cout<<endl;
   for(r=1,l=0,i=0;i<tns;i++){
     r*=a[i][1];
     l++;
   }
   res[l]=r;
    b[l][bi[l]++]=r;
   tp=l;
   for(i=0;i<tns;i++){
     res[i+1]+=res[i]/10;
     res[i]%=10;
   }
}
void calc::disp(){                              //function to display the results
  cout<<endl;
  for(int i=0;i<=tp;i++)
  {
      cout<<pow(10,i)<<"-->";
       for(int j=0;j<bi[i];j++)
            cout<<b[i][j]<<"   ";
      cout<<endl;
  }
  cout<<"\nThe Product Is:\t";
  for(i=tns*tds;res[i]==0;i--);
  cout<<endl;
  if(i<0){cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
  for(int j=tns;j>=0;j--)
     cout<<res[j];
  cout<<endl;
}
void main(){                                    //main method calling all the operations of the calc class
  clrscr();
  calc obj;
 obj.readnums();
 obj.mult();
 obj.disp();
 getch();
 return;
}
```

<u>OUTPUT</u>

```
OUTPUT:
Type "#" if no numbers are to be entered

Enter any Number :99
Enter any Number :99
Enter any Number :99
Enter any Number :99
Enter any Number :#


1-->6561
10-->6561 6561 6561 6561
100-->6561 6561 6561 6561 6561 6561
1000-->6561 6561 6561 6561
10000-->6561

The Product Is:
96059601
```

VI. MOVING MULTIPLICATION

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
class calc{                                 //Class calc to include the method as single object
  int a[100];                               //array to store first number
  int b[100];                               //array to store second number
  int c[10000];                             //arrays to store temporary & final result
  int ai,bi,ci,bd,nc;                       //Variables used as index of arrays
public:
  calc(){ai=bi=ci=bd=nc=0;}                 //constructor to assign 0 to all indeices
  void readnum1();                          //To accept multiplicand
  void readnum2();                          //To accept multiplier
  void eqliz();                             //Method used to equalize the no. of digits of both nos.
  void mult();                              //Method to multiply
  void disp();                              //Method to display the final results
};
void calc::eqliz(){                         //equlizing the size of both the numbers
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readnum1(){                      //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else
    ai++;
   readnum1();
  }
  if(ch!='-')
  a[i++]=(int)ch-48;
}
void calc::readnum2(){                      //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else
    bi++;
   readnum2();
  }
  if(ch!='-')
  b[i++]=(int)ch-48;
}
void calc::mult(){                          //multiplication of two number stored in array a & b, following the method.
   int cno=25;
  {
   clrscr();
   cout<<"OUTPUT:\nMOVING MULTIPLICATION METHOD\n";
   for(int i=ai-1;i>=0;i--){
```

```
    gotoxy(cno+=5,5);
     cout<<a[i];
   }
   cno=25;
   for(int j=bi-1;b[j]==0;j--,cno+=5);
   for(;j>=0;j--){
    bd++;
    gotoxy(cno+=5,7);
     cout<<b[j];
   }
 }
 gotoxy(1,8);
 for(int ln=0;ln<80;ln++) cout<<"-";
 int i=-1,j=0,t=0,s;
 int c1,c2,cnt=0;;
while(j<ai-1){
  i++;
  s=0;
  if (i==ai){
    j++;i--;
  }
  if(i==bi){
   for(int k=bi;k>j;k--)
         b[k]=b[k-1];
  j++;bi++;
  }
  for(c1=i,c2=j;c2<=c1;c1--,c2++)
   if(c1!=c2)
         s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
   else
         s+=a[c1]*b[c2];
   s+=t;
   c[ci++]=s%10;
   t=s/10;
   if(cnt>=bd && cnt<ai){
   gotoxy(1,7);clreol();
    static int cnn=25;
    cnn-=5;
    int cn=cnn;
    for(int j=bi-1;b[j]==0;j--,cn+=5);
   for(;j>=0;j--){
         gotoxy(cn+=5,7);
         cout<<b[j];
   }
   getch();
   }
   if(++cnt<=ai+bd){
   gotoxy(cno,9);
    cout<<c[ci-1];
   gotoxy(cno-2,11);
    cout<<t;
   cno-=5;}
   getch();
   //gotoxy(25,25); cout<<"Press Any Key To Continue...";getch();
}
gotoxy(25,25);
clreol();
gotoxy(24,24);
```

```
}
void calc::disp(){
  for(int j=cl-1;c[j]==0;j--);
  if(j<0) {cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
  for(int i=j;i>=0;i--)
    cout<<c[i];
    cout<<endl;
}
void main(){                              //main method calling all the operations of the calc class
  clrscr();
  calc obj;
  cout<<"\nPlease Don't use any other char like(Back Space)\n";
  cout<<"\nEnter 1st Number \t";  obj.readnum1();
  cout<<"\nEnter 2nd Number \t";  obj.readnum2();
  obj.eqliz();  cout<<"\n";
  obj.mult();
  cout<<"\nThe Product is\t";  obj.disp();
  getch();
  return;
```

OUTPUT

```
OUTPUT:
MOVING MULTIPLICATION METHOD

                1  2  3  4  5  6  7  8

                                  3  2

                                     6
                                   1
OUTPUT:
MOVING MULTIPLICATION METHOD

                1  2  3  4  5  6  7  8

                                  3  2
--------------------------------------
                                  9  6

                                3  1
OUTPUT:
MOVING MULTIPLICATION METHOD

                1  2  3  4  5  6  7  8

                               3  2
--------------------------------------
                               6  9  6

                             3  3  1
```

```
OUTPUT:
MOVING MULTIPLICATION METHOD

                    1   2   3   4   5   6   7   8

                                    3   2
--------------------------------------------------
                                    1   6   9   6

                                  3   3   3   1
OUTPUT:
MOVING MULTIPLICATION METHOD

                    1   2   3   4   5   6   7   8

                                3   2
--------------------------------------------------
                                6   1   6   9   6

                              2   3   3   3   1
OUTPUT:
MOVING MULTIPLICATION METHOD

                    1   2   3   4   5   6   7   8

                            3   2
--------------------------------------------------
                            0   6   1   6   9   6

                          2   2   3   3   3   1
OUTPUT:
MOVING MULTIPLICATION METHOD

                    1   2   3   4   5   6   7   8

                        3   2
--------------------------------------------------
                        5   0   6   1   6   9   6

                      1   2   2   3   3   3   1
OUTPUT:
MOVING MULTIPLICATION METHOD

                    1   2   3   4   5   6   7   8

                    3   2
--------------------------------------------------
                    9   5   0   6   1   6   9   6

                  0   1   2   2   3   3   3   1
```

```
OUTPUT:
MOVING MULTIPLICATION METHOD


                1  2  3  4  5  6  7  8

                3  2
__________________________________________
             3  9  5  0  6  1  6  9  6

            0  0  1  2  2  3  3  3  1

MOVING MULTIPLICATION METHOD


               1  2  3  4  5  6  7  8

               3  2
________________________________________
          0  3  9  5  0  6  1  6  9  6

         0  0  0  1  2  2  3  3  3  1

The Product is  395061696
```

VII. SUM OF PRODUCTS

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
#include <math.h>
class calc{                                   //Class calc to include the method as single object
  int a[100][100];                            //array to store numbers
  int c[100][100];                            //arrays to store temporary & final result
  int ai,aj,ci,cj;                            //Variables used as index of arrays
  int res[20],p[20],ri;                       //arrays to store temporary result
public:
  calc(){                                     //constructor to assign 0 to all indeices
    ai=-1;aj=ci=cj=0;
    for(int i=0;i<20;i++) res[i]=p[i]=0;
  }
  void readnums();                            //function to read numbers into array
  void mult();                                //Method to multiply
  void disp();                                //Method to display the final results
};
void calc::readnums(){                        //Function to read the number as chars
  char *str;
  cout<<"Enter The Expression as +/-(num1*num2)+/-(num3*num4)+/-...\n";
  cin>>str;
  int j,i=0,m=0;
  for(i=0;i<100;i++)
   for(j=0;j<100;j++)
     a[i][j]=-'#';
  i=0;
  int min=1;
  while(str[i]!='\0'){
    aj=0;
    ai++;
   while(str[i]!=')'){
     if(str[i]=='-') min=-1;
     if (m<aj) m=aj;
     if(str[i]=='*') {aj=0;ai++;min=1;}
      If(str[i]>=48 && str[i]<=57)
     a[ai][aj++]=((int)str[i]-48)*min;
      i++;
   }
    i++;
   if(str[i]=='-') min=-1;
  }
  if(m>aj) aj=m;
  for(i=0;i<=ai;i++){
   for(m=j=aj-1;a[i][j]==-'#';j--);
    for(;j>=0;a[i][m--]=a[i][j--]);
   for(;m>=0;a[i][m--]=0);
  }
  int t;
  for(i=0;i<=ai;i++)
   for(j=0,m=aj-1;j<m;j++,m--){
     t=a[i][j];
     a[i][j]=a[i][m];
     a[i][m]=t;
   }
```

```
void calc::mult(){                          //multiplication of two number stored in array a & b, following the method.
  int i,j,t=0,s;
  int c1,c2;
  int cnt=0,m=0;
  for(i=0;i<100;i++)
  for(j=0;j<100;j++)
   c[i][j]=0;
  while(cnt<ai){
   cj=0;i=-1;j=0;
   while(j<aj){
     i++;s=0;
     if (i==aj){j++;i--;}
     for(c1=i,c2=j;c2<=c1;c2++,c1--)
          if(c1!=c2)
            s+=(a[cnt][c1]*a[cnt+1][c2])+(a[cnt][c2]*a[cnt+1][c1]);
          else
            s+=a[cnt][c1]*a[cnt+1][c2];
          c[ci][cj++]=s;
          if(m<cj) m=cj;
   }
   cnt+=2; ci++;
  }
  int k;
   for(j=m-1,k=0;j>=0;j--,k++)
     for(i=ci-1;i>=0;i--)
          res[k]+=c[i][j];
          ri=k;
}
void calc::disp(){
    int cnt=0,i;
    int r=8,c;
    c=5;r++;
    while(cnt<=ai)
    {
     if(a[cnt][0]<0){
          gotoxy(c-2,r+1);cout<<"-";
     }
     else{
          gotoxy(c-2,r+1);cout<<"+";
     }
     for(i=aj-1;i>=0;i--)
     {
          gotoxy(c,r);
          if(a[cnt][i]<0) cout<<-a[cnt][i];else cout<<a[cnt][i];
          gotoxy(c,r+2);
          if(a[cnt+1][i]>=0) cout<<a[cnt+1][i];else cout<<-a[cnt+1][i];
          c+=4;
     }
     c+=5;
     cnt+=2;
    }
     cout<<endl;
     for(i=0;i<79;i++) cout<<"-";cout<<endl;
   for(i=ri-1;i>0;i--) {
     res[i-1]+=(res[i]/10);
     p[i]=(res[i]/10);
     res[i]%=10;
   }
```

```
    gotoxy(5,13);
    int rn=5,cn=13;
    cout<<endl;
    for(i=0;i<ri;i++) {gotoxy(rn+=4,cn);cout<<res[i];
    }
    cout<<endl;
    gotoxy(5,15);
    rn=5,cn=15;
    for(i=0;i<ri;i++){ gotoxy((rn+=4)-1,cn);cout<<p[i]; }
      cout<<endl;
      for(i=0;i<79;i++) cout<<"-";cout<<endl;
    for(int j=0;res[j]==0;j++);
    int flag=0;
    for(i=j;i<ri-1;i++)
      if (res[j]<0 && res[i+1]>0) flag=1;
      else if(res[j]>=0 && res[i+1]<0) flag=-1;
    if (flag==1){
     for(i=ri-1;i>=j;i--)
          if (res[i]>0){
            res[i]-=10;
            res[i-1]++;
          }
    }
    if (flag==-1){
     for(i=ri-1;i>=0;i--)
          if (res[i]<0){
            res[i]+=10;
            res[i-1]-=1;
          }
    }
           cout<<endl<<"The Final Result\n";
    flag=0;
    for(i=0;i<ri;i++) if(res[i]<0) flag=1;
    if(!flag)
     cout<<"+";
    else
      cout<<"-";
    for(i=0;i<ri;i++) cout<<" "<<abs(res[i]);
    getch();
}
void main(){                              //main method calling all the operations of the calc class
  clrscr();
  cout<<"OUTPUT:\n\t\tSUM OF PRODUCTS\n\n";
  calc obj;
 obj.readnums();
  cout<<"\n";
  obj.mult();
  obj.disp();
  getch();
  return;
}
```

```
OUTPUT:
          SUM OF PRODUCTS

Enter The Expression as +/-(num1*num2)+/-(num3*num4)+/-...
-(245*326)+(821*654)-(732*510)


  2  4  5     8  2  1     7  3  2
-          +           -
  3  2  6     6  5  4     5  1  0
--------------------------------------------------
     0  8  4  -2 -5 -6

   0  0  1  0  -2 -2
--------------------------------------------------

The Final Result
+ 0 8 3 7 4 4
```

VIII. VINCULUM RIGHT TO LEFT MULTIPLICATION

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
#include <math.h>
class calc{                                  //Class calc to include the method as single object
  int a[100];                                //array to store first number
  int b[100];                                //array to store second number
  int c[10000],p[100];                       //arrays to store temporary & final result
  int ai,bi,ci,nc;                           //Variables used as index of arrays
public:
  calc(){                                    //constructor to assign 0 to all indeices and all arrays
  for(ai=0;ai<100;a[ai]=b[ai]=0,ai++);
  ai=bi=ci=nc=0;
  }
  void readnum1();                           //To accept multiplicand
  void readnum2();                           //To accept multiplier
  void eqliz();                              //Method used to equalize the no. of digits of both nos.
  void numvenc(int *v,int l);                //Method to covert number to vinculum
  void vencnum();                            //Method to covert vinculum t number
  void mult();                               //Method to multiply
  void disp();                               //Method to display the final results
};
void calc::eqliz(){                          //equlizing the size of both the numbers
 if(ai>bi){
   for(int i=bi;i<ai;i++) b[i]=0;
   bi=ai;
 }
 else if (bi>ai){
   for(int i=ai;i<bi;i++) a[i]=0;
   ai=bi;
 }
}
void calc::readnum1(){                       //recursive function to read the number as chars and store it in reverse
 char ch;
 static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   ai++;
  readnum1();
 }
 if(ch!='-')
 a[i++]=(int)ch-48;
}
void calc::readnum2(){                       //recursive function to read the number as chars and store it in reverse
 char ch;
 static int i=-1;
 if ((ch=getche())!=13){
   if (ch=='-') nc++;
   else
   bi++;
  readnum2();
 }
 if(ch!='-')
 b[i++]=(int)ch-48;
}
```

```
 void calc::numvenc(int *v,int l){
   static int m=0;
   for(int i=0;i<l;i++){
        v[i+1]+=v[i]/10;
        v[i]=v[i]%10;
     if((v[i]%10)>5){
        v[i]=v[i]-10;
        v[i+1]++;
     }
   }
   cout<<"\n\nVinculum Of the "<<++m<<" Number : ";
   for(int bk=l+1;v[bk]==0;bk--);
   int cn,rn;
  for(i=bk;i>=0;i--){
   cn=wherex();
    rn=wherey();
    if (v[i]<0){
      gotoxy(cn,rn);
       cout<<-v[i];
      gotoxy(cn,rn-1);
       cout<<"-";
      gotoxy(cn+=1,rn);
    } else{
     gotoxy(cn,rn);
      cout<<v[i];
    }
   }
   cout<<endl;
 }
void calc::vencnum(){
  for(;c[ci-1]==0;ci--);
  cout<<endl;
  cout<<"\n\nThe Final Result in Vinculum Form : ";
  int cn,rn;
  for(int i=ci-1;i>=0;i--){
   cn=wherex();
    rn=wherey();
   if (c[i]<0){
     gotoxy(cn,rn);
      cout<<-c[i];
     gotoxy(cn,rn-1);
      cout<<"-";
     gotoxy(cn+=1,rn);
   } else{
    gotoxy(cn,rn);
     cout<<c[i];
   }
  }
  cout<<endl;
  for(i=0;i<ci;i++){
    if (c[i]<0){
       c[i]=10+c[i];
      c[i+1]--;
    }
  }
  c[i]=abs(c[i]);
  ci++;
}
```

```
void calc::mult(){                        //multiplication of two number stored in array a & b, following the method.
  numvenc(a,ai);                          //covert First number to vinculum
  numvenc(b,bi);                          //covert Second number to vinculum
  gotoxy(1,12);cout<<"Intermediate Results:";
  gotoxy(45,14);
  int cn=wherex();
  int m=wherey();
  gotoxy(45,16);
  int cn1=wherex()-1;
  int m1=wherey();
   int i=-1,j=0,t=0,s;
   int c1,c2;
  while(j<bi){
    i++;
    s=0;
    if (i==ai+2){
      j++;i--;
    }
    for(c1=i,c2=j;c2<=c1;c1--,c2++)
      if(c1!=c2)
            s+=(a[c1]*b[c2])+(a[c2]*b[c1]);

            s+=a[c1]*b[c2];
      s+=t;
      c[ci++]=s%10;
     t=s/10;
     p[ci]=t;
   {
   gotoxy(cn,m);
   if (c[ci-1]<0){
    gotoxy(cn,m);
     cout<<"   "<<-c[ci-1];
    gotoxy(cn,m-1);
     cout<<"   -";
    gotoxy(cn-=3,m);
   } else{
    gotoxy(cn,m);
     cout<<"   "<<c[ci-1];
    gotoxy(cn-=3,m);
   }}
   {
   gotoxy(cn1,m1);
   if (p[ci]<0){
    gotoxy(cn1,m1);
     cout<<"   "<<-p[ci];
    gotoxy(cn1,m1-1);
     cout<<"   -";
    gotoxy(cn1-=3,m1);
   } else{
    gotoxy(cn1,m1);
     cout<<"   "<<p[ci];
    gotoxy(cn1-=3,m1);
   }}
  }
  vencnum();                              //covert result from  vinculum to number
}
void calc::disp(){
  for(int j=ci-1;c[j]==0;j--);
```

```
  if(!(j>0)){cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
  for(int i=j;i>=0;i--)
    cout<<c[i];
   cout<<endl;
}
void main(){                              //main method calling all the operations of the calc class
 clrscr();
 cout<<"OUTPUT:\nMultiplication of Two Number from Right To Left - Vinculum\n\n";
 calc obj;
 cout<<"Enter 1st Number\t";  obj.readnum1();
 cout<<"\nEnter 2nd Number\t";  obj.readnum2();
 obj.eqliz();  cout<<"\n";
 obj.mult();
 cout<<"\nThe Product is\t";  obj.disp();
 getch();
 return;
}
```

OUTPUT

```
OUTPUT:
Multiplication of Two Number from Right To Left - Vinculum

Enter 1st Number     634869
Enter 2nd Number      721685


Vinculum Of the 1 Number : 1435131


                            -  --
Vinculum Of the 2 Number : 1322325
Intermediate Results:
          -   - -     -       - -
      0 1 6 6 1 9 7 6 6 3 4 3 3 5
             - -     -       -
     0 0 0 1 1 1 2 1 4 0 3 0 1 0


The Final Result in Vinculum Form : 1661976634335
The Product is  458175434265
```

IX. VINCULUM LEFT TO RIGHT MULTIPLICATION

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
#include <math.h>
class calc{                                   //Class calc to include the method as single object
  int a[100];                                 //array to store first number
  int b[100];                                 //array to store second number
  int c[1000],d[1000],e[1000];                //arrays to store temporary & final result
  int ai,bi,ci,di,ei,nc;                      //Variables used as index of arrays
public:
  calc(){                                     //constructor to assign 0 to all indeices and all arrays
    for(ai=0;ai<100;a[ai]=b[ai]=0,ai++);
    ai=bi=ei=nc=0;di=0;ci=1;
    c[0]=0;
  }
  void readnum1();                            //To accept multiplicand
  void readnum2();                            //To accept multiplier
  void eqliz();                               //Method used to equalize the no. of digits of both nos.
  void numvenc(int *v,int l);                 //Method to covert number to vinculum
  void vencnum();                             //Method to covert vinculum t number
  void mult();                                //Method to multiply
  void disp();                                //Method to display the final results
};
void calc::eqliz(){                           //equlizing the size of both the numbers
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readnum1(){                        //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else
    ai++;
   readnum1();
  }
  if (ch!='-')
  a[i++]=(int)ch-48;
}
void calc::readnum2(){                        //recursive function to read the number as chars and store it in reverse
  char ch;
  static int i=-1;
  if ((ch=getche())!=13){
    if (ch=='-') nc++;
    else
    bi++;
   readnum2();
  }
  if (ch!='-')
  b[i++]=(int)ch-48;
}
```

```
  void calc::numvenc(int *v,int l){
    static int m=0;
    for(int i=0;i<l;i++){
       v[i+1]+=v[i]/10;
       v[i]=v[i]%10;
      If((v[i]%10)>5){
         v[i]=v[i]-10;
         v[i+1]++;
      }
    } cout<<endl;
    cout<<"\nVinculum Of the "<<++m<<" Number : ";
   int cn=wherex();  int m=wherey();
    for(int bk=l+1;v[bk]==0;bk--);
   for(i=bk;i>=0;i--)
    {
     cn=wherex(); m=wherey();
     If (v[i]<0){
       gotoxy(cn,m); cout<<-v[i];
       gotoxy(cn,m-1);  cout<<"-";
       gotoxy(cn+=1,m);
     } else{
      gotoxy(cn,m);  cout<<v[i];
     }
    }
    cout<<endl;
  }
 void calc::vencnum(){
   for(;e[ei-1]==0;ei--);
      cout<<endl;
   cout<<"\nThe Final Result in Vinculum Form : ";
   int cn,m;
   for(int i=ei-1;i>0;i--){
   cn=wherex(); m=wherey();
    if (e[i]<0){
      gotoxy(cn,m);  cout<<-e[i];
      gotoxy(cn,m-1);  cout<<"-";
      gotoxy(cn+=1,m);
    } else{
     gotoxy(cn,m); cout<<e[i];
    }
   }
   cout<<endl;
   for(i=0;i<=ei;i++){
     if (e[i]<0){
       e[i]=10+e[i];
       e[i+1]--;
     }
   }
   e[i]=abs(e[i]);
   ei++;
}
void calc::mult(){                                    //multiplication of two number stored in array a & b, following the method.
  numvenc(a,ai);                                      //covert First number to vinculum
  numvenc(b,bi);                                      //covert Second number to vinculum
  gotoxy(1,12);cout<<"Intermediate Results:";
  gotoxy(5,14);
  int cn=wherex();
  int m=wherey();
```

```
  gotoxy(5,16);
  int cn1=wherex()-1;
  int rn1=wherey();
   int i=ai+1,j=bi+1,s;
   int c1,c2;
  while(j>=0){
    i--;  s=0;
    if (i==-1){ j--;i++; }
    for(c1=i,c2=j;c2>=c1;c1++,c2--)
      if(c1!=c2) s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
      else s+=a[c1]*b[c2];
      c[ci++]=s%10;
      d[di++]=s/10;
      if(j>=0)
      {
    {
    gotoxy(cn,rn);
    if (c[ci-1]<0){
     gotoxy(cn,rn);
      cout<<-c[ci-1];
     gotoxy(cn,rn-1);
      cout<<"-";
     gotoxy(cn+=1,rn);
    } else{
     gotoxy(cn++,rn);
     cout<<c[ci-1];
    }}
    {
   gotoxy(cn1,rn1);
   if (d[di-1]<0){
    gotoxy(cn1,rn1);
     cout<<-d[di-1];
    gotoxy(cn1,rn1-1);
     cout<<"-";
    gotoxy(cn1+=1,rn1);
   } else{
    gotoxy(cn1++,rn1);
    cout<<d[di-1];
   }}}
 }
  d[di]=0;
 for(i=ci-1;i>=1;){
  s=c[i]+d[i];
  e[ei++]=s%10;
  if (i>=1)
    c[--i]+=s/10;
  else --i;
  }
  vencnum();//covert result from  vinculum to number
}
void calc::disp(){
 for(;e[--ei]==0;);
 if(ei<0){cout<<"0";return;}
  if(nc%2!=0) cout<<"-";
 for(int i=ei;i>0;i--)
    cout<<e[i];
}
```

```
void main(){                          //main method calling all the operations of the calc class
  clrscr();
  cout<<"OUTPUT:\tPlease Enter Only Digits\nMultiplication of Two Number from Left To Right - Vinculum\n\n";
  calc obj;
  cout<<"Enter 1st Number \t"; obj.readnum1();
  cout<<"\nEnter 2nd Number \t"; obj.readnum2();
  obj.eqliz();  cout<<"\n";
  obj.mult();
  cout<<"\nThe Product is\t"; obj.disp();
  getch();
  return;
}
```

OUTPUT

```
OUTPUT: Please Enter Only Digits
Multiplication of Two Number from Left To Right - Vinculum

Enter 1st Number      634869
Enter 2nd Number      721685


Vinculum Of the 1 Number : 1435131
                             . ..
Vinculum Of the 2 Number : 1322325
Intermediate Results:
        .     .     .       . .
    0 1 7 7 0 1 6 0 6 0 4 4 3 5
           . .     .       .
   0 0 0 1 1 2 2 2 4 0 3 0 1 0
                                     . .   ..  ..
The Final Result in Vinculum Form : 1662184634335

The Product is  458175434265
```

X. MULTIPLICATION OF POLYNOMIAL IN X

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
class calc{                                  //Class calc to include the method as single object
  int a[100],pa[100];                        //array to store first polynomial and the power
  int b[100],pb[100];                        //array to store second polynomial and the power
  int c[10000],p[100];                       //array to store final polynomial and the power
  int ai,bi,ci;                              //Variables used as index of arrays
public:
  calc(){ ai=bi=ci=0;}                       //constructor to assign 0 to all indeices
  void readpoly1();                          //to accept first polynomial and the power
  void readpoly2();                          //to accept second polynomial and the power
  void eqliz();                              //to equlize the number of terms in the polynomial
  void mult();                               //function for multiplication
  void disp();                               //function for displaying the results
};
void calc::eqliz(){                          //equlizing the size of both the polynomials
  if(ai>bi){
    for(int i=bi;i<ai;i++) b[i]=0;
    bi=ai;
  }
  else if (bi>ai){
    for(int i=ai;i<bi;i++) a[i]=0;
    ai=bi;
  }
}
void calc::readpoly1(){                      //function to read first polynomial enter the coeff and its power
  char ch;
  int coef,power;
  printf("\nEnter Polynomial from Highest Degree [ num1 num2 y-stop/ncontinue ]:\n");
  while(1){
    printf("\nEnter Coeff Power End<y-Stop>:");
    scanf("%d %d %c",&coef,&power,&ch);
    pa[ai]=power;
     a[ai++]=coef;
    if (ch=='y') break;
  }
  for(int i=0,j=ai-1;i<ai/2;i++,j--){
   int t=a[i];
   a[i]=a[j];
   a[j]=t;
   t=pa[i];
   pa[i]=pa[j];
   pa[j]=t;
  }
}
void calc::readpoly2(){                      //function to read second polynomial enter the coeff and its power
  char ch;
  int coef,power;
  printf("\nEnter Polynomial from Highest Degree:\n");
  while(1){
    printf("\nEnter Coeff Power End<y-Stop>:");
    scanf("%d %d %c",&coef,&power,&ch);
    pb[bi]=power;
     b[bi++]=coef;
    if (ch=='y') break;
  }
```

```
    for(int i=0,j=bi-1;i<bi/2;i++,j--){
     int t=b[i];
     b[i]=b[j];
     b[j]=t;
     t=pb[i];
     pb[i]=pb[j];
     pb[j]=t;
    }
  }
  void calc::mult(){                              //method to perform multiplication using the procedure specified
   int i=-1,j=0,t=0,s,po;
   int c1,c2;
  while(j<bi){
    i++;
    s=0;
    if (i==ai){
      j++;i--;
    }
    for(c1=i,c2=j;c2<=c1;c1--,c2++)
     if(c1!=c2){
          s+=(a[c1]*b[c2])+(a[c2]*b[c1]);
          po=pa[c1]+pb[c2];
     }
     else{
          s+=a[c1]*b[c2];
          po=pa[c1]+pb[c2];
     }
     p[ci]=po;
      c[ci++]=s;
   }
  }
 void calc::disp(){
   clrscr();
   cout<<"OUTPUT:\n\nMultiplication of Two Numbers (Right to Left)\n\n\n";
   for(int l=0;l<79;l++) cout<<"-";
   cout<<"\n First Polynomial is  : ";
   for(int  k=ai-1;k>=0;k--) cout<<"+("<<a[k]<<"x^"<<pa[k]<<")";
  cout<<"\n Second Polynomial is : ";
   for(k=bi-1;k>=0;k--) cout<<"+("<<b[k]<<"x^"<<pb[k]<<")";
  cout<<endl;
  for(l=0;l<79;l++) cout<<"-";
  cout<<"\n\n The Final Result is :\n";
  for(int j=ci-1,cno=1;c[j]==0;j--);
  gotoxy(1,13);
  for(int i=j;i>=0;i--)
      cout<<"+("<<c[i]<<"x^"<<p[i]<<")";
  cout<<endl;
}
void main(){                                     //main method calling all the operations of the calc class
  clrscr();
  calc obj;
  cout<<"Enter First Polynomial \t"; obj.readpoly1();
  cout<<"\nEnter Second Polynomial\t"; obj.readpoly2();
  obj.eqliz();
  cout<<"\n";
  obj.mult();
  obj.disp();
  getch();
  return;
}
```

OUTPUT

```
Enter First Polynomial
Enter Polynomial from Highest Degree [ num1 num2 y-stop/ncontinue ]:

Enter Coeff Power End<y-Stop>:3 2 n

Enter Coeff Power End<y-Stop>:-2 1 n

Enter Coeff Power End<y-Stop>:-5 0 n

Enter Coeff Power End<y-Stop>:-3 -1 n

Enter Coeff Power End<y-Stop>:1 -2 y

Enter Second Polynomial
Enter Polynomial from Highest Degree:

Enter Coeff Power End<y-Stop>:0 2 n

Enter Coeff Power End<y-Stop>:0 1 n

Enter Coeff Power End<y-Stop>:7 0 n

Enter Coeff Power End<y-Stop>:-4 -1 n

Enter Coeff Power End<y-Stop>:0 -2 y

OUTPUT:
Multiplication of Two Numbers (Right to Left)


 First Polynomial is     : +(3x^2)+(-2x^1)+(-5x^0)+(-3x^-1)+(1x^-2)
 Second Polynomial is  : +(0x^2)+(0x^1)+(7x^0)+(-4x^-1)+(0x^-2)


The Final Result is :

+(21x^2)+(-26x^1)+(-27x^0)+(-1x^-1)+(19x^-2)+(-4x^-3)+(0x^-4)
```

XI. MULTIPLICATION OF GENERAL PLACEMENT & BASE

```
#include <iostream.h>
#include <conio.h>
#include <stdio.h>
#include <math.h>
#include <process.h>
typedef struct tab{                    //structure tab containing the tabular form of the data folding the following fields
 long nums;  long de;  long tb;  long wb;
 double mf;  double mno;  double mde;
 int bpl;
}tab;
void main(){
 clrscr();
 cout<<"OUTPUT:\nMultiplication using general placement and base\n";
 int n,i,j,eb,pos=0;                   //variables used as indecies
 int p[10][10],ps[10];                 //array used to store the placement
 int pd[10][10],pds[10];               //array used to store the temporary results
 long val=1,pl=0,res[100];             //array used to store the final results
 for(i=0;i<10;i++){                     //assign 0 to the elements in the array
   ps[i]=pds[i]=res[i]=0;
  for(j=0;j<10;j++){
     p[i][j]=0;pd[i][j]=0;
  }
 }
 double plval;                         //stores the placement value
 int flag=0;                           //flag is used to indicate whether the placement is correct
 tab t[100];                           //tab type variable
 cout<<"Enter The Maximum Number of Elements:\t";cin>>n;          //reading data into table
 cout<<"Enter The Equilizing Base :\t";cin>>eb;
 for(i=0;i<n;i++){
   cout<<"Enter Number               : "<<i+1<<"  ";
   cin>>t[i].nums;
   cout<<"Enter Theriotical Base for number : "<<i+1<<"  ";
   cin>>t[i].tb;
   cout<<"Enter Working Base for number     : "<<i+1<<"  ";
   cin>>t[i].wb;
   t[i].de=t[i].nums-t[i].wb;    t[i].mf=(double)eb/t[i].wb;
   t[i].mno=t[i].mf*t[i].nums;    t[i].mde=t[i].mf*t[i].de;
   t[i].bpl=0;
 }
  clrscr();
  //display the data in tablular form
  cout<<endl;
  for(i=0;i<79;i++) cout<<"-";
  cout<<endl;
 int row=(wherex()+2);
   gotoxy(1,row);   cout<<"Numbers";
   gotoxy(12,row);       cout<<"T.Base";
   gotoxy(24,row);   cout<<"W.Base";
   gotoxy(36,row);       cout<<"D/E";
   gotoxy(48,row);   cout<<"M.Factor";
   gotoxy(60,row);   cout<<"M.No";
   gotoxy(72,row);   cout<<"M.D/E";
  cout<<endl;
  for(i=0;i<79;i++) cout<<"-";
  cout<<endl;
  row+=2;
  for(i=0;i<n;i++,row++){
```

```
  gotoxy(1,row);cout<<t[i].nums;    gotoxy(12,row);cout<<t[i].tb;
  gotoxy(24,row);cout<<t[i].wb;    gotoxy(36,row);cout<<t[i].de;
  gotoxy(48,row);printf("%8.3f",t[i].mf);    gotoxy(60,row);printf("%8.3f",t[i].mno);//cout<<t[i].mno;
  gotoxy(72,row);printf("%8.3f",t[i].mde);//cout<<t[i].mde;
  cout<<endl;
 }
 for(i=0;i<79;i++) cout<<"-";
 {                                              //calculate the result of first part
 cout<<endl<<"Enter Placement for part : 1 ";cin>>pl;
 for(i=0;i<n;i++)    val*=t[i].de;
 for(i=0;i<n;i++)    if(pl==t[i].tb && t[i].bpl==0){
   t[i].bpl=1;
   flag=1;
   break;
  }
 if (flag==0) {cout<<"\n\n Sorry Placement Value u Entered is Wrong";exit(0);}
 int tmp=pl;
 for(i=0;tmp>0;tmp/=10,pos++);
 int pn=(val>0?1:-1);
 val*=pn;
 for(i=pos-2;i>=0;i--,val/=10)
  p[0][i]=pn*(val%10);
 ps[0]=pos-1;
 if(val>0){
  for(i=0;val>0;i++,val/=10)
   pd[0][i]=pn*val%10;
  pds[0]=i;
 }
 for(i=0;i<ps[0];i++) cout<<p[0][i];cout<<endl;
 for(i=0;i<pds[0];i++) cout<<pd[0][i];cout<<endl;
 }
 double fr=1,dn=1,sum;                          //calculate the results of all the middle parts
 {
 for(int i=1;i<n-1;i++){
  flag=0; sum=0;fr=dn=1;
  for(int c=1;c<pow(2,n)-1;c++){
   int tp=c,cnt=0;double prd=1;
    for(int c1=0;c1<n;c1++,tp>>=1) if(tp&1==1) cnt++;
        else prd*=t[c1].mde; if (cnt==i)    sum+=prd;
  }
  for(int fi=0;fi<n;fi++)
  if (t[fi].bpl==1) fr*=(double)((double)t[fi].wb/(double)t[fi].tb);
  for(int di=0;di<n;di++) if (t[di].bpl==0) dn*=t[di].mf;
 plval=(double)(sum/dn)*fr;    val=(double)plval;
 double rem=plval-val;
 double pv=(rem>0.0?1:-1);
 rem*=pv;
 if (rem>0.0 ) rem+=0.0000001;
 if(rem!=0.0 && !(rem>=1.0)){
   long x,y;
  x=y= (double) (pv*rem)*pl;
   x*=pv;
   for(di=0;x>0;di++,x/=10);
   for(fi=ps[i-1]-1;fi>=0;fi--,y/=10) p[i-1][--di]=p[i-1][fi]+y%10;
   for(;y>0;y/=10) p[i-1][--di]=y%10;
 }
 if(rem>=1.0){
```

```
    if (val!=(double)plval)
     if (((double)(plval-val))>0.0)  val++;
     else if (((double)(plval-val))<0.0)  val--;
    }
    cout<<endl<<"Enter Placement for part :"<< i+1<<" ";cin>>pl;
    for(fi=0;fi<n;fi++)
      if (pl==t[fi].tb && t[fi].bpl==0){
           t[fi].bpl=1;         flag=1;  break;
      }
   if (flag==0) {cout<<"\n\n Sorry Placement Value u Entered is Wrong";exit(0);}
    int tmp=pl;
    for(int ii=0,pos=0;tmp>0;tmp/=10,pos++);
   int pn=(val>0?1:-1);
   val*=pn;
   for(ii=pos-2;ii>=0;ii--,val/=10) p[i][ii]=pn*(val%10);
   ps[i]=pos-1;
   if(val!=0){
     for(ii=0;val!=0;ii++,val/=10)   pd[i][ii]=pn*val%10;
     pds[i]=ii;
   }
  for(int mn=0;mn<ps[i];mn++) cout<<p[i][mn];cout<<endl;
  for(mn=0;mn<pds[i];mn++) cout<<pd[i][mn];cout<<endl;
  }
  }
  {                                              //calculate the result of last part;
  plval=fr=dn=1;
  int tmp;
  plval=t[0].mno;
  for(i=1;i<n;i++)     plval+=t[i].mde;
  for(int fi=0;fi<n;fi++)
   if (t[fi].bpl==1) fr*=((double)t[fi].wb/(double)t[fi].tb);
 for(int di=0;di<n;di++)
   if (t[di].bpl==0) dn*=t[di].mf;
 plval=(plval/dn)*(fr);
 if (plval-(long)plval!=0.0) plval+=0.000001;
 val=(double)plval;
 double rem=plval-val;
  if(rem>0.0||rem<0.0) {
    long y = (double) rem*pl;
    for(fi=ps[n-2]-1;fi>=0;fi--,y/=10)        p[n-2][fi]+=y%10;
  }
 int pn=(val>0?1:-1);
 val*=pn;
 for(int ii=0;val>0;ii++,val/=10)    p[n-1][ii]=pn*(val%10);
 ps[n-1]=ii;
 }
 for(i=0,j=ps[n-1]-1;i<j;i++,j--){
 int temp=p[n-1][i];
 p[n-1][i]=p[n-1][j];
 p[n-1][j]=temp;
 }
 for(int mn=0;mn<ps[n-1];mn++) cout<<p[n-1][mn];cout<<endl;
 for(mn=0;mn<pds[n-1];mn++) cout<<pd[n-1][mn];cout<<endl;
 {                                               //calculating the final results
 for(j=0,i=1;i<n;i++,j++){
  int a=ps[i]-1,b=pds[j]-1;
  if((a-b)>=0)
    for(a-=b;a<ps[i] && b>=0;a++,b--)
```

```
            p[i][a]+=pd[j][b];
    else{
      for(int k=0;k<=a;k++){
            p[i][a-k]+=pd[j][k];
            if (k<a){
             if(abs(p[i][a-k])>9){
               p[i][(a-k)-1]+=p[i][a-k]/10;
                p[i][a-k]=p[i][a-k]%10;
             }
             }
            else{
             if(abs(p[i][a-k])>9){
               pd[i][0]+=p[i][a-k]/10;
               p[i][a-k]=p[i][a-k]%10;
             }
            }
      }
       for(int l=0;k<=b;k++,l++) pd[j+1][l]+=pd[j][k];
    }
    for(int k=0;k<b;k++){
     if(k<b-2){
           if (abs(pd[i][k])>9){
             pd[i][k+1]+=pd[i][k]/10;
             pd[i][k]=pd[i][k]%10;
           }
     }
     else{
           if (abs(pd[i][k])>9){
             //pds[i]++;
             pd[i][k+1]+=pd[i][k]/10;
             pd[i][k]=pd[i][k]%10;
           }
     }
   }
  }
  }  for(i=0;i<n;i++){                              //displaying the temporary results
   for(j=0;j<ps[i];j++) cout<<p[i][j]<<" ";
  cout<<endl;
   for(j=0;j<pds[i];j++) cout<<pd[i][j]<<" ";
  cout<<endl;
  }
  int k=0;
 for(j=n-1;j>=0;j--)
    for(i=0;i<ps[j];i++)      res[k++]=p[j][i];
 for(i=k-1;i>0;i--)
 if (res[i]>9){
   res[i-1]+=res[i]/10;
   res[i]=res[i]%10;
 }   for(i=k-1;i>=0;i--){
     if (res[i]<0){ res[i]=10+res[i];  res[i-1]--;
     }
   }
 cout<<"The Final Result :\n";
 for(i=0;res[i]==0;i++);
 for(;i<k;i++) cout<<res[i];  getch();
 return;
}
```

OUTPUT

```
OUTPUT:
Multiplication using general placement and base
Enter The Maximum Number of Elements:  3
Enter The Equilizing Base :    200
Enter Number                        : 1  41
Enter Theriotical Base for number : 1  10
Enter Working Base for number      : 1  40
Enter Number                        : 2  48
Enter Theriotical Base for number : 2  10
Enter Working Base for number      : 2  50
Enter Number                        : 3  199
Enter Theriotical Base for number : 3  100
Enter Working Base for number      : 3  200


Numbers    T.Base     W.Base     D/E      M.Factor    M.No       M.D/E
-----------------------------------------------------------------------
41         10         40         1        5.000       205.000    5.000
48         10         50         -2       4.000       192.000    -8.000
199        100        200        -1       1.000       199.000    -1.000
-----------------------------------------------------------------------
Enter Placement for part : 1 10
2


Enter Placement for part :2 10
-7
-3
3920



-7
-3
3 9 2 -3

The Final Result :
391632
```

XII. APPLYING DIRECT FORMULA

```
#include <iostream.h>
#include <conio.h>
#include <stdio.h>
#include <math.h>
#include <process.h>
typedef struct tab{                              //structure tab containing the tabular form of the data folding the following fie
  long nums;
  long tb;
  long wb;
  long de;
  double mf;
  double mno;
  double mde;
}tab;
void main(){
  clrscr();
  cout<<"OUTPUT:\n\tMultiplication using general Formula\n";
  int n,i,j,eb;//variables used as indecies
  long double p[10];                    //array used to store the placement
  long double res=0.0;                  //variable to store the results
  long double plval=1.0;                //variable to store placement value
  tab t[100];                           //tab type structure variable
  cout<<"Enter The Maximum Number of Elements:\t";cin>>n;            //Reading the data in the tabular format
  cout<<"Enter The Equilizing Base :\t";cin>>eb;
  for(i=0;i<n;i++){
    cout<<"Enter Number                    : "<<i+1<<"  ";cin>>t[i].nums;
    cout<<"Enter Theriotical Base for number : "<<i+1<<"  ";cin>>t[i].tb;
    cout<<"Enter Working Base for number    : "<<i+1<<"  ";cin>>t[i].wb;
    t[i].de=t[i].nums-t[i].wb;
    t[i].mf=(double)eb/t[i].wb;
    t[i].mno=t[i].mf*t[i].nums;
    t[i].mde=t[i].mf*t[i].de;
  }  clrscr();
    cout<<endl;                         //display the data in the tabular format
    for(i=0;i<79;i++) cout<<"-";
    cout<<endl;
   int row=(wherex()+2);
     gotoxy(1,row);   cout<<"Numbers";
     gotoxy(12,row);      cout<<"T.Base";
     gotoxy(24,row);   cout<<"W.Base";
     gotoxy(36,row);      cout<<"D/E";
     gotoxy(48,row);   cout<<"M.Factor";
     gotoxy(60,row);   cout<<"M.No";
     gotoxy(72,row);   cout<<"M.D/E";
    cout<<endl;
    for(i=0;i<79;i++) cout<<"-";
    cout<<endl;
    row+=2;
    for(i=0;i<n;i++,row++){
     gotoxy(1,row);   cout<<t[i].nums;
     gotoxy(12,row);      cout<<t[i].tb;
     gotoxy(24,row);   cout<<t[i].wb;
     gotoxy(36,row);   cout<<t[i].de;
     gotoxy(48,row);   cout<<t[i].mf;
     gotoxy(60,row);   cout<<t[i].mno;
     gotoxy(72,row);   cout<<t[i].mde;
     cout<<endl;
}
```

```
 for(i=0;i<79;i++) cout<<"-";
 {                                          //calculate the result of first part
 for(i=0;i<n;i++)
  pival*=t[i].de;
 p[0]=pival;
 cout<<endl<<"Part 1 : "<<p[0];
 }
 cout<<endl;
 long double den=1.0,num=1.0;               //calculate the results of all the middle parts
 {int i=0;
  for(int q=0;q<n-2;q++)
  {
    num*=t[q].wb;
    den=1;
    for(int j=q+1;j<n;j++)
         den*=t[j].mf;
    double sum;
    {
         { i++;
           sum=0;
         for(int c=1;c<pow(2,n)-1;c++){
           int tp=c,cnt=0;
          double prd=1;
           for(int c1=0;c1<n;c1++,tp>>=1)
             if(tp&1==1) cnt++;
             else prd*=t[c1].mde;
           if (cnt==i)
             sum+=prd;
         }
         pival=(num/den)*sum;
         p[i]=pival;
         cout<<endl<<"Part "<<i+1<<" : "<<p[i]<<"\n";
    }
   }
   }
 }
 num=1;                                     //calculate the result of last part;
 long double sum=0;
 { for(int i=0;i<n-1;i++)
     num*=t[i].wb;
   den=t[n-1].mf;
   for(i=0;i<n;i++)
    if(eb==t[i].wb) sum+=t[i].nums;
    else sum+=t[i].mde;
  pival=(num/den)*sum;
  p[n-1]=pival;
   cout<<endl<<"Part "<<n<<" : "<<p[n-1];
 }
 res=0.0;
 cout<<endl;
 cout<<endl;
 for(i=0;i<n;i++)
   res+=p[i];
 cout<<"\nThe Final Result :";
 printf("\n%.0f",(float)res);
 getch();
 return;
}
```

OUTPUT:

```
     Multiplication using general Formula
Enter The Maximum Number of Elements:  3
Enter The Equilizing Base :    200
Enter Number                          : 1  41
Enter Theriotical Base for number : 1  10
Enter Working Base for number       : 1  40
Enter Number                          : 2  48
Enter Theriotical Base for number : 2  10
Enter Working Base for number       : 2  50
Enter Number                          : 3  199
Enter Theriotical Base for number : 3  100
Enter Working Base for number       : 3  200

--------------------------------------------------------------------------------
Numbers    T.Base     W.Base     D/E        M.Factor     M.No       M.D/E
--------------------------------------------------------------------------------
41         10         40         1          5            205        5
48         10         50         -2         4            192        -8
199        100        200        -1         1            199        -1
--------------------------------------------------------------------------------
Part 1 : 2

Part 2 : -370

Part 3 : 392000


The Final Result :
391632
```

XIII. ALPHA METHOD

```
#include <iostream.h>
#include <conio.h>
#include <stdio.h>
#include <math.h>
#include <process.h>
typedef struct tab{                          //structure tab containing the tabular form of the data folding the following fields
  long nums;
  long de;
  long wb;
  double mf;
  double mno;
  double mde;
}tab;
void main(){
  clrscr();
  cout<<"OUTPUT:\n\tMultiplication using Alpha\n";
  int n,i,j,eb;                              //variables used as indecies
  long double p[10];                         //array used to store the placement
  long double res=0.0;                       //variable to store the results
  long double plval=1.0;                     //variable to store placement value
  tab t[100];                                //tab type structure variable
  cout<<"Enter The Maximum Number of Elements:\t";cin>>n;          //Reading the data in the tabular format
  cout<<"Enter The Equilizing Base :\t";cin>>eb;
  for(i=0;i<n;i++){
    cout<<"Enter Number                  : "<<i+1<<"  ";cin>>t[i].nums;
    cout<<"Enter Working Base for number    : "<<i+1<<"  ";cin>>t[i].wb;
    t[i].de=t[i].nums-t[i].wb;
    t[i].mf=(double)eb/t[i].wb;
    t[i].mno=t[i].mf*t[i].nums;
    t[i].mde=t[i].mf*t[i].de;
  }
   clrscr();
   cout<<endl;
   int row=1;
   for(i=0;i<79;i++) cout<<"-";
   cout<<endl;
     gotoxy(1,row);   cout<<"Numbers";                  //display the data in the tabular format
     gotoxy(12,row);   cout<<"W.Base";
     gotoxy(24,row);   cout<<"D/E";
     gotoxy(36,row);   cout<<"M.Factor";
     gotoxy(48,row);   cout<<"M.No";
     gotoxy(60,row);   cout<<"M.D/E";
   cout<<endl;
   for(i=0;i<79;i++) cout<<"-";
   cout<<endl;
   row+=2;
   for(i=0;i<n;i++,row++){
     gotoxy(1,row);   cout<<t[i].nums;
     gotoxy(12,row);       cout<<t[i].wb;
     gotoxy(24,row);   cout<<t[i].de;
     gotoxy(36,row);   cout<<t[i].mf;
     gotoxy(48,row);   cout<<t[i].mno;
     gotoxy(60,row);   cout<<t[i].mde;
     cout<<endl;
}
```

```
  for(i=0;i<79;i++) cout<<"-";
  {                                       //calculate the result of first part
  for(i=0;i<n;i++)
   pival*=t[i].de;
  p[0]=pival;
  }
  cout<<endl;
 double alpha=1.0;                        //calculate the results of all the middle parts
  int flag=0;
 for(i=0,alpha=1.0;i<n;i++){
   if(flag==0)
     if (eb==t[i].wb){
           flag=1;
           continue;
     }
   alpha*=t[i].wb;
 }
 double fr,sum;
 {
  for(i=1;i<n-1;i++){
    fr=1.0;
   for(int mn=n-(i+1);mn>0;mn--) fr*=eb;
    sum=0;
   for(int c=1;c<pow(2,n)-1;c++){
          int tp=c,cnt=0;
          double prd=1;
          for(int c1=0;c1<n;c1++,tp>>=1)
            if(tp&1==1) cnt++;
            else prd*=t[c1].mde;
          if (cnt==i)
            sum+=prd;
    }
   pival=(sum*alpha)/fr;
    p[i]=pival;
  }
 }
 {                                        //calculate the result of last part;
 pival=fr=1;
 int tmp;
 pival=t[0].mno;
 for(i=1;i<n;i++)
   pival+=t[i].mde;
 pival=pival*alpha;
 p[n-1]=pival;
 }
 res=0.0;
 cout<<endl;
 for(i=0;i<n;i++)
 printf("%.0f  ",(float)p[i]);
 cout<<endl;                              //cout<<p[i]<<"  ";
 for(i=0;i<n;i++)
   res+=p[i];

 cout<<"Aplha : "<<alpha;
 cout<<"\nThe Final Result :";
 printf("\n%.0f",(float)res);
 getch();
 return;
}
```

```
OUTPUT:
      Multiplication using Alpha
Enter The Maximum Number of Elements:  6
Enter The Equilizing Base :    4000
Enter Number                        1  110
Enter Working Base for number       1  100
Enter Number                        2  204
Enter Working Base for number       2  200
Enter Number                        3  38
Enter Working Base for number       3  40
Enter Number                        4  46
Enter Working Base for number       4  50
Enter Number                        5  2005
Enter Working Base for number       5  2000
Enter Number                        6  3999
Enter Working Base for number       6  4000

Numbers    W.Base     D/E        M.Factor    M.No       M.D/E
---------------------------------------------------------------
110        100        10         40          4400       400
204        200        4          20          4080       80
38         40         -2         100         3800       -200
46         50         -4         80          3680       -320
2005       2000       5          2           4010       10
3999       4000       -1         1           3999       -1
---------------------------------------------------------------

-1600 5716000 2720320000 63490000000 -3079400000000 317520000000000
Aplha : 8e+10
The Final Result :
314506816034400
```

XIV. NUMBER IN TERMS OF BASE & D/EX — SERIES MULTIPLICATION

```
#include <iostream.h>
#include <stdio.h>
#include <conio.h>
#include <string.h>
#include <math.h>
class calc{                              //Class calc to include the method as single object
  long double a[50][50];                 //a two dim array to store first part of the values
  long double b[50][50];                 //a two dim array to store second part of the values
  long double bi[50];                    //arrays to store the temporary results
public:
  int tns,tds;                           //variables to store no. of values
  int tp;                                //variables to store no. of +ve values and -ve values
  calc(){                                //constructor to assign 0 to all variables and arrays
    tns=tds=0;
    int i,j;
    for(i=0;i<50;i++)
    { bi[i]=0;
      for(j=0;j<50;j++)
           b[i][j]=0;
    }
  }
  void readnums();                       //function to read the numbers
  void mult();                           //function for performing multiplication
  void disp();                           //function to display final results
};
void calc::readnums(){                   //function to read the numbers
  char *str;
  char ch;
  cout<<endl<<"OUTPUT:\nType \"[0 0 y] to stop\" \n";
  do{
    cout<<"Enter any Number as [Base D/E]  ";
    cin>>a[tns][1];
    cin>>a[tns][0];
    cin>>ch;
   if (ch=='y') break;
    tns++;
  }while(ch=='n');
  return;
}
void calc::mult(){                       //function to multiple n numbers of two digits
  long double n,i,k,l;
  int j;
  long double r;
  n=pow(2,tns);
  for(i=n-1;i>0;i--){
    r=1;l=0;j=i;
    for(k=tns-1;k>=0;k--){
      if(j&1)
           r*=a[k][0];
      else{
           r*=a[k][1];
           l++;
      }
      j>>=1;
```

```
      }
       b[l][bi[l]++]=r;
    }
    for(r=1,l=0,i=0;i<tns;i++){
      r*=a[i][1];
      l++;
    }
     b[l][bi[l]++]=r;
    tp=l;
  }
  void calc::disp(){                                    //function to display the results
    clrscr();
    {long double resll=0.0;
     cout<<endl;
     for(int i=0;i<=tp;i++)
     {
         cout<<"Part "<<i<<" --> ";
         resll=0.0;
         for(int j=0;j<bi[i];j++)
        {     cout<<b[i][j]<<"   ";
              resll+=b[i][j];
        }
        cout<<" ----->   "<<resll;
        cout<<endl;
   }}
   getch();
   long double  res=0.0;
   cout<<"The Product Is    :\t";
   for(int i=0;i<=tp;i++)
   {
       for(int j=0;j<bi[i];j++){
              //cout<<b[i][j];
             //printf("%15.0f + ",/*cout<<*/(float)b[i][j]);/*<<" + ";*/
             res+=b[i][j];
       }
   }
   printf("\b\b\b = ");
   //cout<<endl;
  printf("\n%20.0f",(float)res);
   cout<<endl;
}
void main(){
  clrscr();
  calc obj;
 obj.readnums();
  obj.mult();
  obj.disp();
  getch();
  return;
}
```

OUTPUT

```
OUTPUT:
Type "[0 0 y] to stop"
Enter any Number as [Base D/E]  40 -2 n
Enter any Number as [Base D/E]  50 -4 n
Enter any Number as [Base D/E]  100 10 n
Enter any Number as [Base D/E]  200 4 n
Enter any Number as [Base D/E]  400 -1 n
Enter any Number as [Base D/E]  2000 5 n
Enter any Number as [Base D/E]  4000 -1 n
Enter any Number as [Base D/E]  0 0 y

Part 0 --> 1600  ----->  1600

Part 1 --> -6400000 640000 -640000 80000 16000 -20000 -32000  ----->  -6
356000

Part 2 --> -2.56e+09 2.56e+09 -2.56e+08 -3.2e+08 3.2e+07 -3.2e+07 -6.4e+07
 6400000 -6400000 800000 8e+07 -8000000 8000000 -1000000 -200000 1.28e+
08 -12800000 12800000 -1600000 -320000 400000  ----->  -4.3392e+08

Part 3 --> 1.024e+12 -1.28e+11 1.28e+11 -1.28e+10 -2.56e+10 2.56e+10 -2.56
e+09 -3.2e+09 3.2e+08 -3.2e+08 3.2e+10 -3.2e+10 3.2e+09 4e+09 -4e+08 4e
+08 8e+08 -8e+07 8e+07 -10000000 5.12e+10 -5.12e+10 5.12e+09 6.4e+09 -6
.4e+08 6.4e+08 1.28e+09 -1.28e+08 1.28e+08 -16000000 -1.6e+09 1.6e+08 -1
.6e+08 2e+07 4000000  ----->  1.024638e+12

Part 4 --> 5.12e+13 1.024e+13 -1.28e+12 1.28e+12 -1.28e+11 -1.28e+13 1.6e+
12 -1.6e+12 1.6e+11 3.2e+11 -3.2e+11 3.2e+10 4e+10 -4e+09 4e+09 -2.048e
+13 2.56e+12 -2.56e+12 2.56e+11 5.12e+11 -5.12e+11 5.12e+10 6.4e+10 -6.4
e+09 6.4e+09 -6.4e+11 6.4e+11 -6.4e+10 -8e+10 8e+09 -8e+09 -1.6e+10 1.6
e+09 -1.6e+09 2e+08  ----->  2.84754e+13

Part 5 --> 5.12e+14 -6.4e+14 -1.28e+14 1.6e+13 -1.6e+13 1.6e+12 -1.024e+15
 -2.048e+14 2.56e+13 -2.56e+13 2.56e+12 2.56e+14 -3.2e+13 3.2e+13 -3.2e+
12 -6.4e+12 6.4e+12 -6.4e+11 -8e+11 8e+10 -8e+10  ----->  -1.22928e+15

Part 6 --> -6.4e+15 -1.024e+16 1.28e+16 2.56e+15 -3.2e+14 3.2e+14 -3.2e+13
 ----->  -1.312e+15

Part 7 --> 1.28e+17  ----->  1.28e+17
The Product Is  :  =
 125488219597725600
```

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